# An Evaluation of Algebraic Turbulence Length Scale Formulations using Budget-Based Diagnostics

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# Outline

# Motivation

- The subgrid parameterization of turbulence should adapt to the resolution of the model in the **gray zone of turbulence**.
- A key component of turbulence kinetic energy (TKE) turbulence parameterizations is the turbulence length scale.
- Algebraic formulations of the turbulence length scale are commonly used in models.
- Classical formulations of the turbulence length scale don't take the cross-scale transfer of TKE into account.
- To overcome this issue a new scale aware turbulence length scale diagnostic was developed by Bastak Duran et al. (2020).

# Aim

The aim of this study is to evaluate a series of existing algebraic turbulence length scale formulations, using LES based scale-aware turbulence length scale diagnostics as a reference.

# Turbulence Length Scale Diagnostics

Classical Diagnostics:

$$L_{\epsilon,e_k} = \frac{(e_k)^{\frac{3}{2}}}{\epsilon} C_\epsilon, \quad (1)$$

$$L_{\epsilon,\theta_l} = (e_k)^{\frac{1}{2}} \frac{\overline{\theta_l'^2}}{\epsilon_{\theta_l}} \frac{C_\epsilon}{C_p}, \quad (2)$$

$$L_{\epsilon,q_t} = (e_k)^{\frac{1}{2}} \frac{\overline{q_t'^2}}{\epsilon_{q_t}} \frac{C_\epsilon}{C_p}, \quad (3)$$

New Diagnostics:

$$L_{C,e_k} = \frac{(e_k)^{\frac{3}{2}}}{\widetilde{\epsilon}} C_\epsilon, \quad (4)$$

$$L_{C,\theta_l} = (e_k)^{\frac{1}{2}} \frac{\overline{\theta_l'^2}}{\widetilde{\epsilon}_{\theta_l}} \frac{C_\epsilon}{C_p}, \quad (5)$$

$$L_{C,q_t} = (e_k)^{\frac{1}{2}} \frac{\overline{q_t'^2}}{\widetilde{\epsilon}_{q_t}} \frac{C_\epsilon}{C_p}, \quad (6)$$

where $\epsilon$, $\epsilon_{\Theta_l}$ and $\epsilon_{q_t}$ are the viscous and molecular dissipation terms derived from the prognostic equations of TKE, and the scalar variances of $\Theta_l$ and $q_t$, $e_k$ is the TKE, $\widetilde{\epsilon}$, $\widetilde{\epsilon}_{\theta_l}$, and $\widetilde{\epsilon}_{q_t}$ are the effective dissipation terms of the TKE and scalar variances respectively, and $C_\epsilon$ is a closure constant.

# Algebraic Formulations

A total of 5 existing algebraic formulations were evaluated.

**Height dependent**

Blackadar (1962) **($L_B$)**
Bastak Duran et al. (2018) **($L_{BD1}$)**

**Dependent on TKE and stratification**

Bougeault and Lacarrere (1989) **($L_{BL}$)**
Nakanishi and Niino (2009) **($L_{NN}$)**
Honnert et al. (2021) **($L_{H21}$)**

# LES cases

Using the Micro-HH LES model 5 idealized boundary layer cases are simulated.

| Case | Domain Size ($km^3$) | Grid Size ($m^2$) |
|---|---|---|
| ARM | 12.8x12.8x4.4 | 12.5x12.5x34.4 |
| BOMEX | 12.8x12.8x3.0 | 12.5x12.5x23.4 |
| RICO | 12.8x12.8x1.5 | 12.5x12.5x2.93 |
| DYCOMS-II | 12.8x12.8x4.4 | 12.5x12.5x31.25 |
| GABLS1 | 0.4x0.4x0.4 | 0.78x0.78x0.78 |

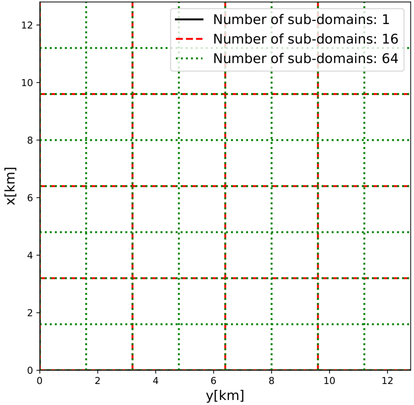
Table: Setup of the 5 LES cases (Reilly et al. 2022).

