synergistic use of tide gauges, satellite altimetry and GPS data for sea level studies

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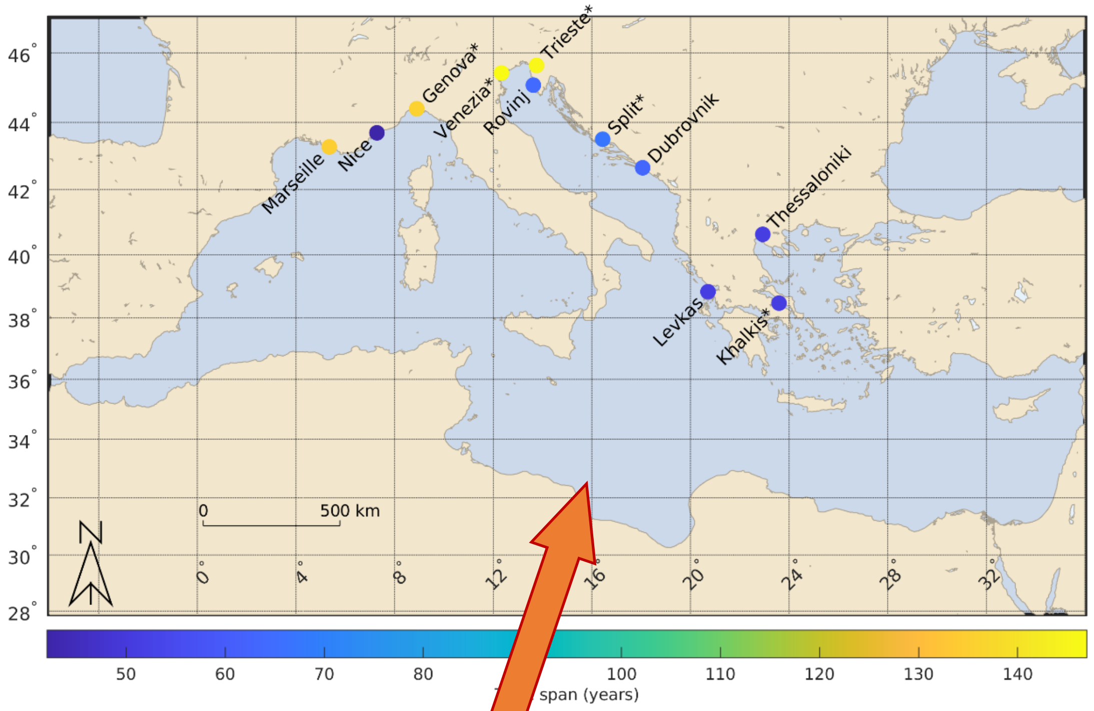
OUTLINE:

- objectives
- background
- data sources
- methods
- results
- conclusions



Objectives:

1. estimation of the vertical land motion (VLM) at eleven tide gauges (TGs)
2. the VLM estimates have to be derived optimally
3. demonstrate the SYNERGY of altimetry, TG observations and Global Positioning System (GPS) data

Where?
Mediterranean Sea: one of the coastal zone most populated and exposed to sea level rise and storm surge related risks.

background

[Cazenave et al. (1999)] observed that the rate of absolute (geocentric) vertical land movement u_i at a tide gauge i, is given by $u_i = g_i - S_i^{Alt}$ where g_i is the absolute (geocentric) sea level change rate measured by satellite altimetry, and S_i^{Alt} is the sea level change rate measured by TG, relative to the ground.

[Kuo et al. (2004)] extended the approach to a network of tide gauges (TG) in Fennoscandia, grouping the N equations $u_i = g_i - S_i^{Alt}$ in a linear system $A \cdot X = Y$. Moreover, they constrained the system with a set of equations enforcing relative vertical land movement rate differences between pairs of tide gauges: $ru_{ij} = u_i - u_j = g_i - S_i^{Alt} - (g_j - S_j^{Alt})$. The system is said a linear inverse problem with constraints (LIPWC) and gives accurate VLM estimates **if each pair of TGs in the constraints observes the same absolute sea level rate: $g_i = g_j$ (g-g condition / g-g limitation).**

Improvements to the Kuo method were made by [Wöppelmann and Marcos (2012)] and [De Biasio et al. (2020)] . The latter overcame the g-g limitation, permitting its application to **a generic network of TGs, regardless the absolute sea level rates** experienced by the TGs.

