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EGU22-8971

A Numerical Study of Wormhole Formation and Growth in Homogeneous and Heterogeneous Carbonate Rocks

May 26, 2022

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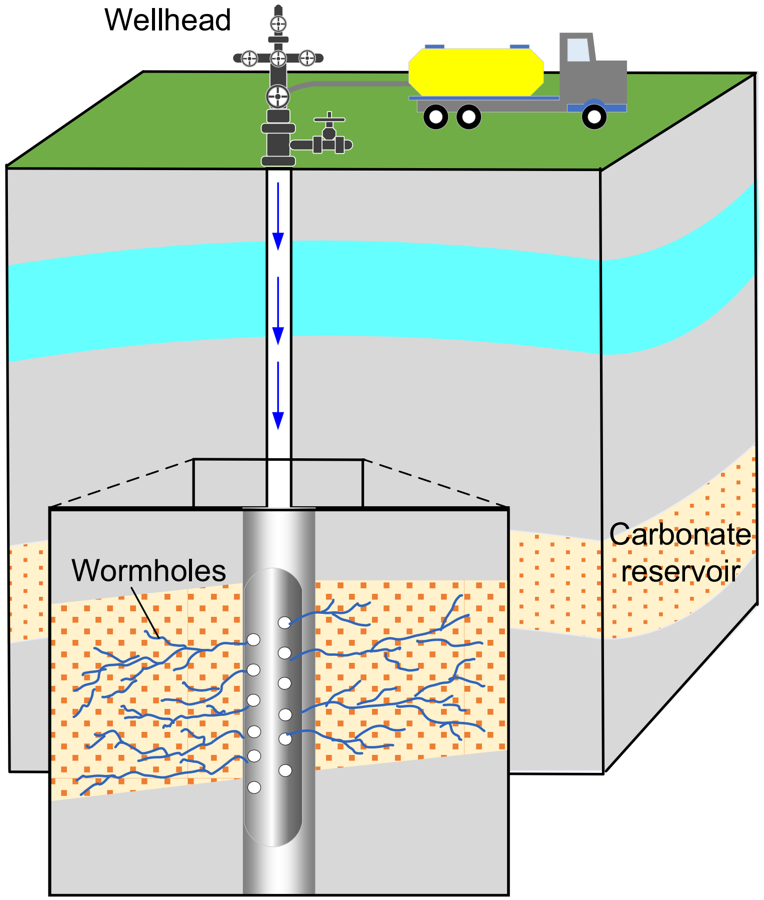
Introduction

Carbonate Matrix Acidizing and Wormholing

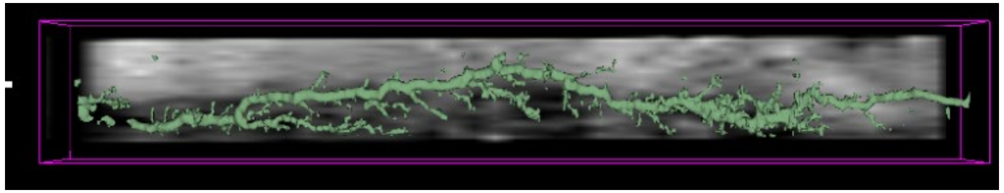
- Acid injection into carbonate rock enhances matrix permeability features or natural fractures by dissolving the rock to form larger conduits called wormholes.
- The wormhole growth is a function of acid concentration, injection rate, rock mineralogy, and temperature.
- Extrapolation of laboratory test results and direct indications from field treatment responses show that wormholes commonly penetrate distances on the order of 5 to 6 m (Burton et al. 2020).



Carbonate Matrix Acidizing Treatment

Introduction

- Wormholing mechanisms
  - Rock heterogeneity
  - Reactive infiltration instability