The scale dependence is analyzed using a coarse graining method of the LES domain.

# Coarse Graining

| Case | $1^2$ $(m^2)$ | $2^2$ $(m^2)$ | $4^2$ $(m^2)$ | $8^2$ $(m^2)$ | $16^2$ $(m^2)$ | $32^2$ $(m^2)$ |
|---|---|---|---|---|---|---|
| ARM | $12.8^2$ | $6.4^2$ | $3.2^2$ | $1.6^2$ | $0.8^2$ | $0.4^2$ |
| BOMEX | $12.8^2$ | $6.4^2$ | $3.2^2$ | $1.6^2$ | $0.8^2$ | $0.4^2$ |
| RICO | $12.8^2$ | $6.4^2$ | $3.2^2$ | $1.6^2$ | $0.8^2$ | $0.4^2$ |
| DYCOMS-II | $12.8^2$ | $6.4^2$ | $3.2^2$ | $1.6^2$ | $0.8^2$ | $0.4^2$ |
| GABLS1 | $0.4^2$ | $0.2^2$ | $0.1^2$ | $0.05^2$ | $0.025^2$ | $0.0125^2$ |

Table: Sizes of the subdomains.




A schematic of the LES grid with the different subdomains (Bastak Duran et al. 2020)

# Methods for Evaluation: Root Mean Square Error

Two methods are used to evaluate the algebraic formulations.
The normalized root mean square error:

$$\mathrm{RMSE}_n = \sqrt{\frac{1}{n_z\, n_t} \sum_{j=1}^{j=n_t} \sum_{i=1}^{i=n_z} \left( \frac{L_F\left(z_i, t_j\right) - L_D\left(z_i, t_j\right)}{H_{pbl}} \right)^2}, \tag{7}$$

where $z_i$ is the height at point i, $n_z$ is the number of model levels within the boundary layer, $n_t$ is the number of time steps averaged over, $L_F$ is the algebraic length scale, $L_D$ is the diagnostics, and $H_{abl}$ is the boundary layer height.

# Methods for Evaluation

A three-component non-local technique including the height and magnitude of profile peak ($S_l$ and $S_m$) within the boundary layer and the mean difference in the normalized length scale paths ($A$)

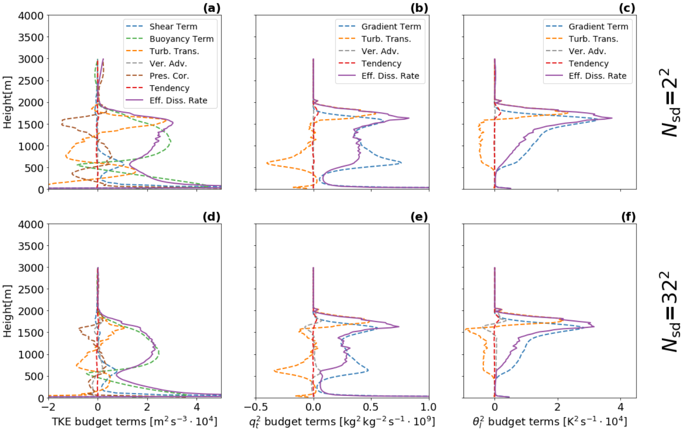
$$\mathrm{A} = \frac{1}{n_t}\sum_{j=1}^{j=n_t}\left(\mathrm{L_F P}\left(t_j\right) - \mathrm{L_D P}\left(t_j\right)\right), \tag{8}$$

$$\mathrm{S_m} = \frac{1}{n_t}\sum_{j=1}^{j=n_t}\frac{\mathrm{L_{F,max}}\left(t_j\right) - \mathrm{L_{D,max}}\left(t_j\right)}{H_{pbl}}, \tag{9}$$

$$\mathrm{S_l} = \frac{1}{n_t}\sum_{j=1}^{j=n_t}\frac{\mathrm{z_{F,max}}\left(t_j\right) - \mathrm{z_{D,max}}\left(t_j\right)}{H_{pbl}}, \tag{10}$$

