Long-term development of triadic resonance instability in a finite-width internal wave beam

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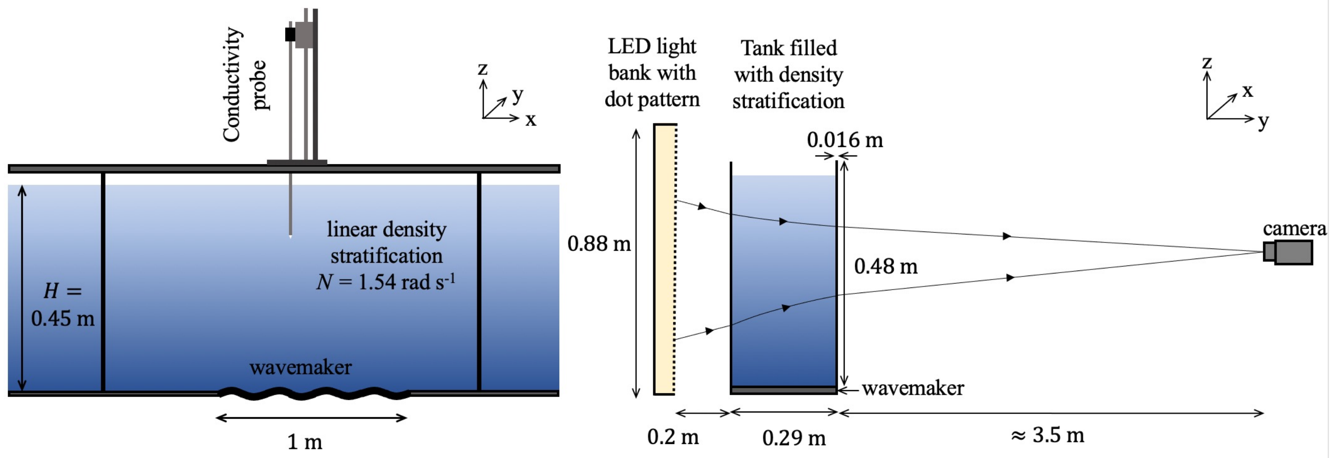
Experimental Set-up

- 11 m long tank
- A flexible bottom 1 m boundary called the Arbitrary Spectrum Wave Maker (ASWaM) or **magic carpet**



Front view of tank — Side view of tank

- Generate a primary wave from the magic carpet moving with a group velocity up and to the left

Spatial variability of secondary wave beams

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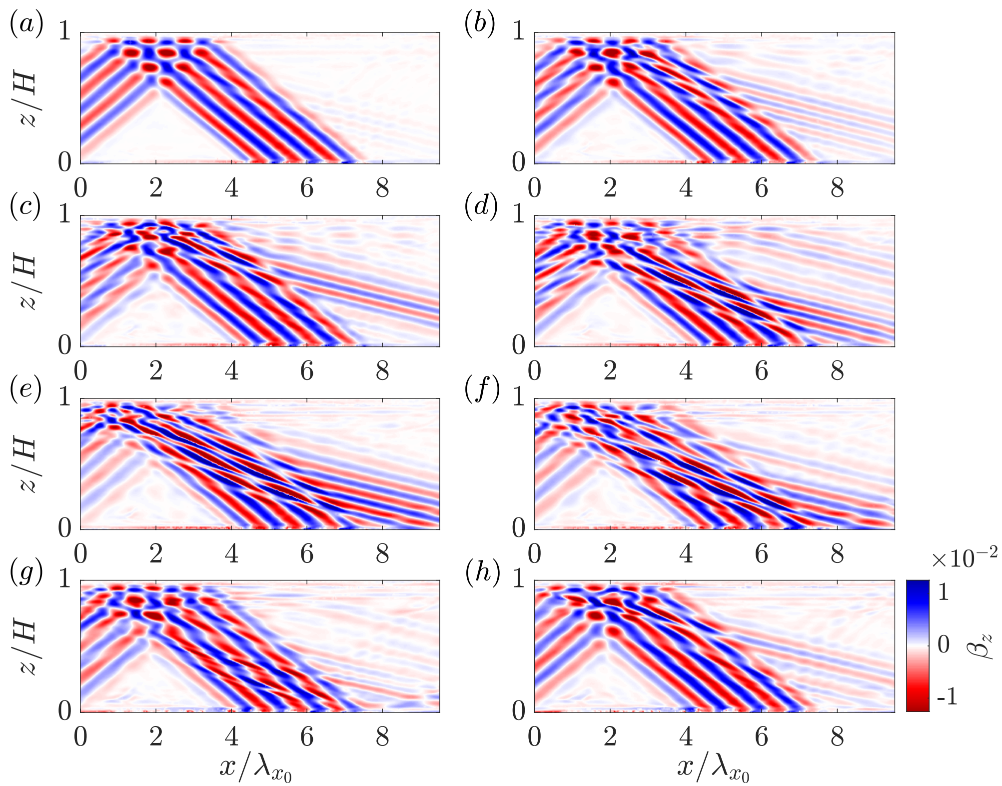
Triadic resonance instability (TRI): weakly non-linear instability where a primary wave (0) becomes unstable and generates two secondary waves (1,2) that resonant as a triad and satisfy:

$$\omega_0 = \omega_1 + \omega_2$$
$$\boldsymbol{k}_0 = \boldsymbol{k}_1 + \boldsymbol{k}_2$$

Experimentally observed modulations during TRI to the:

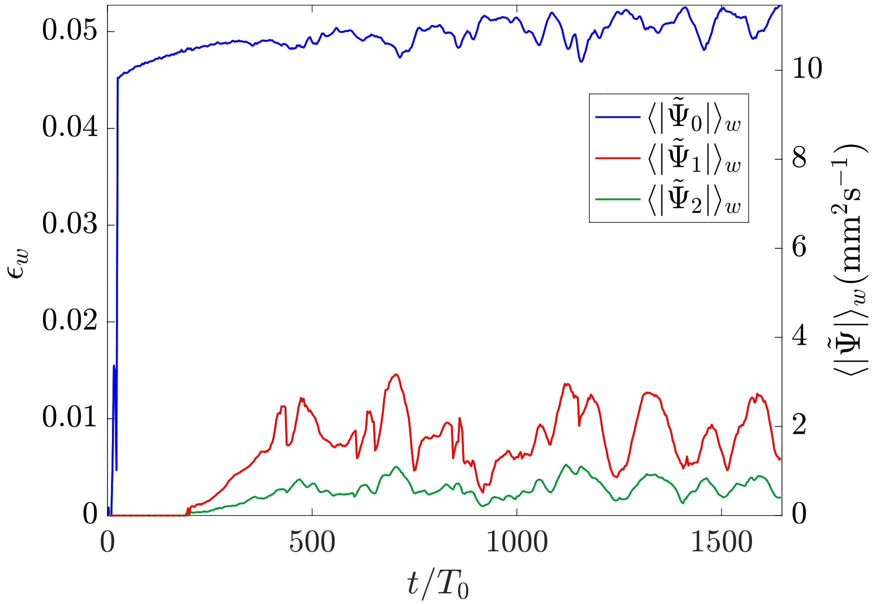
- Spatial location of the secondary beams
- Amplitude of all three beams
- Frequency and wavenumber of the two secondary beams



Experimental snapshots, approximately 8 minutes apart, showing the primary wave beam becoming unstable to TRI.