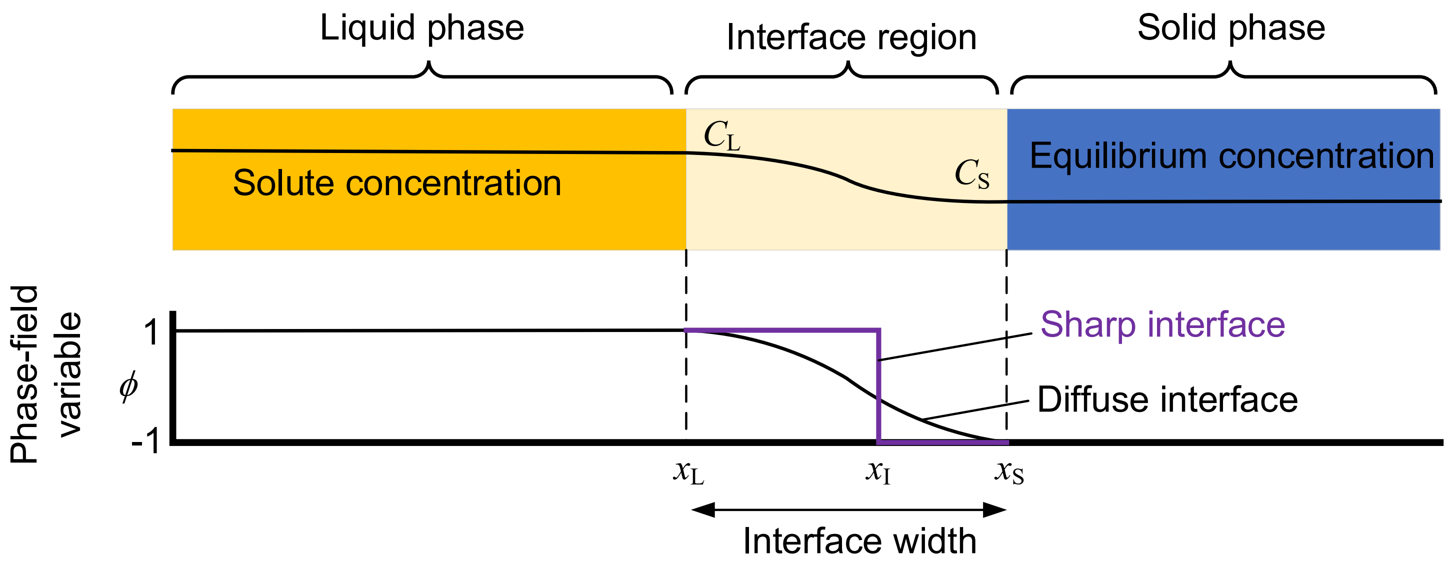


CT image of wormholes created in Austin Chalk core sample.

- Several numerical models have been proposed to investigate matrix acidizing process and wormhole formation.
  - Network model（Fredd & Fogler, 1998）
  - Lattice-Boltzmann（Kang, 2003）
  - Particle-based model（Pereira Nunes, 2016）
  - Two-scale continuum model（Schwalbert et al., 2017）
- The above numerical models require artificial heterogeneity to form wormholes.

Phase Field Approach for Wormhole Modeling

- A phase-field approach may be used to simulate reaction infiltration-instability problems.
  - A moving solid-liquid interface is assumed to have a finite width forming a diffuse zone where the phase-field variable changes smoothly.
  - The model is applicable to dendritic growth due to solute precipitation/dissolution.
  - The solution converges to the sharp-interface limit when the interface width approaches zero.



Governing Equations of Phase-Field-Based Wormhole Model

Acid wormholing processes can be simulated by the following equations (Furui et al. 2022):

- Phase field equation

$$\tau \frac{\partial \phi}{\partial t} = \varepsilon^2 \nabla^2 \phi + \left(1 - \phi^2\right)(\phi - \lambda c_D) - \varepsilon^2 \kappa |\nabla \phi|$$

- Acid transport equation

$$\frac{\partial c_D}{\partial t} = D \nabla^2 c_D - \nabla \cdot (\mathbf{v} c_D) + \alpha \frac{\partial \phi}{\partial t} \quad \text{for large } Da$$

- Continuity equation & Darcy's law

$$\nabla \cdot \mathbf{v} = 0, \mathbf{v} = -\frac{K(\phi)}{\mu} \nabla p$$

ϕ: phase-field variable
ε : interface width
λ : coupling factor of ϕ & c_D
τ : phase field mobility
K: permeability
μ : viscosity
D: diffusion coefficient
c_D : acid concentration ($= C/C_{ref}$)
D_a: Damköhler number
α : reaction parameter defined by

$$\alpha = \frac{(1-\varphi)\rho_s}{2 C_{ref} \beta_{100} \rho_a}$$

Reaction-Infiltration Instability Conditions

- Chadam & Ortoleva (1990) investigated the morphological stability of the reaction interface using bifurcation and stability theory on the Stefan problem and showed that the critical injection velocity to induce the instability is found to be:

$$\nu_c = \nu_c(\Gamma) = \frac{(3-\Gamma)(1+\Gamma)}{2(1-\Gamma)}$$

where the measure of the porosity change, Γ, is defined by

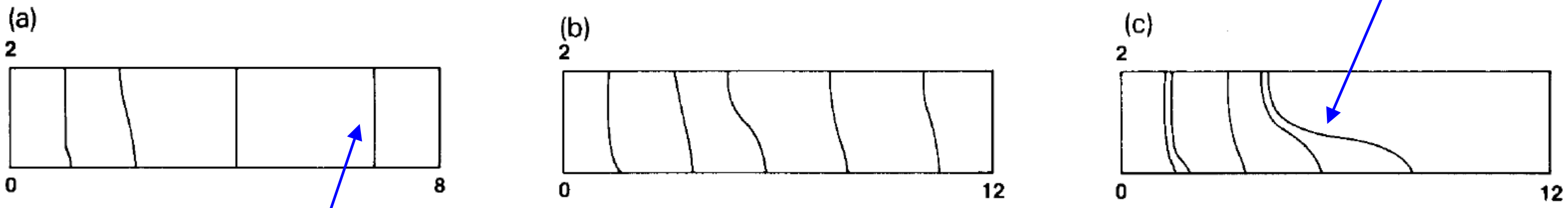
$$\Gamma = \frac{\phi_0 k_0}{\phi_f k_f}$$

ϕ_0 = initial porosity (pre-dissolution)
ϕ_f = final porosity after dissolution
k_0 = initial permeability ($k_0 = k(\phi_0)$)
k_f = final permeability ($k_f = k(\phi_f)$)
$\nu_c = u_c\left(\frac{L}{D_f \pi}\right)$
D_f = diffusion coefficient
L = original transverse dimension of the formation

- The critical value for the instability to the various modes, $\cos(my)$ $(m = 1,2,\ldots)$ is calculated by

$$\nu_{c,m} = \frac{(3-\Gamma)(1+\Gamma)}{2(1-\Gamma)}|m|$$

The many instable modes couple to produce an elongating finger.