In this work, we investigate on the use of **Global Positioning System (GPS)** observations in direct **synergy** with the traditional tools, **tide gauges and satellite altimetry** for VLM and sea level studies. This approach is **not subject to the g-g limitation.**

GEOPHYSICAL RESEARCH LETTERS, VOL. 26, NO. 14, PAGES 2077-2080, JULY 15, 1999

Sea level changes from Topex-Poseidon altimetry and tide gauges, and vertical crustal motions from DORIS

A. Cazenave[1], K. Dominh[1], F. Ponchaut[1], L. Soudarin[2], J.F. Cretaux[1] and C. Le Provost[1]

GEOPHYSICAL RESEARCH LETTERS, VOL. 31, L01608, doi:10.1029/2003GL019106, 2004

Vertical crustal motion determined by satellite altimetry and tide gauge data in Fennoscandia

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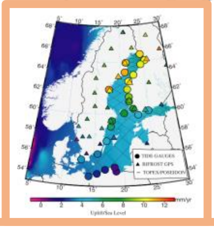

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 117, C01007, doi:10.1029/2011JC007469, 2012

Coastal sea level rise in southern Europe and the nonclimate contribution of vertical land motion

G. Wöppelmann[1] and M. Marcos[2]

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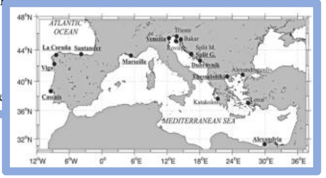
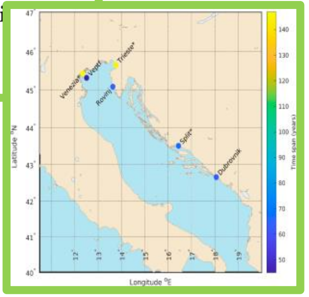


Journal of Marine Science and Engineering — J. Mar. Sci. Eng. 2020, 8, 949; doi:10.3390/jmse8110949

Article

Revisiting Vertical Land Motion and Sea Level Trends in the Northeastern Adriatic Sea Using Satellite Altimetry and Tide Gauge Data

Francesco De Biasio [1,*], Giorgio Baldin [2] and Stefano Vignudelli [3]



data sources

Satellite altimetry gridded (0.25°) daily means of Sea Level Anomaly (1993-2020)

from Copernicus Climate Change Service (C3S)

- state-of-the-art, climate-oriented dataset of SLA
- processing: editing, cross-calibration, homogeneous corrections, removal of global and regional biases, homogenization of long-spatial-scale errors, optimal interpolation gridding
- Dynamic Atmospheric Correction (DAC) from CNES AVISO+ re-added to SLA to obtain a sea level comparable to TG monthly means

Tide Gauge hourly and monthly means sea level time series (1978-2020)

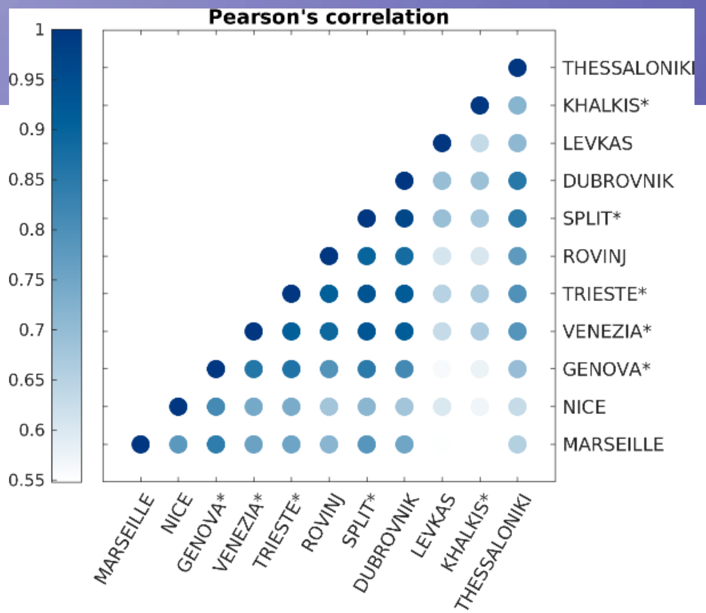
from Permanent Service for Mean Sea Level (PSMSL), Tide Forecast and Early Warning Center of the Venice Municipality, Trieste section of the CNR-ISMAR

