Extreme waves on vertically sheared flows: Statistical analysis of weakly nonlinear waves

Zibo Zheng, Yan Li, Simen Å. Ellingsen

Department of Energy and Process Engineering, Norwegian University of Science and Technology

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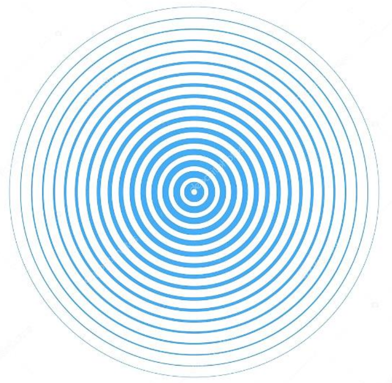
Contents of the talk

- Motivation
- Description of the problem and Methodology
- Input and cases chosen
- Results
- Summary

Effects of a depth-dependent current on wave dispersion relation

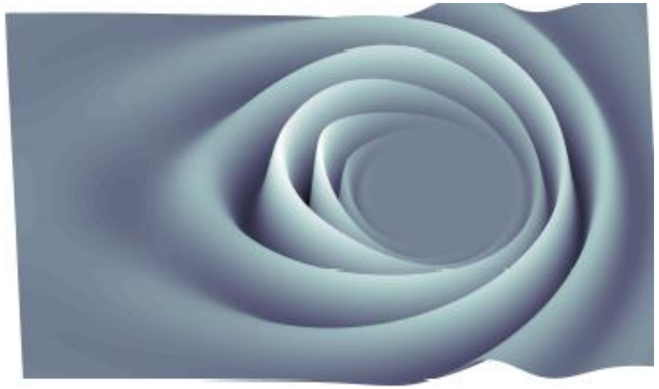
Surface elevation of ring waves from an initial value problem in the absence and presence of a uniform vorticity (i.e. Linearly sheared current)

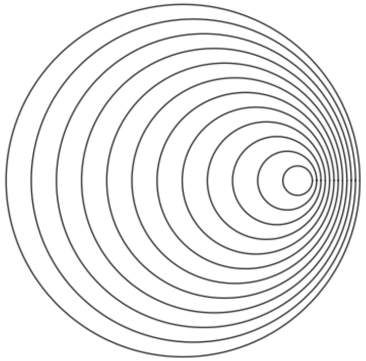
Symmetrical propagation



In the absence

Asymmetrical propagation





In the presence of shear (note: surface is at rest)

(Ellingsen 2014b, Akselsen & Ellingsen 2019)

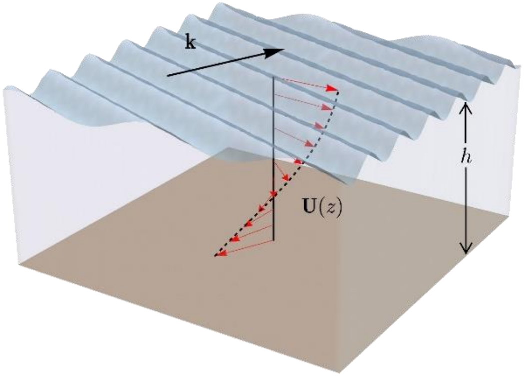


Diagram of wave-current system (Li & Ellingsen, 2017)

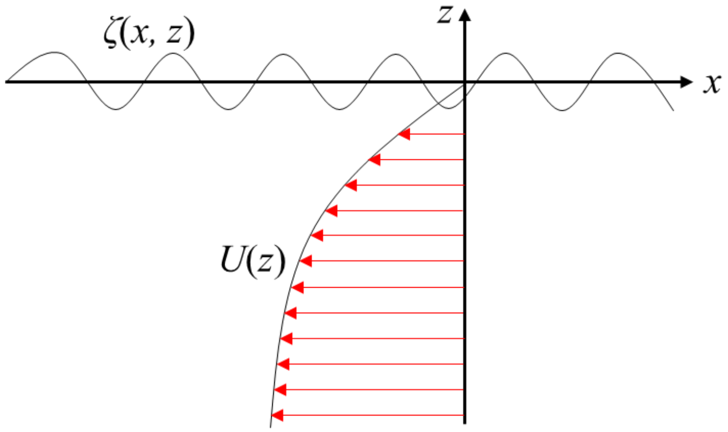
Contents of the talk

What are effects of a background depth-dependent flow on weakly nonlinear waves?