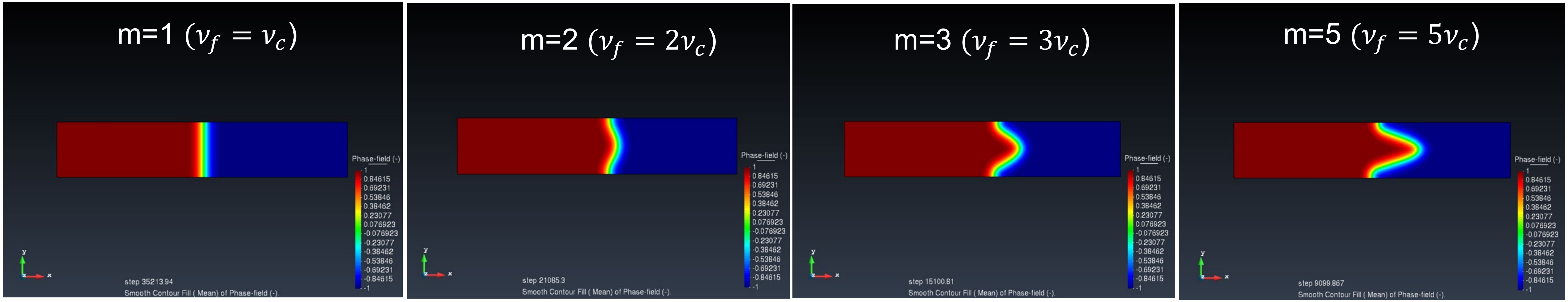


(a) $\nu_f \ll \nu_c$ (b) $\nu_f = \nu_c$ (c) $\nu_f \ll 5\nu_c$

The perturbation dies out leaving a stable front.

Verification of the Phase-Field-Based Wormholing Model

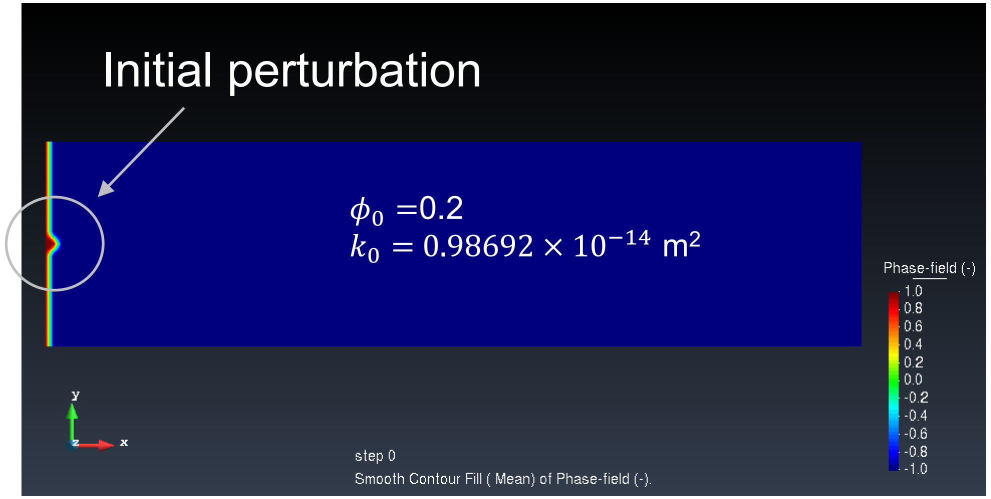
- The phase-field simulation was performed at different acid injection velocities, v_f.

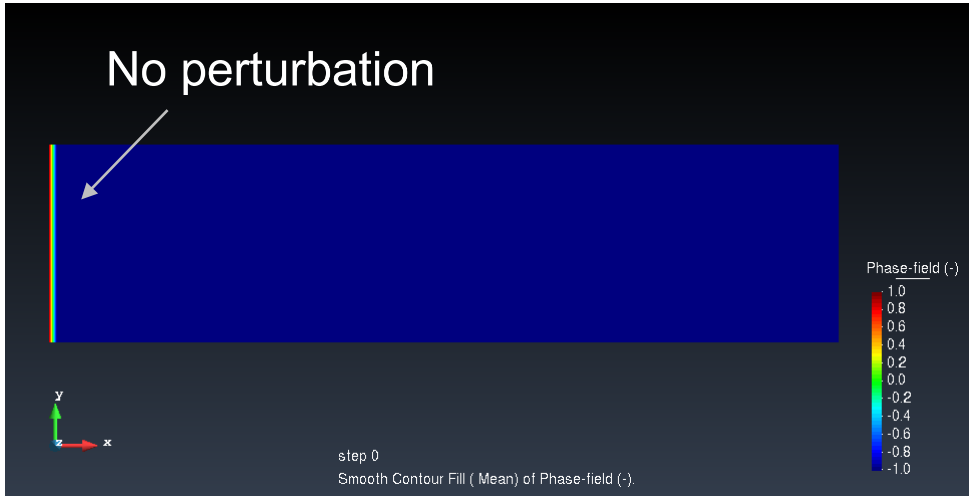


- The results showed
  - A wormhole started forming when the injection velocity becomes greater than the critical velocity of the bifurcation parameter, v_c.
  - An elongating finger is produced when the many instable modes are coupled (i.e., high injection velocity).