- the sea level observed by tide gauges is measured with respect to a benchmark on the ground
- hourly data processed with X0-filtering
- trends errors calculated with 95% confidence interval. Serial correlation is accounted for.
- the mutual Person's linear correlation coefficient is always > 0.55

Vertical land motion rates and errors derived from GPS (mean ~2004-2021)

from EUREF (NL), NGL Nevada Geodetic Lab. (USA), SONEL-Univ. La Rochelle (FR)

- Only two GPS are less than 10 m from TG; three in range 10 – 4000 m; six over 4000 m



TG name	Lat	Lon	Gap (%)	Span	GPS (mm yr^{-1})	Span / TG distance (m)
MARSEILLE	43.279	5.354	3.47	1885-2019	-0.93±0.10	1998-2022 / 5
NICE	43.696	7.285	12.22	1978-2019	-0.20±0.45	2003-2022 / 4764
GENOVA*	44.4	8.9	18.38	1884-2019	-0.51±0.10	1998-2022 / 1000
VENEZIA*	45.431	12.336	3.46	1872-2018	-1.58±0.14	2009-2022 / 1384
TRIESTE*	45.647	13.76	10.52	1875-2020	-0.69±0.45	2003-2022 / 6707
ROVINJ	45.083	13.628	1.18	1955-2018	-1.11±0.19	2011-2022 / 16069
SPLIT*	43.507	16.442	0.00	1952-2019	-0.25±0.34	2004-2012 / 1
DUBROVNIK	42.658	18.063	0.93	1956-2018	-1.42±0.33	2000-2022 / 4168
LEVKAS	38.835	20.712	13.38	1969-2019	-0.54±0.73	2007-2022 / 6807
KHALKIS*	38.472	23.593	11.17	1969-2019	1.23±1.09	2010-2022 / 1151
THESSALONIKI	40.633	22.935	10.33	1969-2019	-3.15±0.23	2005-2022 / 9326

methods

Satellite altimetry measures the rate of absolute sea level change g w.r.t. earth center of mass.

Tide gauges measure the rate of relative sea level change S w.r.t. ground.

Subtracting the second from the first gives the absolute vertical motion of the ground u:

$$u = g - S$$

In the Kuo's approach for a network of TGs, the LIPWC solution is enforced to reproduce the rates of relative VLM between pairs of TGs.

$$ru_{ij} = u_i - u_j = g_i - S_i^{Alt} - (g_j - S_j^{Alt})$$

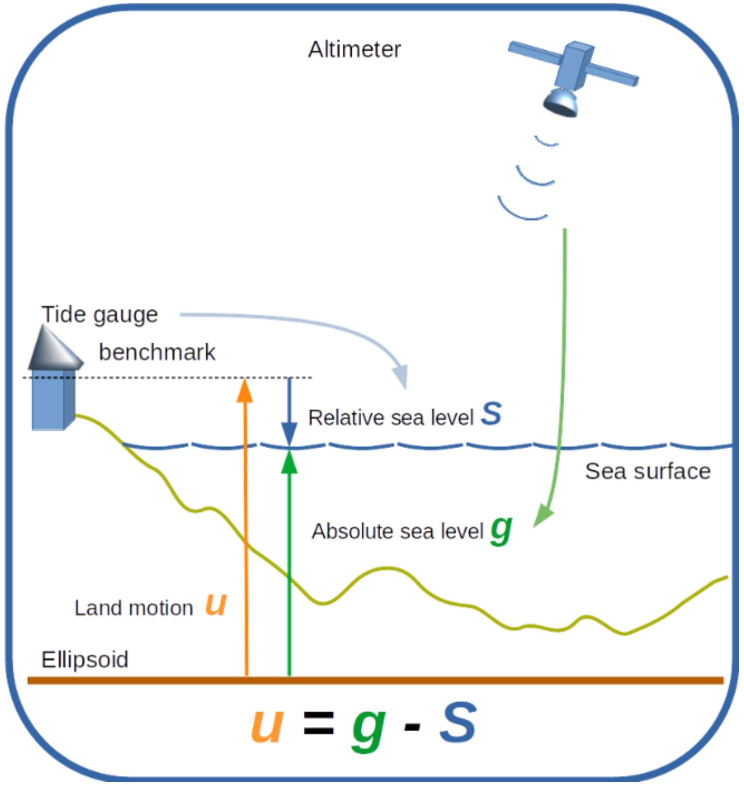
The constraints are consistently enforced **if each pair of TGs observes the same absolute sea level change rate** (g-g condition):

$$u_i - u_j = S_j^{Alt} - S_i^{Alt} \cong S_j^{tg} - S_i^{tg}$$

As several TGs have very long time series, the error on such quantities is usually very low, determining low errors in the system solution too.

