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Finite-Volume Flux Reconstruction and Semi-Analytical Particle Tracking for Finite-Element-Type Models of Variably Saturated Flow in Porous Media

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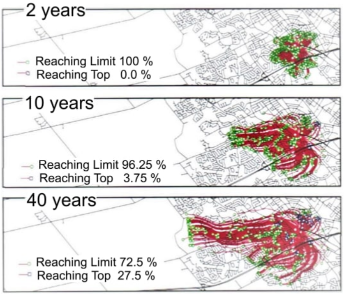
CRC 1253 CATCHMENTS AS REACTORS CAMPOS

Research Training Group 1829
Integrated Hydrosystem Modelling

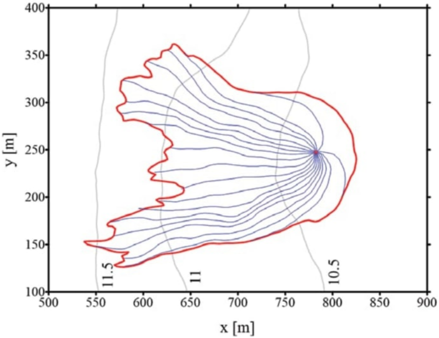
DFG

Motivation: Why Particle Tracking

- Computing residence and travel times, streamlines and path-lines
- Delineation of capture zones



(Frind et al., 2002)



(Feo et al., 2017)

⇒ Advantages of PT: computational efficiency, no numerical dispersion

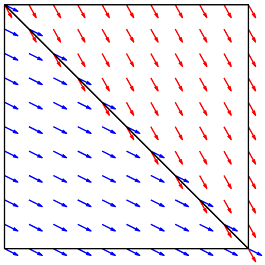
Motivation: Why Postprocessing of FEM Velocities?

Emerging problem:

- Existence of mature finite-element-type codes for simulating variably-saturated flow, and integrated hydrosystem modeling

$\Rightarrow$ FE-type-velocities are not conforming

Example: non-conforming FEM-velocity field



But, finite-element-type methods

- yield a continuous solution of hydraulic heads
- a full FEM can handle full tensors of material coefficients
- can easily handle unstructured grids including anisotropy

Attributes of the solution:

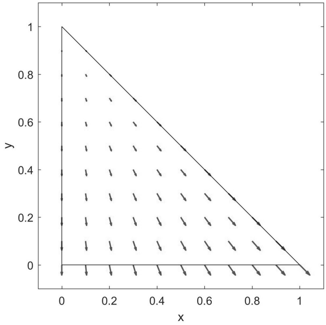
$$[h]_F = (h|_{E_i})|_F - (h|_{E_j})|_F = 0 \wedge [\boldsymbol{q} \cdot \boldsymbol{\nu}_F]_F = (\boldsymbol{q}|_{E_i})|_F \cdot \boldsymbol{\nu}_F - (\boldsymbol{q}|_{E_j})|_F \cdot \boldsymbol{\nu}_F \neq 0$$

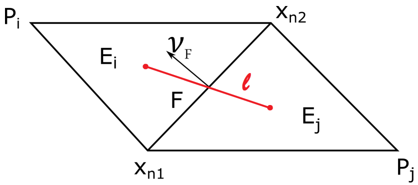
The FVM reconstruction for Richards' equation

A centroid-centered finite-volume flux reconstruction on the primal grid

$$\sum_{F \in \mathcal{F}_E} \int_F \mathbf{n}_{E,F} \cdot \mathbf{q} \, ds = \int_E f_* \, d\mathbf{x} \quad \forall E \in \mathcal{T} \wedge f_* = f - \left(\frac{\theta(\psi)}{\theta_s} S_s \frac{\partial \psi}{\partial t} + \frac{\partial \theta(\psi)}{\partial t} \right)$$

$$\boldsymbol{q}_E^{RTN_0} = (a_j + b x_j)_{j=1,\ldots,d} \,, \quad [\boldsymbol{q} \cdot \boldsymbol{\nu}_F]_F = (\boldsymbol{q}|_{E_i})|_F \cdot \boldsymbol{\nu}_F - (\boldsymbol{q}|_{E_j})|_F \cdot \boldsymbol{\nu}_F = 0$$

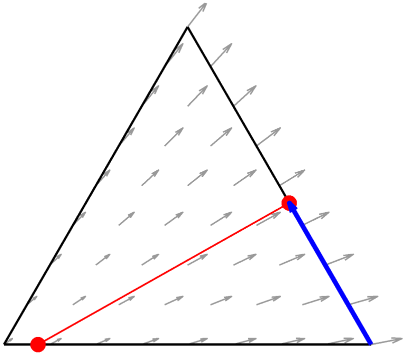




Adapted from Selzer & Cirpka (2020)

The resulting system resembles those of the steady-state groundwater flow equation: $\mathbf{A}\hat{\mathbf{h}} = \mathbf{r}$.