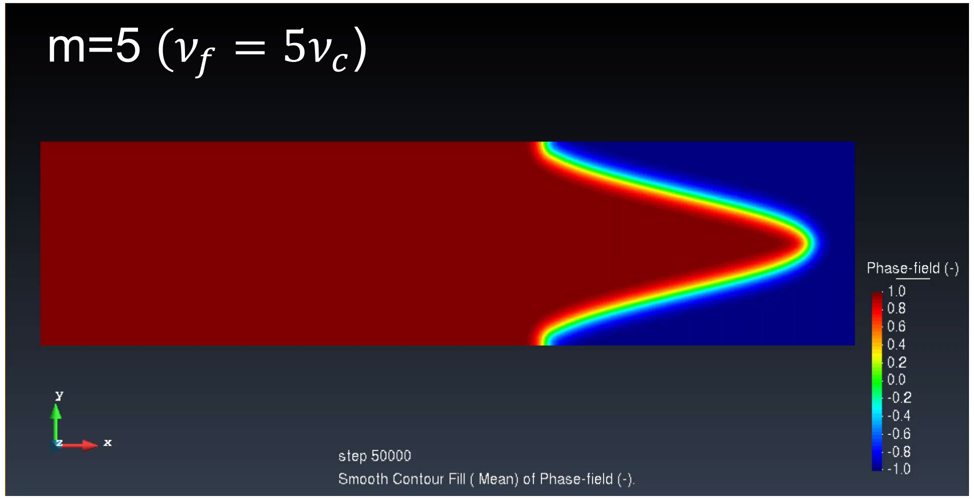
Effect of Perturbation in Phase-Field Based Wormhole Model

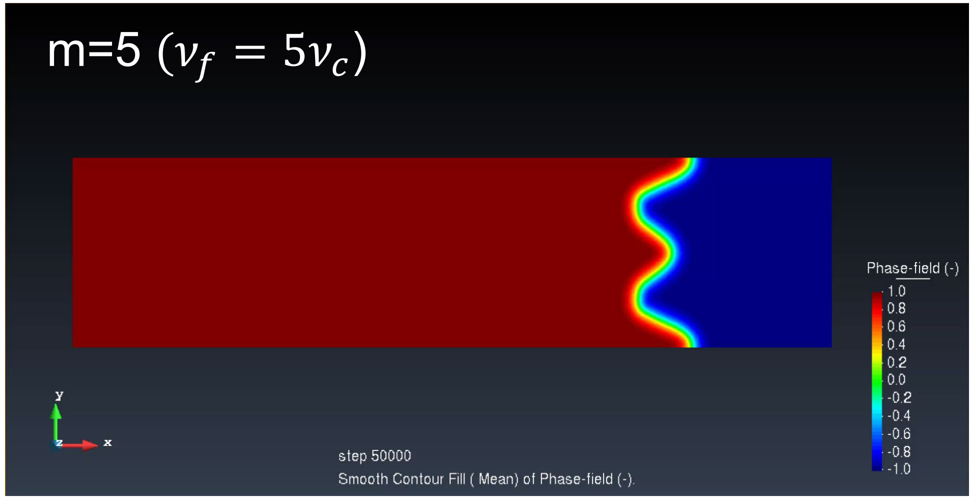
Initial conditions





Simulation Results



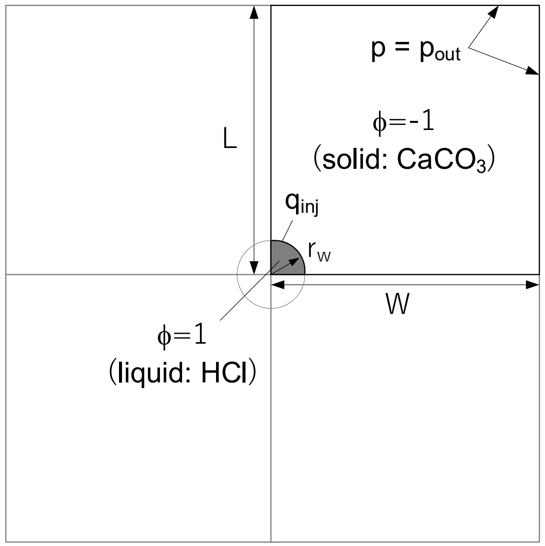


In the phase-field model, the reaction infiltration instability occurs without introducing the initial perturbation.