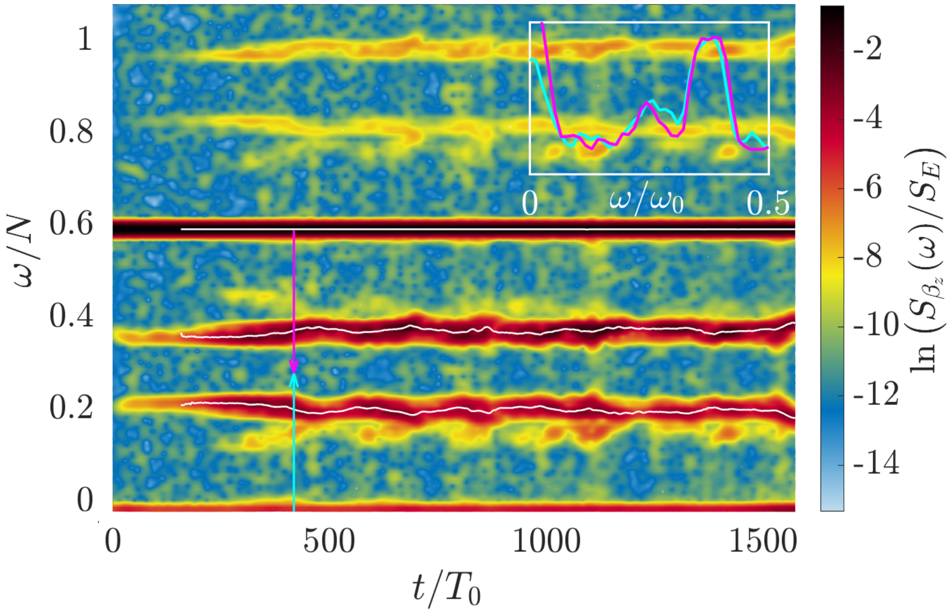
Amplitude and frequency modulations of secondary beams

- Coupled amplitude modulations of all three triadic beams beams



Non-dimensional amplitude of the primary beam (blue) and secondary beams (green and red) over three hour experiment.

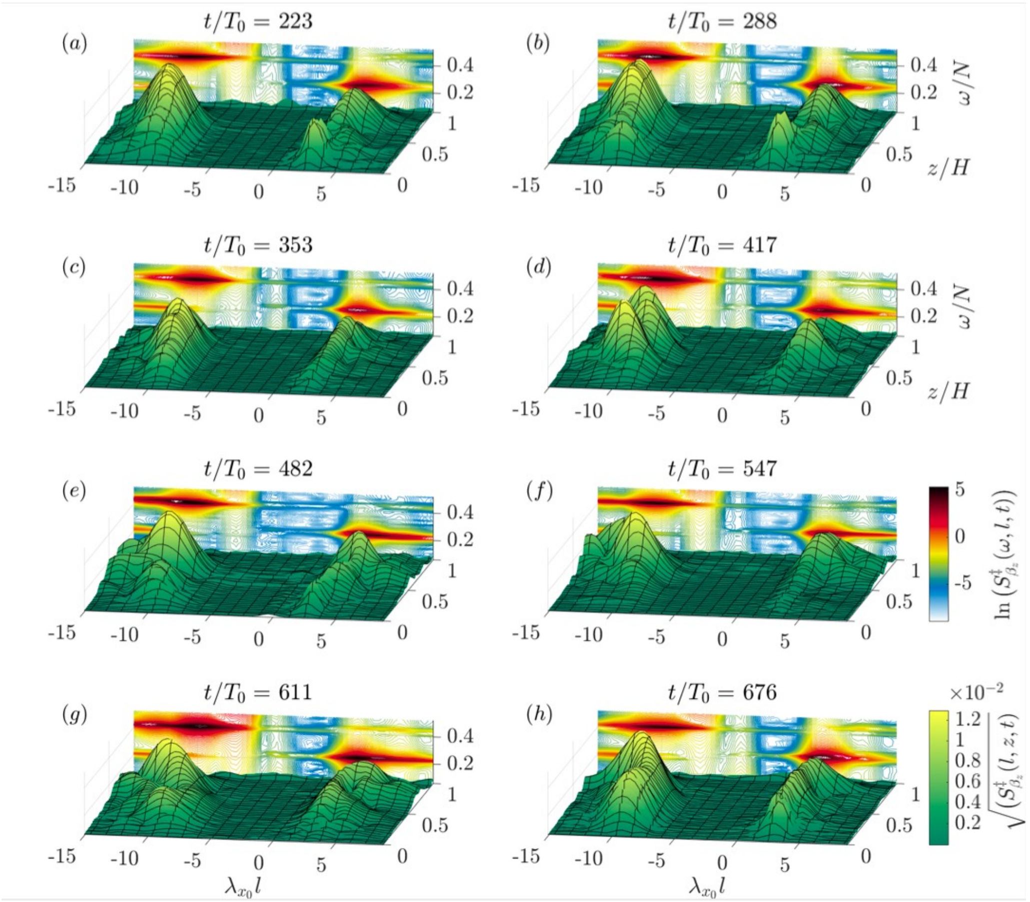
- Frequency and wavenumber of the two resonant beams