Often the g-g condition is not satisfied. In such cases, we propose to use **independent, direct VLM determinations obtained from GPS data**:

$$ru_{ij} = u_i - u_j = VLM_i^{gps} - VLM_j^{gps}$$



results

We solved the LIPWC for a network of eleven TGs in the Mediterranean Sea in two experimental set-up:

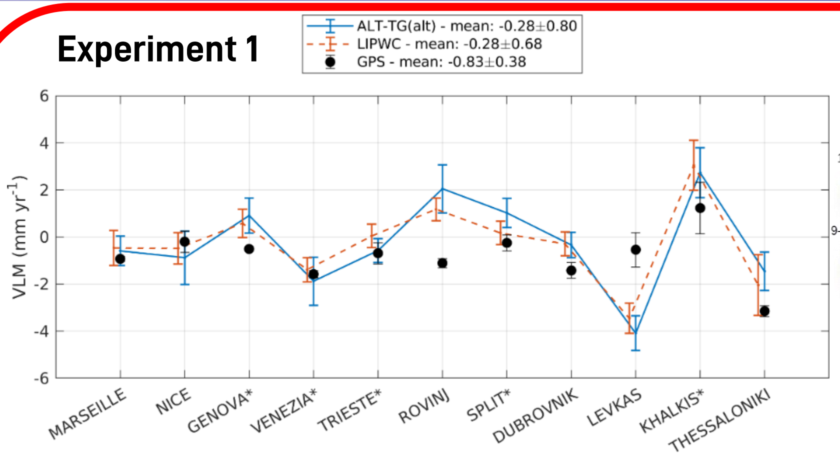
- **Experiment 1** (upper plot): the system is constrained using **only TG-TG differences**: $u_i - u_j \cong S_j^{tg} - S_i^{tg}$
- **Experiment 2** (lower plot): as Exp. 1, but two constraints are realized with **GPS derived VLM rates**: $u_i - u_j = VLM_i^{gps} - VLM_j^{gps}$

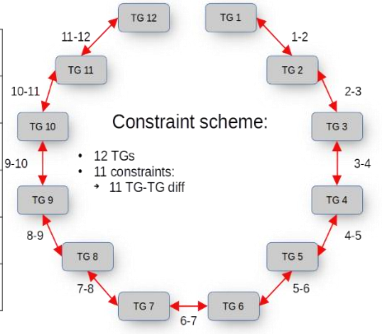
In both plots the VLM estimates are marked by:

- **Black dots for GPS**
- **Blue lines for ALT-TG** classical estimates
- **Orange lines for the LIPWC solutions**

Enforcing GPS-GPS constraints implies that the relative VLM differences in the LIPWC solution equals those measured by GPS