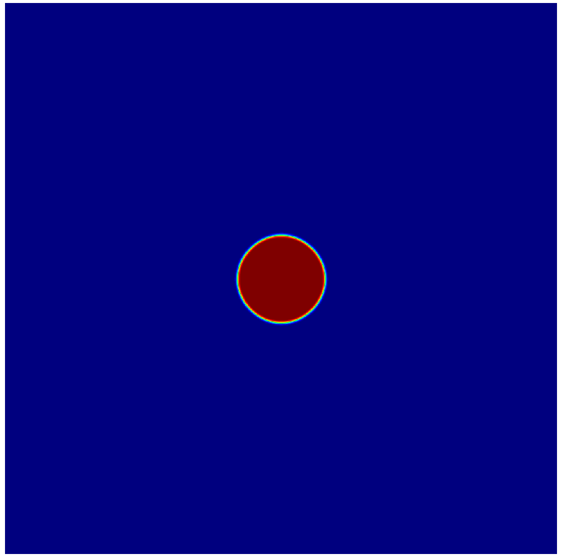
Numerical Experiment: Radial Acid Injection in Homogeneous Rock

Simulation Conditions

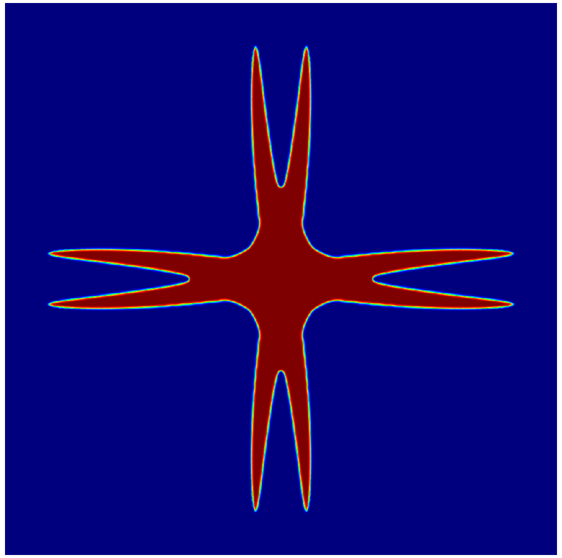
Rock type	Calcite	Permeability	0.98692×10^{-14} m^2	Borehole radius, r_w	0.020 m
Injection fluid	28 wt% HCl	Fluid viscosity	1.0 cp	Interface width, ε	1.0 mm
Domain size, L, W	0.25 m	Porosity	0.2	Grid block size, $\Delta x = \Delta y$	0.5 mm
P_{out}	1.0133×10^5 Pa	Diffusion coefficient	1.0133×10^{-8} m^2/s		



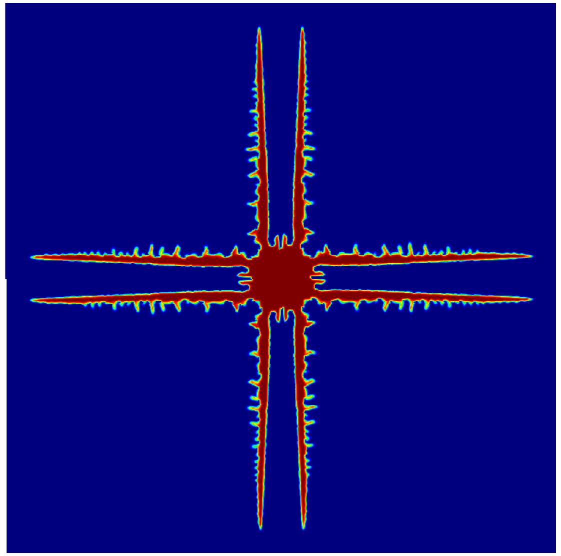
Case 1 (Low injection rate)
$q_{inj} = 1 \times 10^{-7}$ m^3/s (t=5x10^4 s)



Case 2 (Moderate injection rate)
$q_{inj} = 6 \times 10^{-6}$ m^3/s (t=6000 s)