Elementwise Analytical Particle Tracking



On ∂E_i every face is embedded in a vectorized line equation:

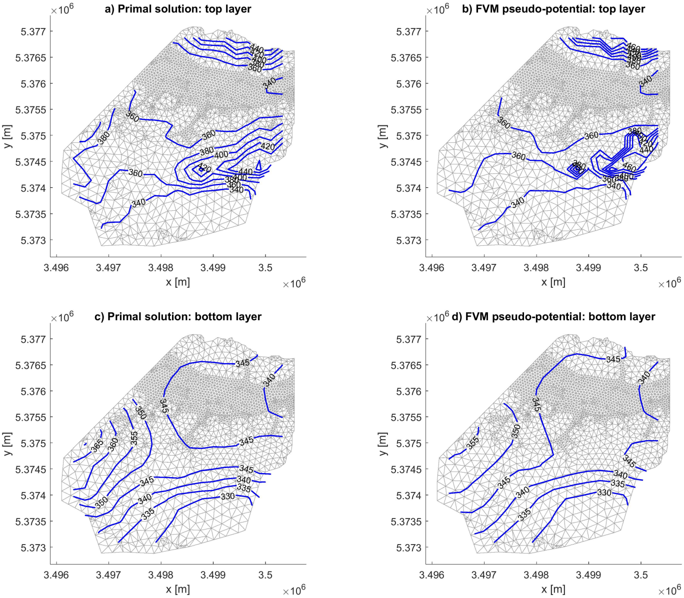
$\mathbf{x}_\ell = \mathbf{x}_{n1} + s\mathbf{t}_\ell$

$\rightarrow$ solve for s such that the exit coordinates of the particle are recovered

$$s = \left(\frac{a_y + by_{n1}}{a_y + by_p} - \frac{a_x + bx_{n1}}{a_x + bx_p} \right) \cdot \left(\frac{bt_{x,\ell}}{a_x + bx_p} - \frac{bt_{y,\ell}}{a_y + by_p} \right)^{-1}$$

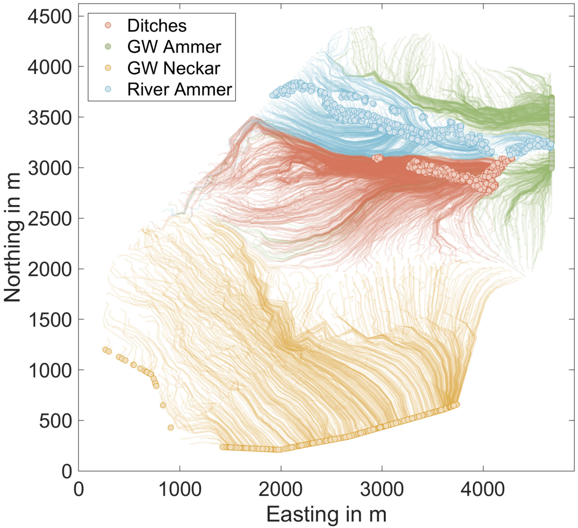
$\Rightarrow$ Extendible to hyperplanes in $\mathbb{R}^{n-1}$

At the Catchment Scale: FVM reconstruction on prisms



(Selzer et al., 2021)

Application to the Catchment Scale: Trajectories



(Selzer et al., 2021)

Outcome and Conclusions

- ☺ Definition and implementation of a flux postprocessing technique for heterogeneous models of variably saturated flow in porous media on the catchment-scale linked to HydroGeoSphere
- ☺ The finite-volume flux reconstruction is robust and fast to compute
- ☺ Very effective, parallelized code for particle tracking in Matlab (10^6 particle tracks in an unstructured, variably saturated, and transient catchment-scale model (without pre-computations) take a few minutes on a Standard-PC on the CPU)
- ☹ The FVM flux reconstruction comes with a comparably high coding effort ☺ but is available on GitHub

Thank you for your attention!

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All codes are available on GitHub:

`https://github.com/PhilippSelzer/FluxCorr_ParticleTracking`

Related Publications:

- Selzer, P., Cirpka, O.A.: Postprocessing of standard finite element velocity fields for accurate particle tracking applied to groundwater flow. *Computational Geosciences*, 24 (4), 1605–1624 (2020). `https://doi.org/10.1007/s10596-020-09969-y`
- Selzer, P., Allgeier, J., Therrien, R., and Cirpka, O.A.: Finite-volume flux reconstruction and semi-analytical particle tracking on triangular prisms for finite-element-type models of variably-saturated flow. *Advances in Water Resources*, 154:103944 (2021). `https://doi.org/10.1016/j.advwatres.2021.103944`

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