Experiment 1

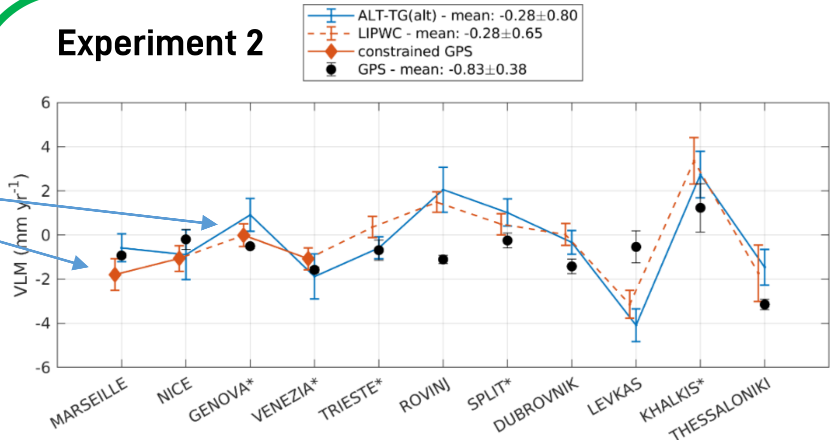


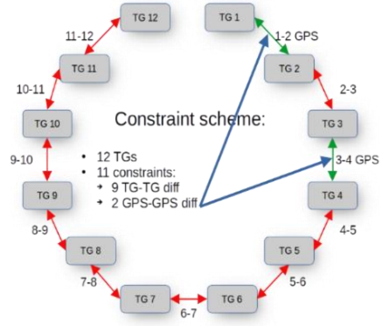


Scheme of constraints

↑ Experiment 1: all the constraints are enforced by $S_j^{tg} - S_i^{tg}$ differences

Experiment 2





Scheme of constraints

↑ Experiment 2: as above, but two constraints are enforced by $VLM_i^{gps} - VLM_j^{gps}$ differences

example: results for VENICE

GPS Individual estimate	ALT-TG Individual estimate	LIPWC Experiment 1 (Kuo)	LIPWC Experiment 2 (study)
-1.58±0.14	-1.88±1.02	**-1.39±0.51**	**-1.08±0.50**

conclusions

We proposed a methodology for using **Global Positioning System (GPS)** observations in **direct synergy** with the traditional tools, **tide gauge and satellite altimetry observations**, for vertical land motion and sea level studies

We estimated vertical land motion trends at eleven locations in the Mediterranean Sea using **observations from three independent instruments,** namely: **satellite altimetry**, **tide gauges** and **global positioning system** devices

Pros

- Our approach is **not subject to the g-g limitation**, as no assumptions are made about the ASL change rates
- VLM trends and errors are obtained simultaneously by least square minimization, and reproduce observed relationships between TG RSLR and/or GPS VLM trends
- The VLM solution has less variability than the direct method, and a more realistic distribution for the presence of the constraints
- The errors on the VLM rates are of the order of 0.6 mm yr^{-1}, lower than the direct TG-ALT method

Cons

- Reliability of GPS rates (Centers give different numbers for the same position time series)
- Representativeness of TG vertical position from uncollocated GPS observations