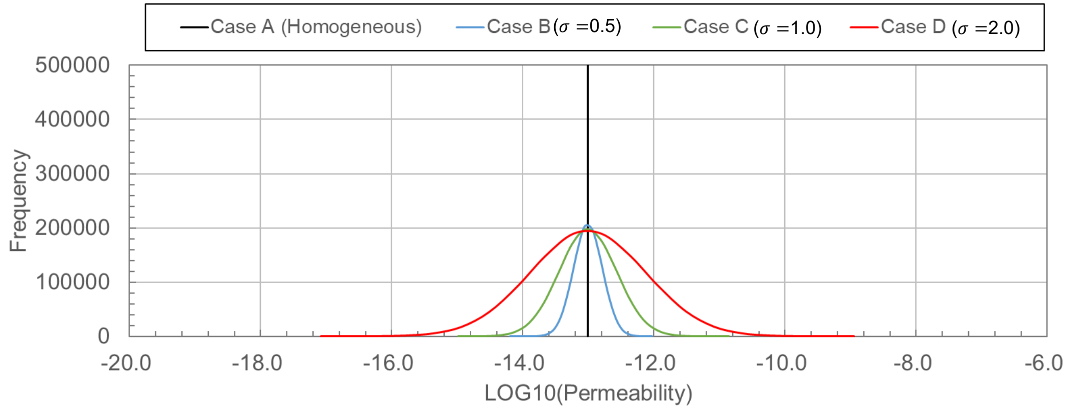


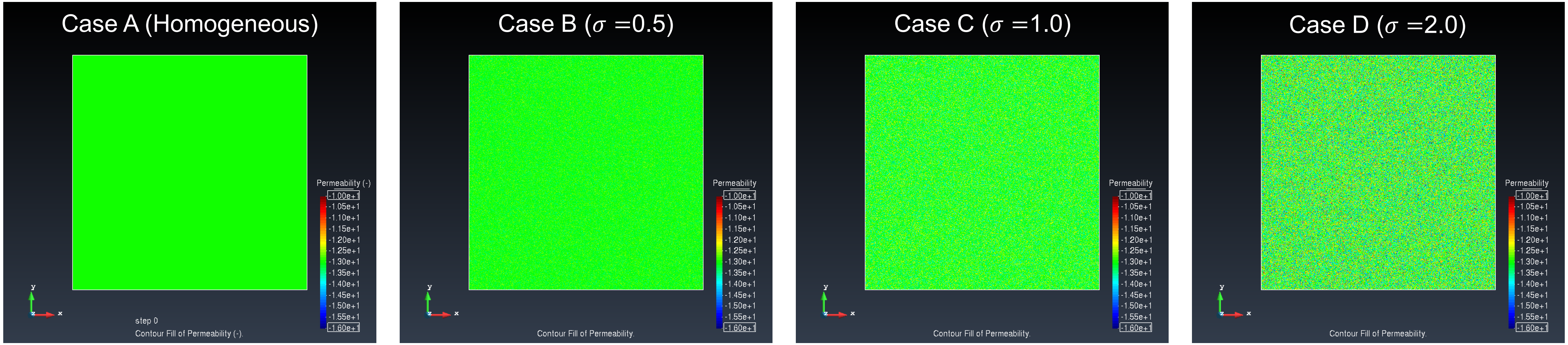
Case 3 (High injection rate)
$q_{inj} = 6 \times 10^{-3}$ m^3/s (t=4 s)



Numerical Experiment: Radial Acid Injection in Heterogeneous Rock

- Assuming that permeability is log-normally distributed in a rock, the following heterogeneous permeability fields were created with different widths of permeability distribution, σ.

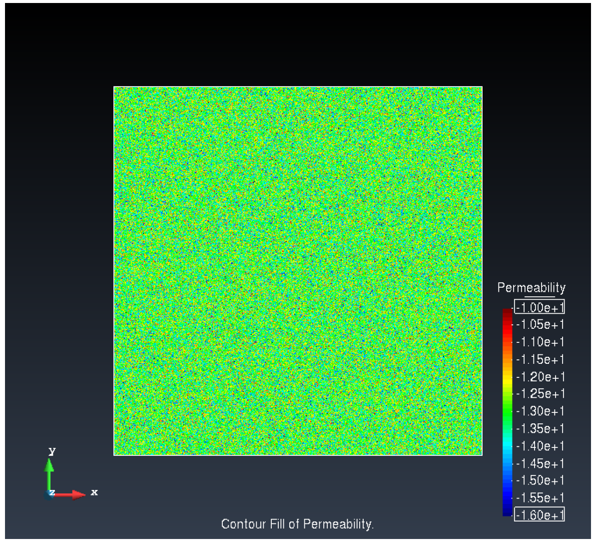




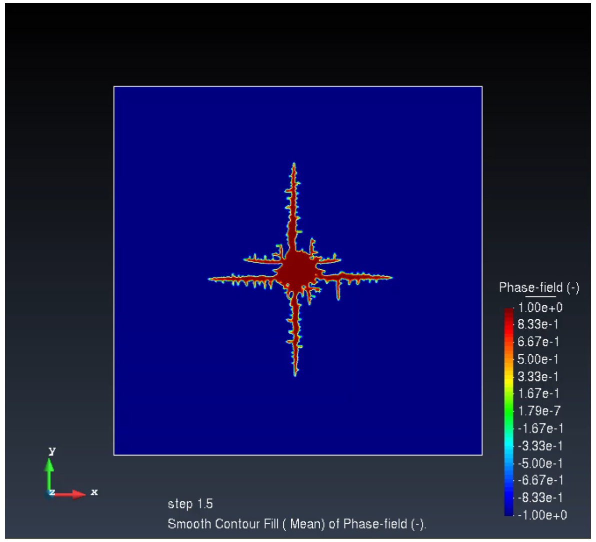
Numerical Experiment: Radial Acid Injection in Heterogeneous Rock

- **Example Acid Injection Simulation Results (Case D,σ =2.0)**
  - Injection Rate: $2.4 \times 10^{-2}\ \mathrm{m}^3/s$ (Constant rate)
  - Injection time: 1.5 s