- focus on **deep-water** surface waves in a **depth-dependent** background flow $U(z)$ up to **second order** in wave steepness;
- develop a **semi-analytical framework** for the analysis,
- present numerical results.

Description of the problem

- Waves on a vertical shear current
  - potential wave theory
  - Euler and Continuity equations
- Linear dispersion relation $\omega = \sqrt{gk}$
- Computationally expensive for CFD simulations;



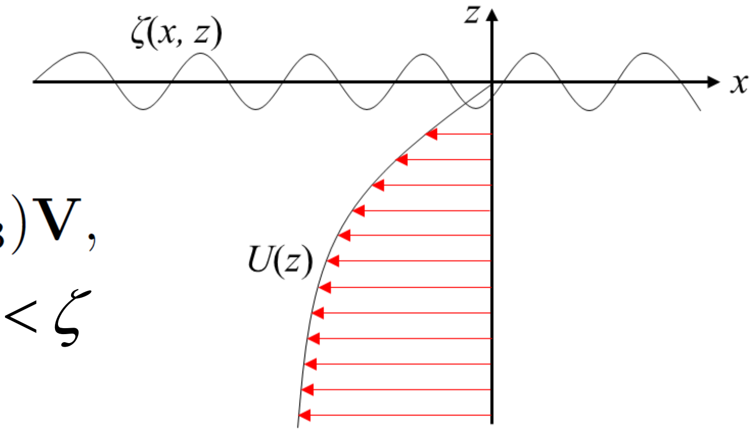
Diagrams of a wave-current system

Description of the problem

Continuity and momentum equations:

$$\nabla \cdot \mathbf{V} = 0 \quad \text{for } -\infty < z < \zeta$$

$$\partial_t \mathbf{V} + (\mathbf{V} \cdot \nabla_3)\mathbf{U}_3 + (\mathbf{U}_3 \cdot \nabla_3)\mathbf{V} + \nabla_3 P = -(\mathbf{V} \cdot \nabla_3)\mathbf{V}, \quad \text{for } -\infty < z < \zeta$$



Diagrams of a wave-current system

Boundary conditions on the free water surface:

$$P - \zeta = 0 \quad \text{and} \quad w = \partial_t \zeta + (\mathbf{u} + \mathbf{U}) \cdot \nabla \zeta \qquad \text{for } z = \zeta$$

where

- ***U***(*z*): background current velocity in the absence of waves;
- ***v***(x,y,z,t)=(*u*(x,y,z,t), *v*(x,y,z,t)), ***V***(x,y,z,t)=(*u*(x,y,z,t), *v*(x,y,z,t), w(x,y,z,t)) and *P*(x,y,z,t): wave perturbed velocity and pressure
- *ζ*(*x*,y,*t*) : the surface elevation.

Approximate solution of the problem

Solutions for unknowns

- in a form of a perturbation expansion in wave steepness and
- they are obtained in sequence from the first order up to second order.

$$\zeta(x,z,t) = \epsilon\zeta^{(1)} + \epsilon^2\zeta^{(2)} + \mathrm{h.o.t.}$$
$$u(x,z,t) = \epsilon u^{(1)} + \epsilon^2 u^{(2)} + \mathrm{h.o.t.}$$
$$v(x,z,t) = \epsilon v^{(1)} + \epsilon^2 v^{(2)} + \mathrm{h.o.t.}$$
$$w(x,z,t) = \epsilon w^{(1)} + \epsilon^2 w^{(2)} + \mathrm{h.o.t.}$$
$$p(x,z,t) = \epsilon p^{(1)} + \epsilon^2 p^{(2)} + \mathrm{h.o.t.}$$

Approximate solution of the problem

Linear wave elevation in three dimensions:

- Amplitude, phase and frequency of an individual wave are assumed known,
- the dispersion relation is solved through the Direct integration method (DIM) by Li & Ellingsen (2019).