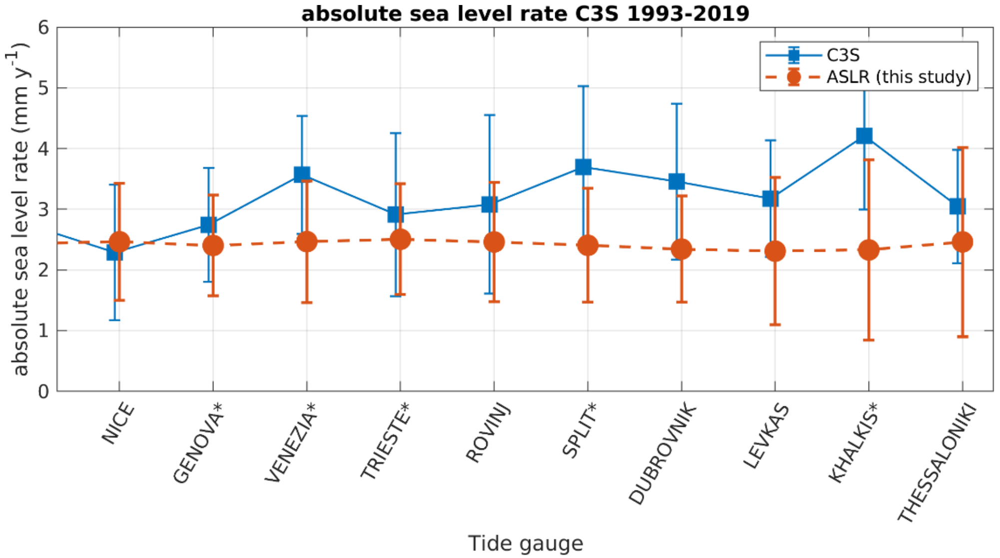
End

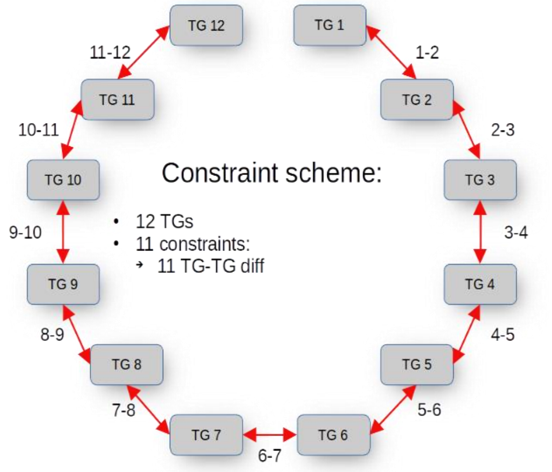
EMPTY SLIDE

Supplementary Material 1.
Absolute Sea Level Change rates from LIPWC

Upper plot: solution of the LIPWC system constrained by using the *Kuo et al., Wöppelmann and Marcos* approach, using TG-TG differences:

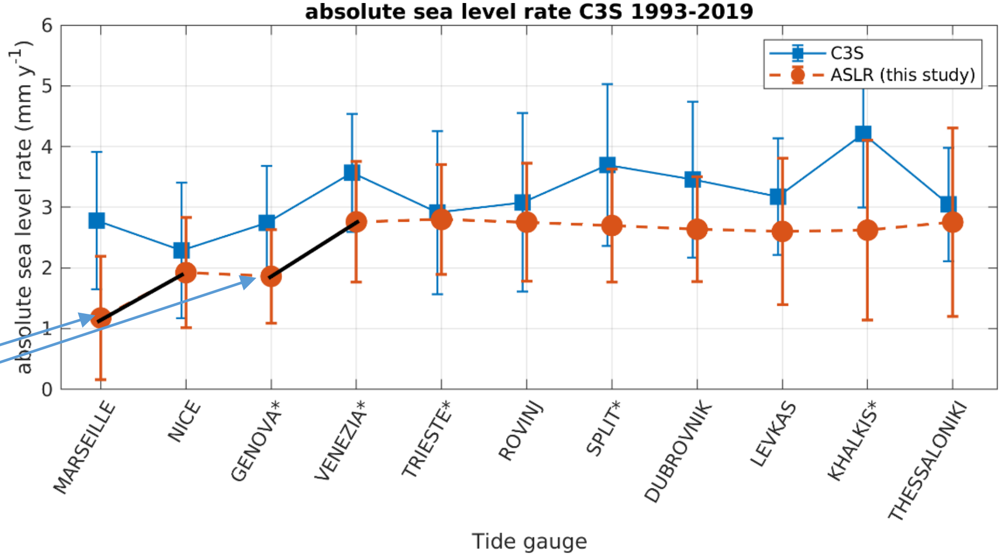
$$u_i - u_j \cong S_j^{tg} - S_i^{tg}$$

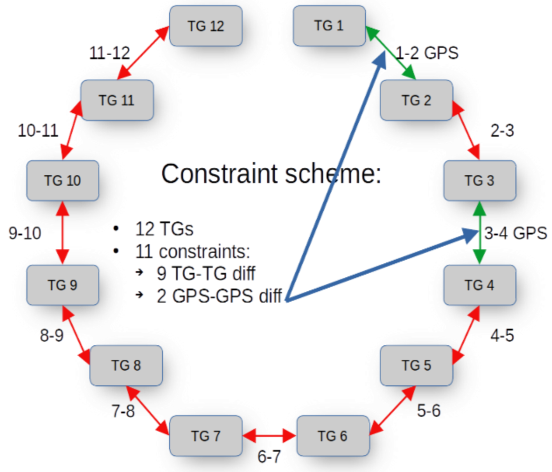




Lower plot: solution of the LIPWC system constrained by using GPS derived VLM rates for two pairs of TGs:

$$u_i - u_j = VLM_i^{gps} - VLM_j^{gps}$$





In both plots:

- **GPS** VLMs (black dots)
- **ALT-TG** VLMs (blue lines)
- **LIPWC solution** VLMs (orange lines)

Enforcing GPS-GPS constraints implies that relative VLM differences in the LIPWC solution equals those measured by GPS

Supplementary Material 2.
Linear inverse problem with constraints (LIPWC)

The rate of absolute vertical land movement at tide gauge *i* is given by the difference between the absolute sea level change rate and the relative sea level change rate at the same place and time:

a) $\dot{\boldsymbol{u}}_i = \dot{\boldsymbol{g}}_i - \dot{\boldsymbol{S}}_i^{Alt}$

$\dot{g}_i, \dot{S}_i^{Alt}$ = **absolute** and **relative** sea level change **rates** at the tide gauge / **in the same period**; dot means time differentiation. In practice, $\dot{g}_i$ is measured by the altimeter, and $\dot{S}_i^{Alt}$ by the tide gauge.