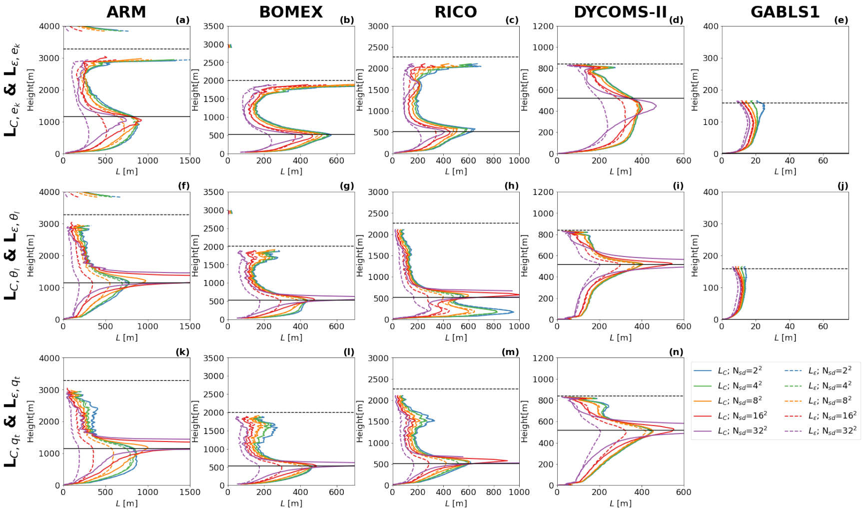
# Results: Scale Dependence of Diagnostics

BOMEX case:



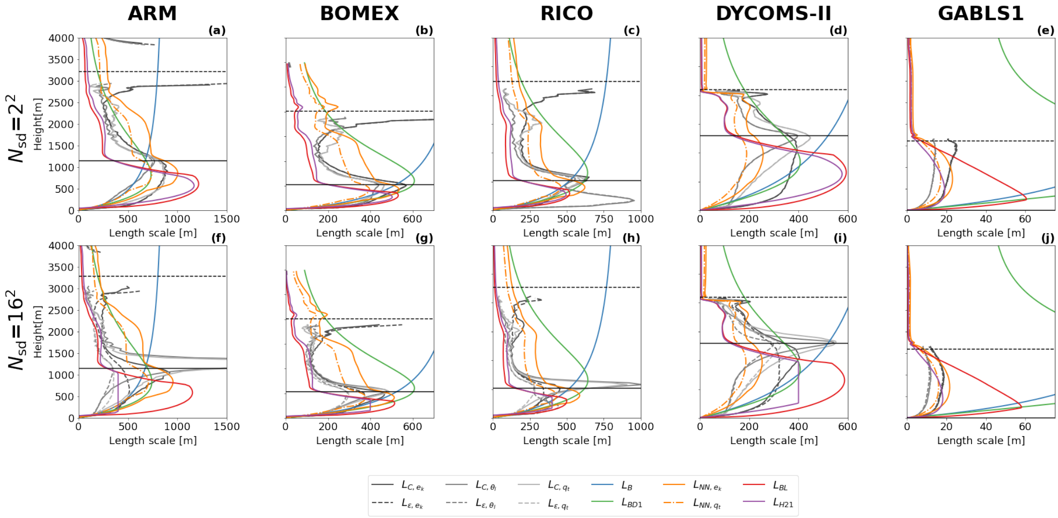

The budget terms for the TKE, scalar variance of $q_t$ and the scalar variance of $\Theta_l$ (columns) for the second largest subdomain size and the smallest subdomain sizes (rows) (Reilly et al. 2022).

# Results: Scale Dependence of Diagnostics



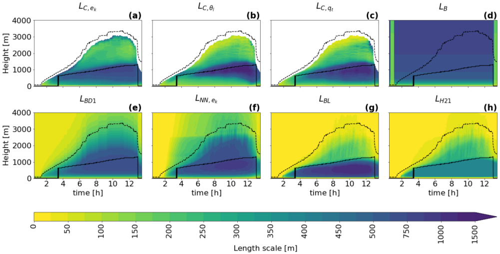

The profiles of the classical (dashed lines) and new (solid lines) turbulence length scale diagnostics for all subdomain sizes (colors) for the 5 idealized LES cases. (Reilly et al. 2022)

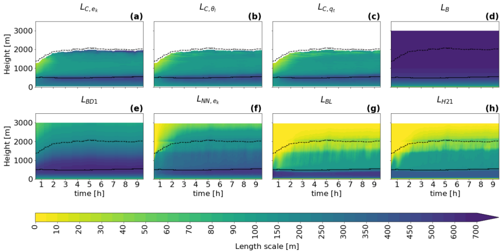
# Results: Profiles of the Turbulence Length Scale