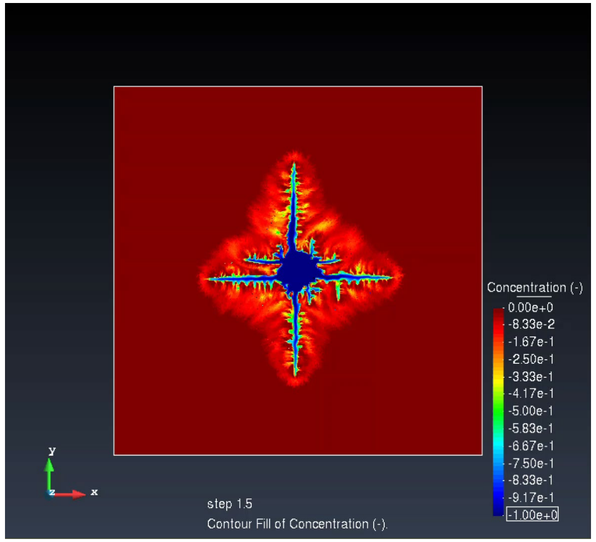
Permeability field



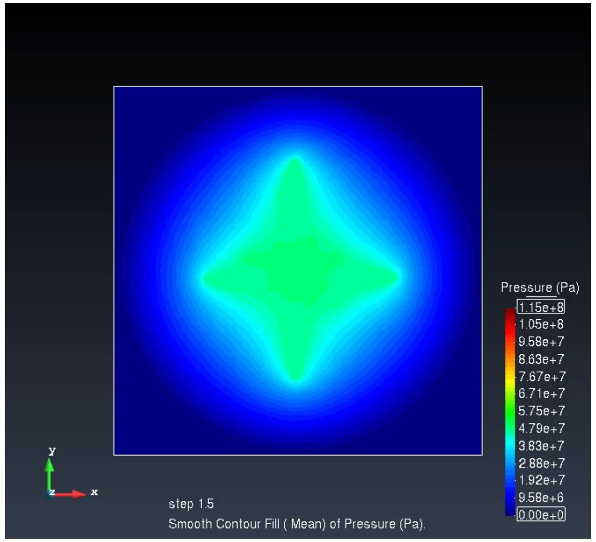
Wormhole structure (Phase-field value)



