



A numerical and experimental study of Galilei-transformed nonlinear wave groups

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› Nonlinear Schrödinger equation (NLSE)

$$i \left(\psi_x + \frac{1}{c_g} \psi_t \right) + \delta \psi_{tt} + \nu |\psi|^2 \psi = 0$$

$$c_g = \frac{\omega}{2k}, \delta = -\frac{1}{g}, \nu = -k^3, \omega = \sqrt{gk}$$

V. E. Zakharov, Stability of periodic waves of finite amplitude on the surface of a deep fluid. *Journal of Applied Mechanics and Technical Physics* 9(2), 190-194 (1968).

› Complex wave envelope ψ

$$\psi(x, t) \in \mathbb{C}$$

› Wave elevation η

$$\eta(x, t) = \text{Re}(\psi(x, t)e^{i(kx - \omega t)}) \in \mathbb{R}$$

› Sech envelope soliton – **stable**

$$\psi_S(x, t) = a \operatorname{sech} \left(-\sqrt{2} \alpha k c_g \left(t - \frac{x}{c_g} \right) \right) \exp \left(-\frac{i \alpha^2 k x}{2} \right)$$

H. C. Yuen and B. M. Lake, Nonlinear deep water waves: theory and experiment. *The Physics of Fluids* 18(8), 956-960 (1975).

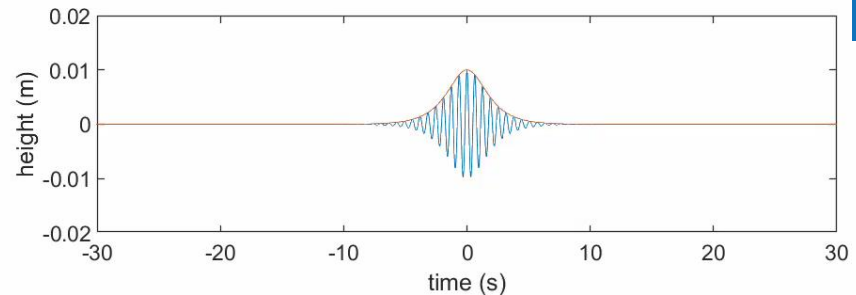
A. Slunyaev, G. F. Clauss, M. Klein and M. Onorato, Simulations and experiments of short intense envelope solitons of surface water waves. *Physics of Fluids* 25(6), 067105 (2013).

› Multi-soliton – **breathe**

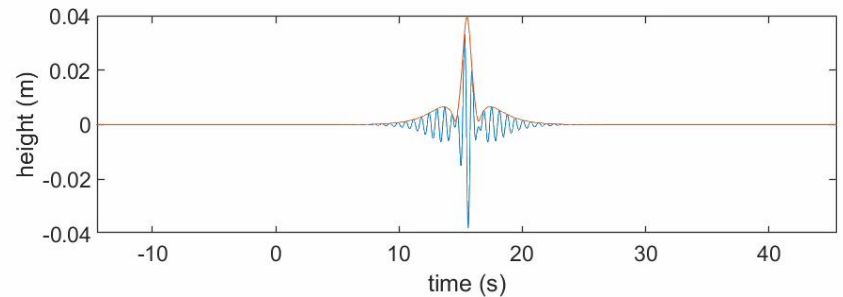
$$\psi_{MS}(x = 0, t) = N \psi_S(x = 0, t), N \in \mathbb{N} \text{ (initial condition)}$$

L. F. Mollenauer, R. H. Stolen and J. P. Gordon, Experimental observation of picosecond pulse narrowing and solitons in optical fibers. *Physical Review Letters* 45, 1095 (1980).

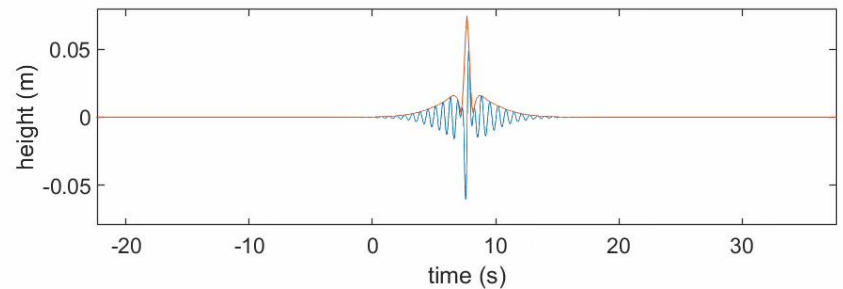
A. Chabchoub, N. Hoffmann, M. Onorato, G. Genty, J. M. Dudley and N. Akhmediev, Hydrodynamic supercontinuum. *Physical Review Letters* 111, 054104 (2013).