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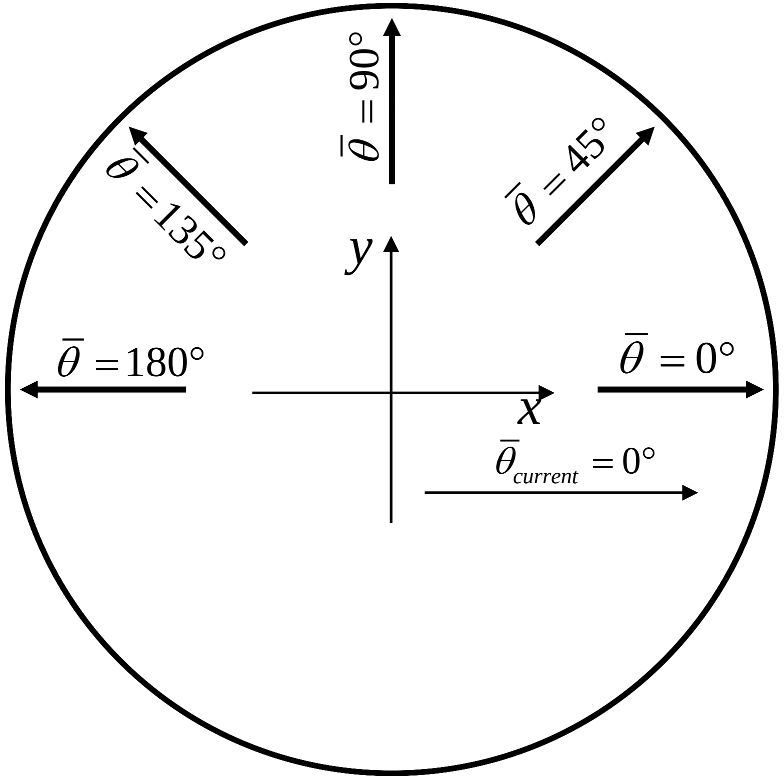
Input and cases chosen



Power energy spectrum:

- JONSWAP spectra, enhancement peak factor 3.3.

Directional spreading function:

- Gaussian distribution

- Current (with **opposing shear**) propagates in x direction
- 5 cases with different $\bar{\theta}$, which refers to the angle between current direction and wave group direction

