Bayesian parameter inference in hydrological modelling using a Hamiltonian Monte Carlo approach with a stochastic rain model

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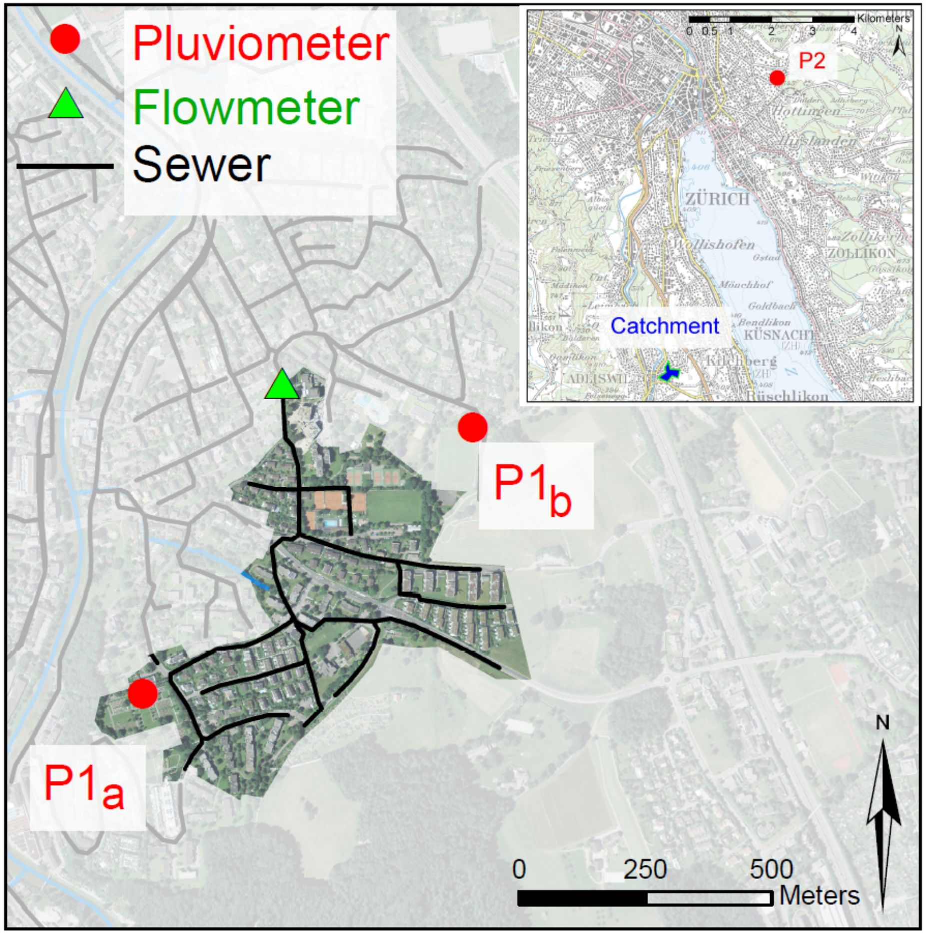
Inference Problem



Input uncertainty

↓

Stochastic model:

deterministic hydrological model

+ stochastic rain generator

↓

High-dimensional inference:

infer model parameters

+ true rainfall time-series

from rain and runoff observations

Stochastic Model

Stochastic prior for the rainfall potential

$$\dot{\xi}(t) = -\,\xi(t)\tau^{-1} + \sqrt{2\tau^{-1}}\eta(t)$$

$\xi(t)$

Stochastic rain observation model

$P_{obs}(t_I)\,,$

$I = 1,\ldots,n_P$

Deterministic hydrological model

$Q(t)$

Stochastic runoff observation model

$Q_{obs}(t_J)\,,$

$J = 1,\ldots,n_Q$

$$\dot{S}(t) = AP(t) + Q_{\mathrm{gw}} - Q(t)\,,$$

$$P(t) = h(\xi(t), \boldsymbol{\theta}_h)\,, \quad Q(t) = S(t)/K$$

To infer:

$$\boldsymbol{\theta} = (S_0, K, Q_{gw}, \sigma_P, \sigma_Q, \boldsymbol{\theta}_h)$$

$$\boldsymbol{\xi} = (\xi(t_1), \ldots, \xi(t_N))$$

Bayesian inference

Prior: $f(\boldsymbol{\xi},\boldsymbol{\theta}) = f(\boldsymbol{\xi})f(\boldsymbol{\theta})$

Observational

likelihood functions: $f(\mathbf{P}_{obs} \mid \boldsymbol{\xi},\boldsymbol{\theta})$, $f(\mathbf{Q}_{obs} \mid \boldsymbol{\xi},\boldsymbol{\theta})$

Posterior: $f(\boldsymbol{\xi},\boldsymbol{\theta} \mid \mathbf{P}_{obs}, \mathbf{Q}_{obs}) \propto f(\mathbf{P}_{obs} \mid \boldsymbol{\xi},\boldsymbol{\theta}) f(\mathbf{Q}_{obs} \mid \boldsymbol{\xi},\boldsymbol{\theta}) f(\boldsymbol{\xi},\boldsymbol{\theta})$

Methods

Three ways of sampling the high-dimensional posterior:

- **Metropolis with Gibbs sampling**

  Prior: proposal; data: accept/reject

- **Metropolis (θ) with Particle filter (ξ)**

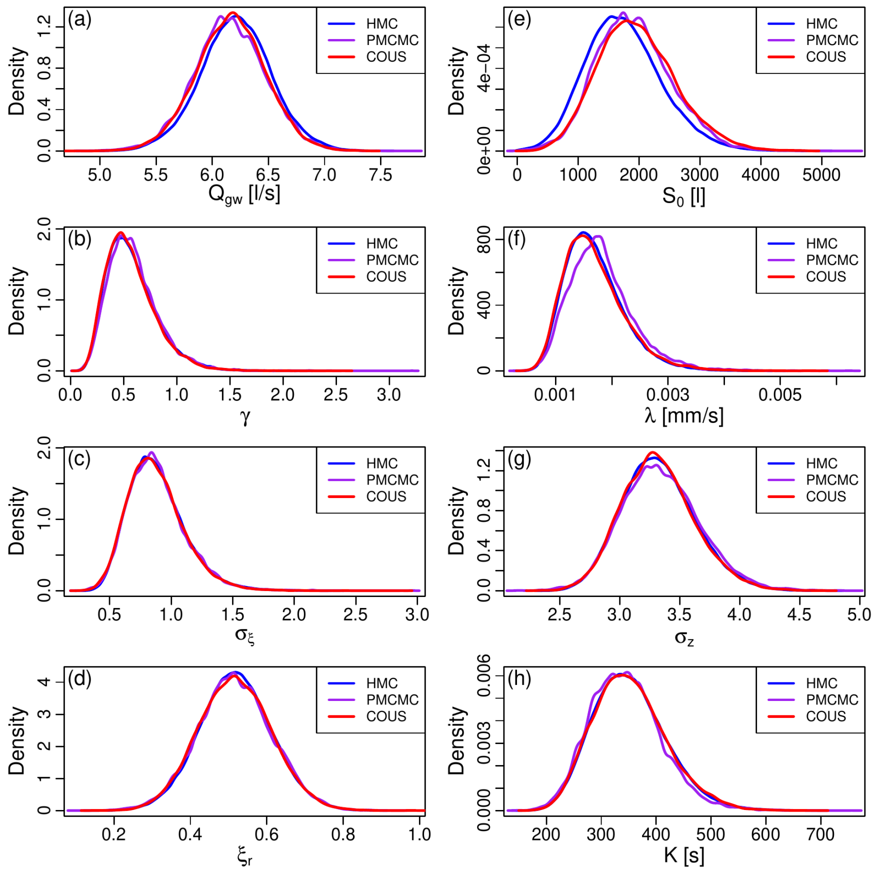
  Prior: proposal; data: importance sampling

- **Hamiltonian Monte Carlo**

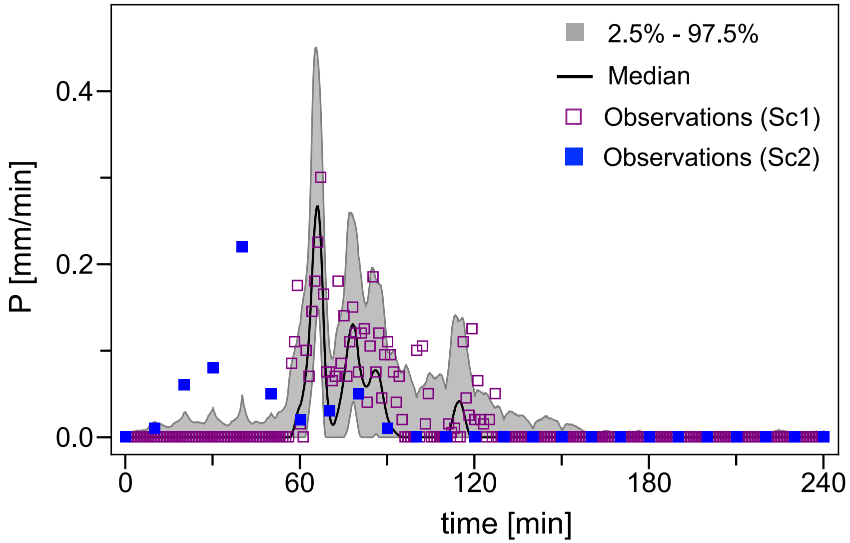
  Simulating particle in a potential, defined by the prior *and* the data:

$$V(\boldsymbol{\xi},\boldsymbol{\theta}) = V_{prior}(\boldsymbol{\xi},\boldsymbol{\theta}) + V_{P_{obs}}(\boldsymbol{\xi},\boldsymbol{\theta}) + V_{Q_{obs}}(\boldsymbol{\xi},\boldsymbol{\theta})$$

Results



The three methods give consistent results!



Inference corrects for rain measurement errors!

Conclusions

- Solving high-dimensional inference problems resulting from stochastic models is feasible, using available sampling algorithms (and parallel computing infrastructure).
- It should be common practice to apply *different* inference methods to the *same* inference problem.

References

Del Giudice, D., Albert, C., Rieckermann, J., & Reichert, P. (2016). *Describing the catchment-averaged precipitation as a stochastic process improves parameter and input estimation*. Water Resources Research, 52(4), 3162-3186.

Albert, C., Ulzega, S., & Stoop, R. (2016). *Boosting Bayesian parameter inference of nonlinear stochastic differential equation models by Hamiltonian scale separation*. Physical Review E, 93(4), 043313.

Šukys, J., & Kattwinkel, M. (2017). SPUX: *Scalable particle Markov chain monte Carlo for uncertainty quantification in stochastic ecological models*. Parallel Computing is Everywhere, edited by: Bassini, S., Danelutto, M., Dazzi, P., Joubert, GR, and Peters, F, 159-168.

…Two more to come.