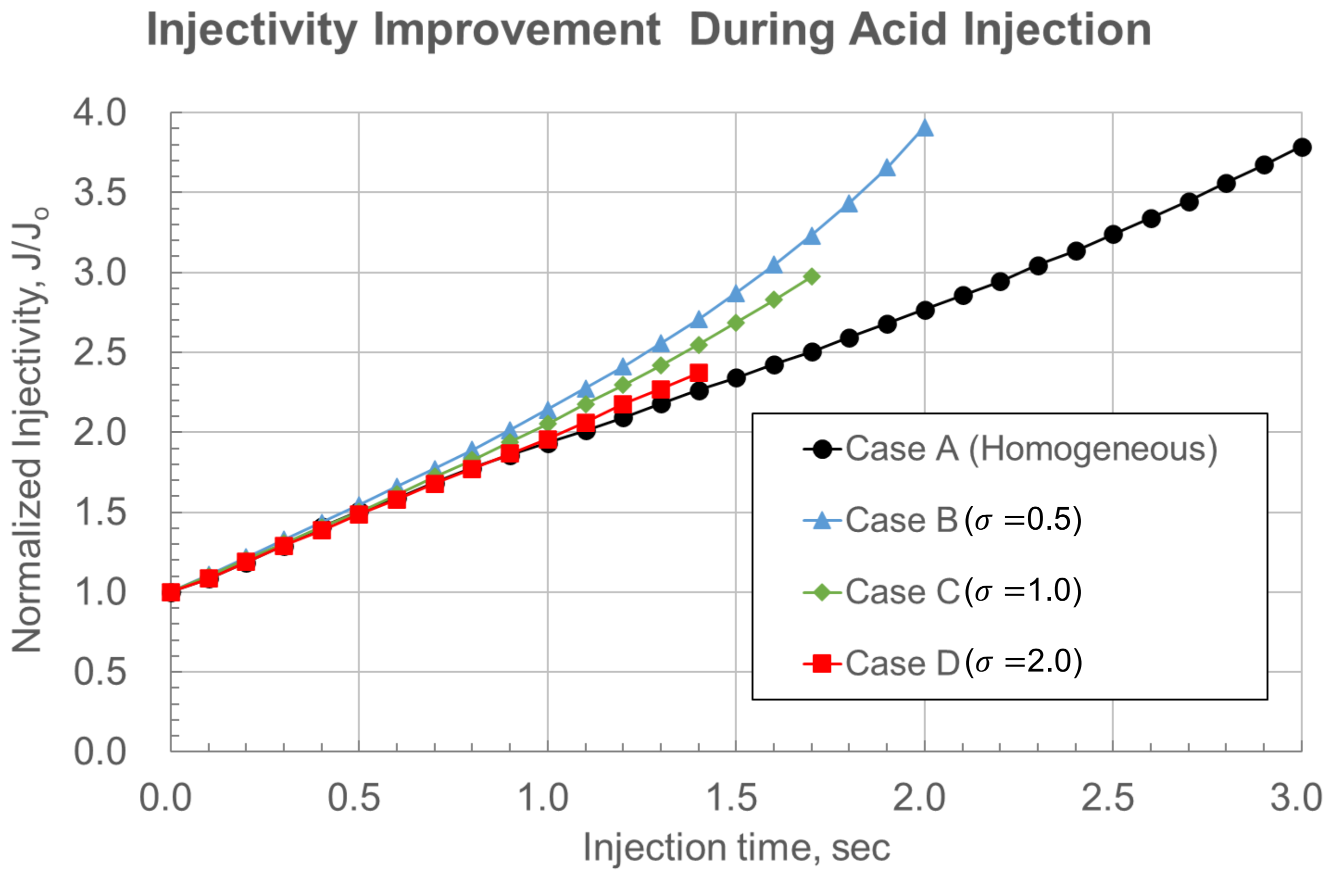
Acid concentration field

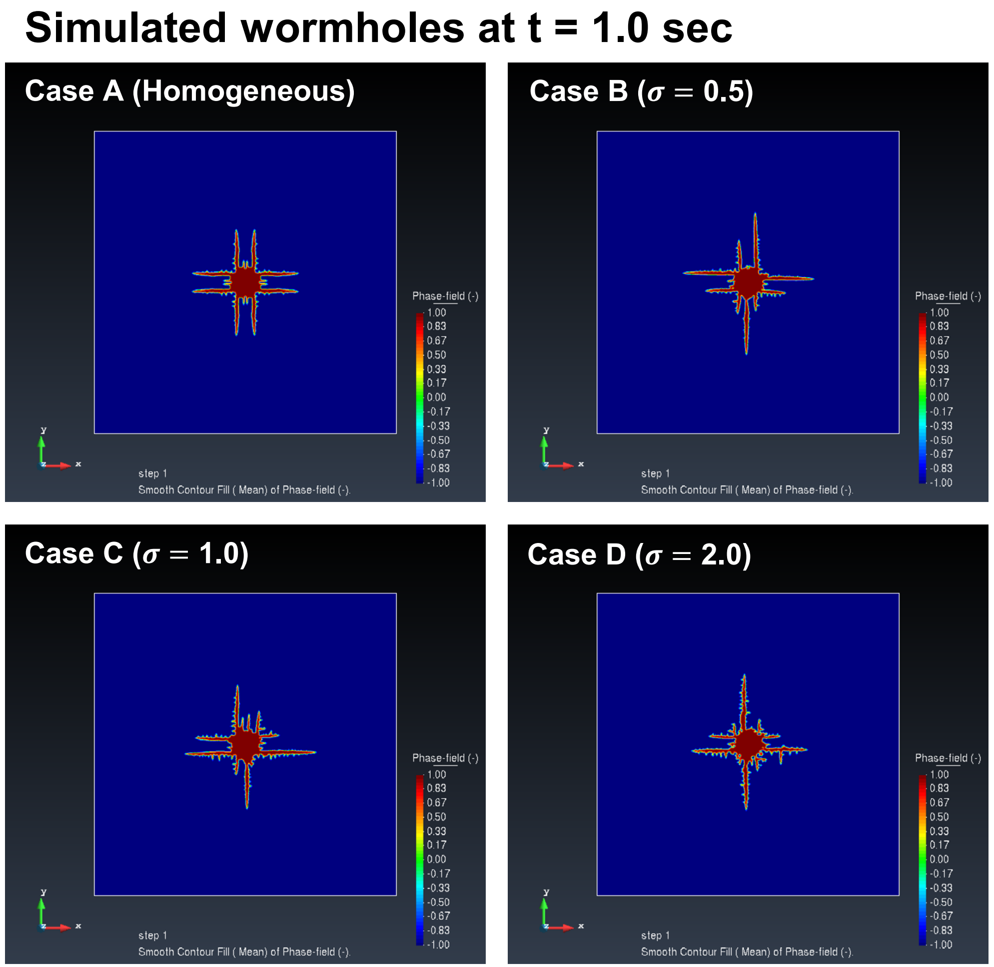


Pressure field



Numerical Experiment: Radial Acid Injection in Heterogeneous Rock





- The heterogeneous permeability fields cause better injectivity improvement during acid injection.
- The fold improvement in injectivity vary depending on the distribution of the permeability.

Conclusions

- The phase-field wormhole model was compared with the reaction infiltration instability solution presented by Chadam & Ortoleva (1990) for model verification.
- Wormholes were numerically simulated through injection of 28 wt% HCl under both homogeneous and heterogeneous permeability fields.
- Heterogeneous permeability fields localize the flow in high-permeability domains and enhance the splitting and branching of wormholes. The length of the dominant wormholes can be suppressed as an increasing amount of acid infiltrates into the branched wormholes.
- Our findings indicate that material heterogeneities should not be treated as a trigger for wormholes in the numerical simulation but as one of the parameters to control their nucleation and growth.

Acknowledgement

A part of the present work was performed as a part of activities of Research Institute of Sustainable Future Society, Waseda Research Institute for Science and Engineering, Waseda University. Also, this work has been funded through Experimental and Numerical Study of Wormhole Formation In Carbonate Acidizing research project by Japan Oil, Gas and Metals National Corporation (JOGMEC) being conducted collaboratively by the following research institute and universities.