Time-frequency spectra of the same experiment, with dominant frequencies obtained from Dynamic Mode Decomposition[1] overlaid in white.

1. Schmid, P. J. (2010). Dynamic mode decomposition of numerical and experimental data. *Journal of Fluid Mechanics, 656,* 5–28. https://doi.org/10.1017/S0022112010001217

Wavenumber modulations of secondary wave beams



Surface plot shows horizontal wavenumber of the two secondary beams as a function of height in the domain.

We see that horizontal wavenumber changes in both:

- time (between snapshots)
- space (moving peaks across height in the domain)

Background contour plot showing l_1 and l_2 as a function of frequency ω

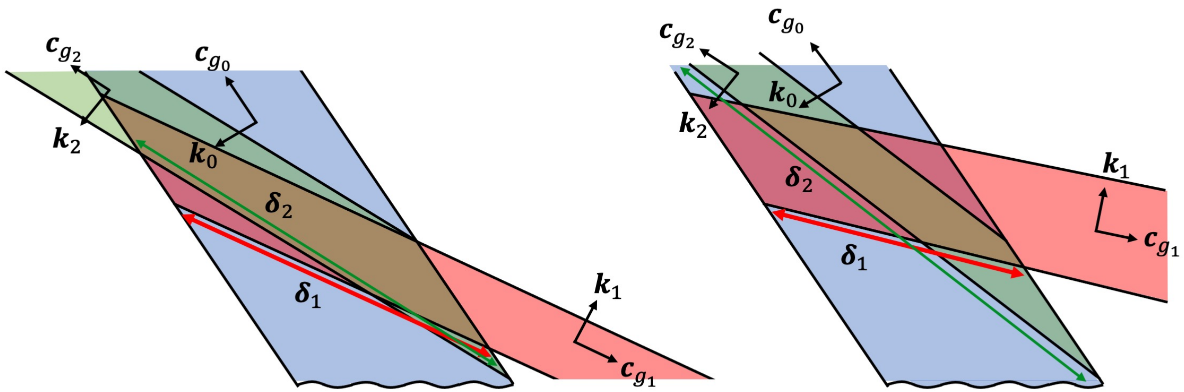
WHY?

The reasons for these modulations are due to the **finite-width primary beam**.

All secondary beam perturbations have a finite-distance δ, and hence finite-time in which to extract energy from the primary beam. This time is defined as

$$R = \frac{\delta}{\boldsymbol{c}_g},$$

where δ is a function of frequency and $\boldsymbol{c}_g$ is a function of wavenumber $\boldsymbol{k} = (l, m)$. When the residence time, R, is approximately equal to the development time (the inverse of the linear growth rate σ^{-1}, of the form $e^{\sigma t}$), we see this unsteady behaviour.



Schematic showing the effect of different ω_1 and ω_2 combinations on δ_1 (red) and δ_2 (green), the distances over which the secondary beams can extract energy from the primary beam.