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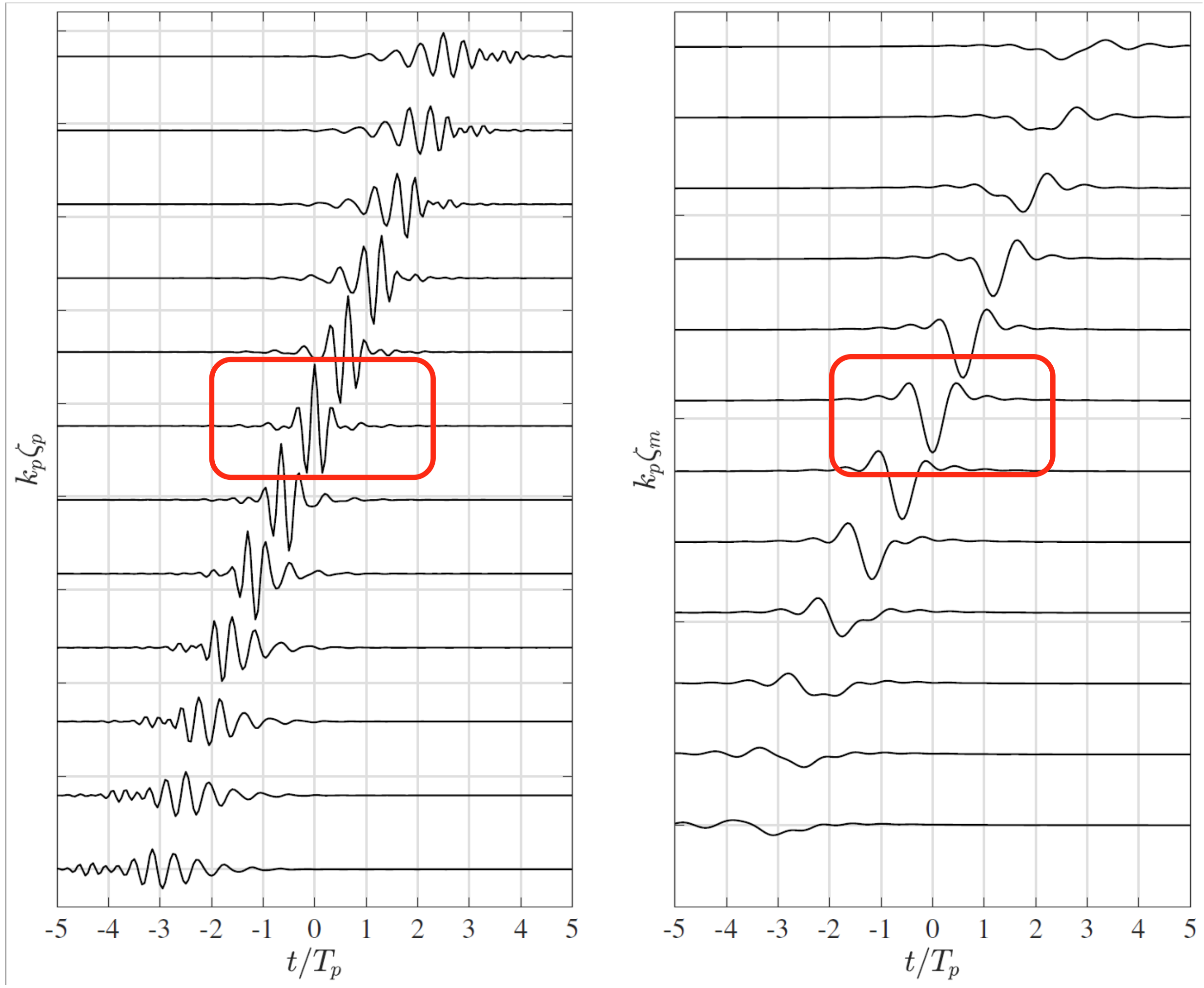