The diagnosed turbulence length scales and the algebraic turbulence length scales for the second largest subdomain size and the second smallest subdomain size. (Reilly et al. 2022)

# Results: Temporal Evolution of the Turbulence Length Scales



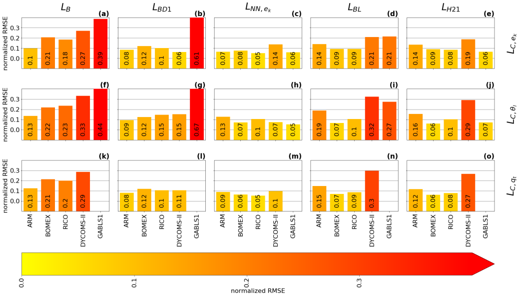




The evolution of both the diagnostics and the algebraic turbulence length scales for the ARM and BOMEX cases. The length scales are computed for the second smallest sub-domain sizes. (Reilly et al. 2022)

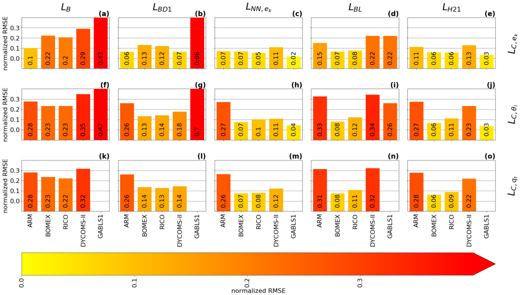
# Results: Evaluation of the Turbulence Length Scale

Evaluation of the algebraic turbulence length scale formulations by means of a root mean square error method.

For $2^2$ subdomains:




For $16^2$ subdomains:

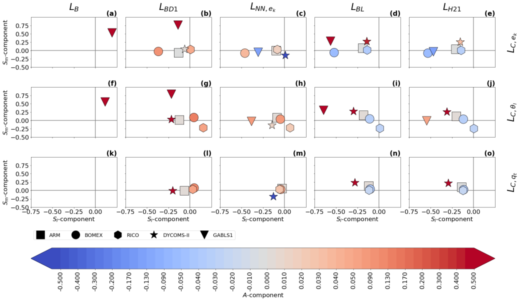



The normalized root mean square errors between the three diagnostics (rows) and the algebraic formulations (columns) for the second largest subdomain size and the second smallest subdomain size (Reilly et al. 2022)

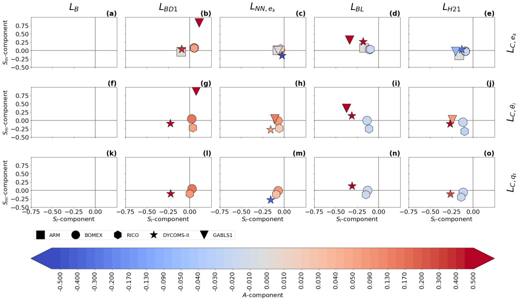
# Results: Evaluation of the Turbulence Length Scale

Evaluation of the algebraic turbulence length scale formulations by means of a three component technique.

For $2^2$ subdomains:




For $16^2$ subdomains:




The differences in the normalized magnitude and the normalized height of the peaks for each of the algebraic formulations (columns) compared to the diagnostics (rows) for the second largest subdomain size and the second smallest subdomain size. The different colors represent temporally averaged amplitude, and the different marker shapes represent the different LES cases. (Reilly et al. 2022)

# Conclusions & Outlook

Conclusions:

- An LES based turbulence length scale diagnostic is used for evaluating the algebraic formulations.
- The profiles of the diagnostic show a peak in the length scale within the boundary layer.
- The diagnostic is dependent on the resolution (subdomain size) and the dependence changes with height.
- $L_{H21}$ and $L_{NN}$ show the best ability overall.
- $L_{H21}$ has the best scale-awareness, but it is limited.

Outlook:

The outlook is to develop an algebraic turbulence length scale that can scale with the ABL, stratification, TKE, and the cloud base height.

Paper: An Evaluation of Algebraic Turbulence Length Scale Formulations, Atmosphere, 2022