This equation is sufficient to obtain an estimate of the VLM rates at each tide gauge[1], but often with strong uncertainties. Solution: introduce the rate of relative vertical motion between two nearby tide gauges: $r\dot{u}_{ij} = (\dot{g}_i - \dot{S}_i^{Alt})$ - $(\dot{g}_j - \dot{S}_j^{Alt})$ which reduces to:

b) $\boldsymbol{r\dot{u}_{ij}} = \dot{\boldsymbol{S}}_j^{Alt}$- $\dot{\boldsymbol{S}}_i^{Alt}$ if $\dot{g}_i = \dot{g}_j$, or calculated as: $\boldsymbol{r\dot{u}_{ij}} = \boldsymbol{vl\dot{m}_i^{gps}}$- $\boldsymbol{vl\dot{m}_j^{gps}}$(regardless of $\dot{g}$)

Since tide gauge time series often go back in time before the altimeter era begins, we can substitute to $\boldsymbol{r\dot{u}_{ij}}$ the term $\dot{\boldsymbol{S}}_j^{TG}$- $\dot{\boldsymbol{S}}_i^{TG}$, where the differenced rates are those of the complete lifespan of the coupled TGs. As in general the rates $\boldsymbol{r\dot{u}_{ij}}$ have much smaller errors, they reduce the overall error in the unknown $\dot{\boldsymbol{u}}_i$. This is done by putting the N equations (**a**) in matrix form:

c) $G \cdot \dot{\boldsymbol{u}} = \boldsymbol{d}; \quad \dot{\boldsymbol{u}} = \begin{pmatrix} \dot{u}_1 \\ \vdots \\ \dot{u}_N \end{pmatrix}; \qquad \boldsymbol{d} = \begin{pmatrix} \dot{g}_1 - \dot{S}_1^{Alt} \\ \vdots \\ \dot{g}_N - \dot{S}_N^{Alt} \end{pmatrix}; \qquad G = Identity$

(1) Cazenave, A., K. Dominh, F. Ponchaut, L. Soudarin, J. F. Crétaux, and C. Le Provost (1999), Geophys. Res. Lett., 26, 2077–2080, doi:10.1029/1999GL900472

and adding the M<N equations (**b**) as Lagrange constraints to the linear system:

The constraints can be chosen arbitrarily, but they have to be linearly independent so that the rank of the matrix F is <=N-1, and that the condition expressed in b) is true ($\dot{g}_i - \dot{g}_j = 0$).

The linear system **c) + d)** is simoultaneously solved with the use of Lagrange multipliers[1], $\begin{bmatrix} G^T \cdot G & F^T \\ F & 0 \end{bmatrix}\begin{pmatrix} \dot{\boldsymbol{u}} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \boldsymbol{d} \\ \boldsymbol{h} \end{pmatrix}$ by direct inversion.

Note that the system is of the type $AX = Y$ and has solution $X = A^{-1}Y$, with A = $\begin{bmatrix} G^T \cdot G & F^T \\ F & 0 \end{bmatrix}$. Errors are calculated via the associated covariance matrix.

Where reliable GPS are colocated with TGs, the constraint $h_{ij} = \dot{S}_j^{TG} - \dot{S}_i^{TG}$ can be substituted by

$h_{ij} = v\dot{l}m_i^{gps}$- $v\dot{l}m_j^{gps}$.

$$d)\quad F \cdot \dot{\boldsymbol{u}} = \boldsymbol{h}; \quad \boldsymbol{h} = -F \cdot \begin{pmatrix} \dot{S}_1^{TG} \\ \vdots \\ \dot{S}_N^{TG} \end{pmatrix}; \qquad F = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 \\ & \cdots & & & \cdots & \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}$$

(1) *Menke, W. (1989), Geophysical Data Analysis: Discrete Inverse Theory,289 pp., Academic, San Diego, Calif.*