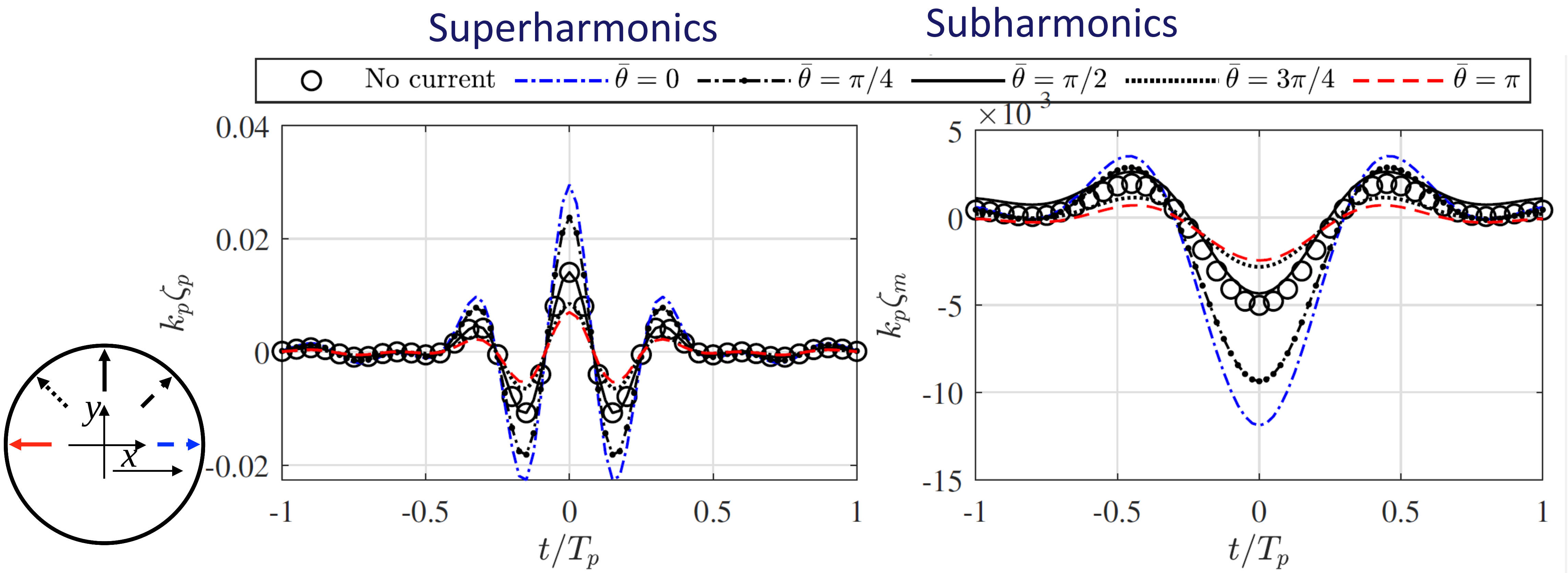
Results: Process of wave focusing



- Left panel: superharmonics
- Right panel: subharmonics

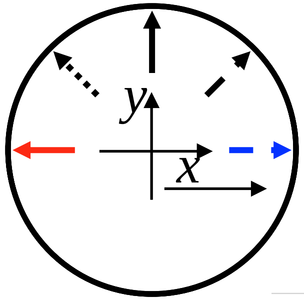
- Each line refers to wave surface elevation at different location
- The wave packet propagates from left to right and linear focus happens at t=0

Results: Focused wave group

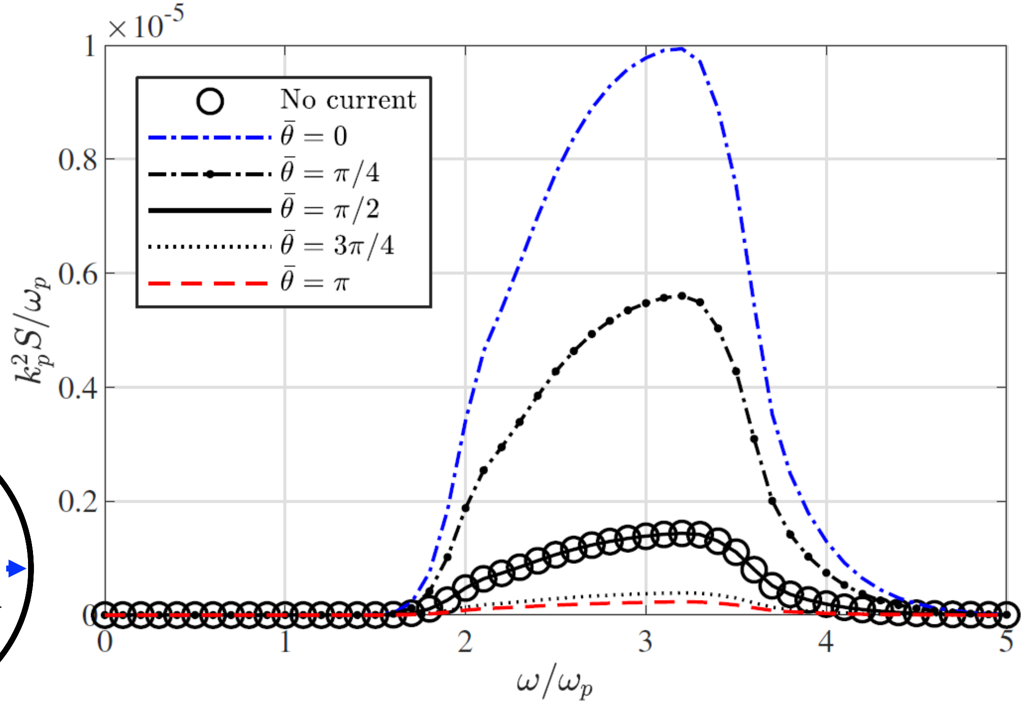


- Wave crests and troughs grow higher or deeper with smaller $\bar{\theta}$
- The superharmonics of no current case is identical with case $\bar{\theta} = \frac{\pi}{2}$
- Shear current has substantial influence on wave surface elevation.

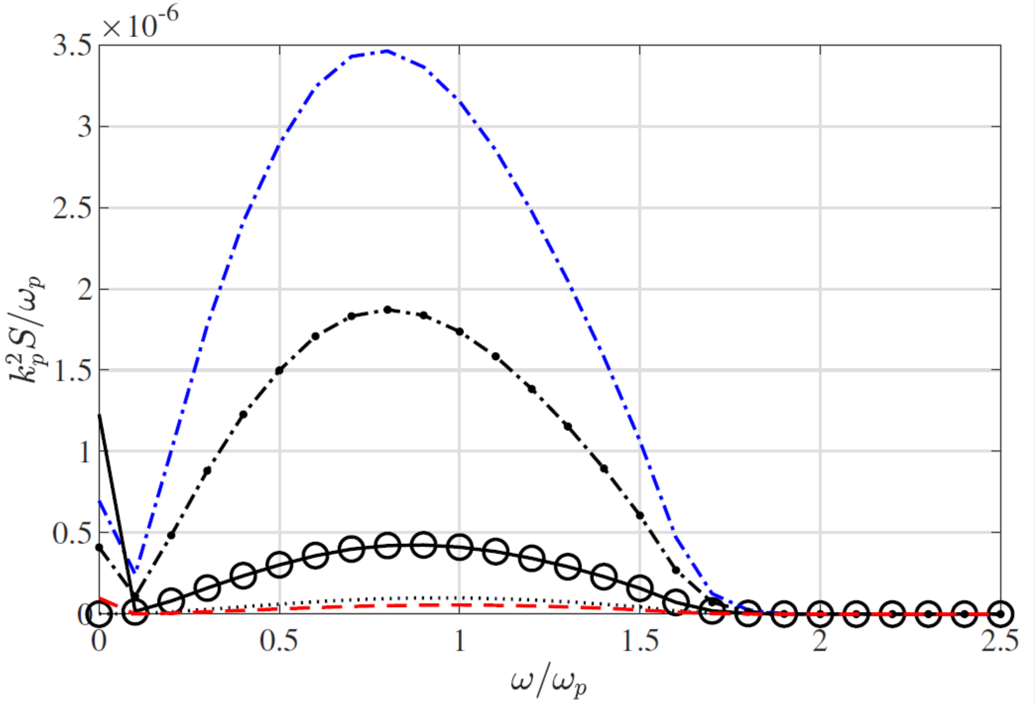
Results: Power energy spectrum



Superharmonics



Subharmonics



- With $\bar{\theta}$ getting larger, wave energy gets smaller and smaller
- The power energy spectra of subharmonics become unsmooth due to the presence of shear current.

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Summary

I. A semi-analytical framework for weakly nonlinear surface gravity waves in a vertically sheared current has been developed, corrected to second order in wave steepness

II. For opposing shear, the maximum effect happens when the direction of shear current and wave group is identical.

III. For subharmonics the presence of shear current can lead to unsmooth power energy spectrum

Thank you very much for your attention