N=1



N=2



N=3

Fundamental properties of NLSE

✓ › NLSE amplitude scaling transformation

✓ › NLSE time reversal symmetry

A. Chabchoub and M. Fink, Time-reversal generation of
rogue waves, Physical Review Letters 112, 124101 (2014).

? › NLSE Galilean symmetry

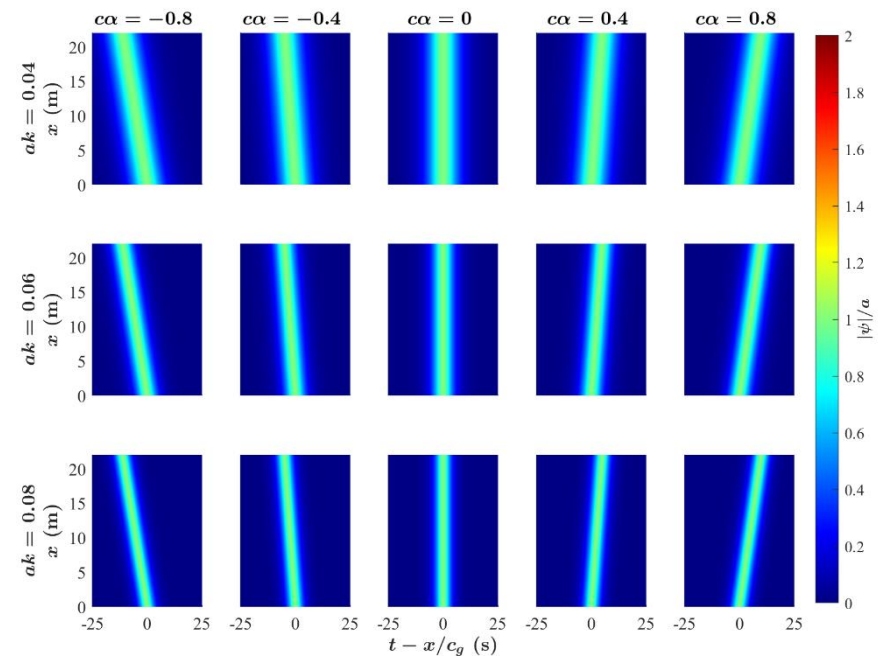
› Galilean transformation

$$\psi_G(x, t) = \psi \left(x, t - \frac{\sqrt{2}c\alpha}{4c_g}x \right) \exp \left(\frac{i\sqrt{2}c\alpha k(x - c_g t)}{2} + \frac{ic^2\alpha^2 kx}{8} \right)$$

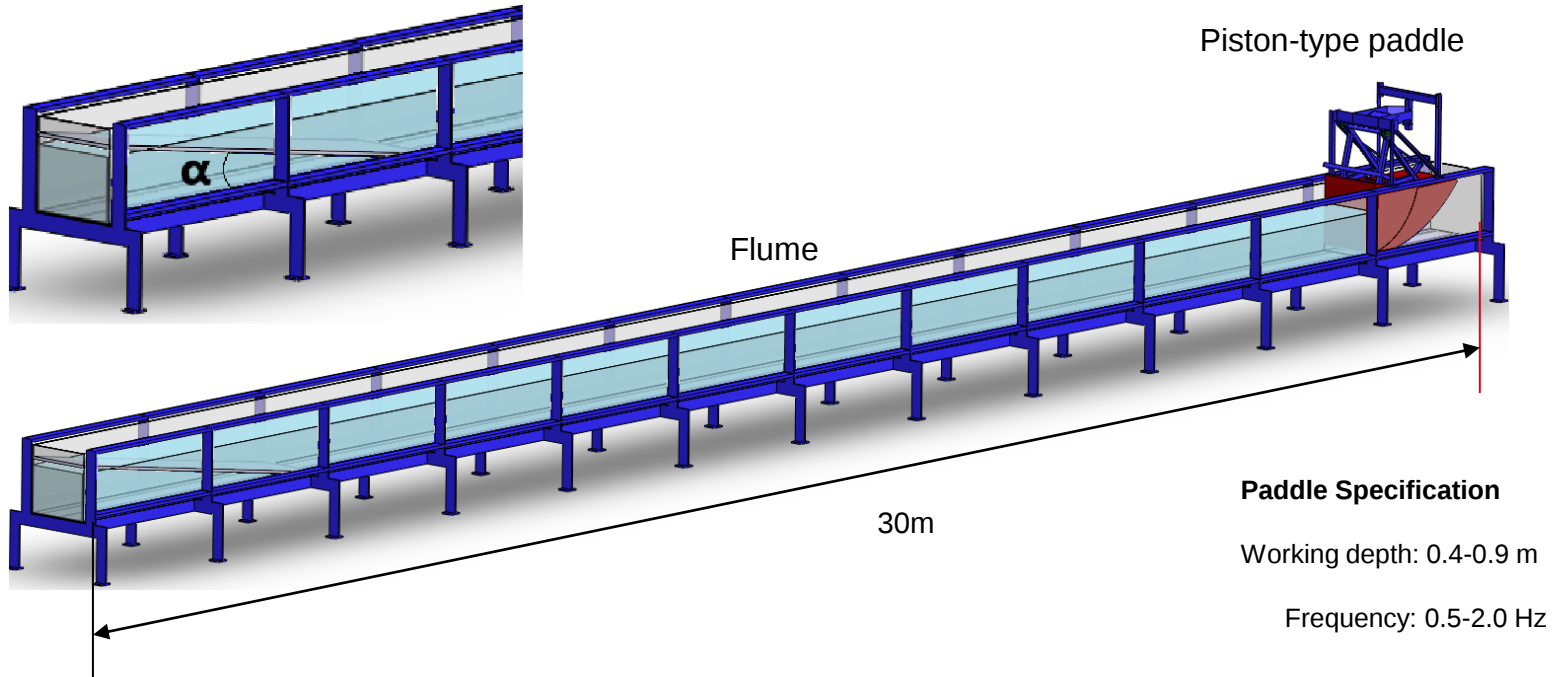
› Galilean-transformed soliton

$$\psi_{GS}(x, t) = a \operatorname{sech} \left(-\sqrt{2}\alpha k c_g \left(t - \frac{x}{c_g} \right) + \frac{c}{2}\alpha^2 kx \right) \exp \left(-\frac{i\alpha^2 kx}{2} \right) \exp \left(\frac{i\sqrt{2}c\alpha k(x - c_g t)}{2} + \frac{ic^2\alpha^2 kx}{8} \right)$$

Theory (NLSE)



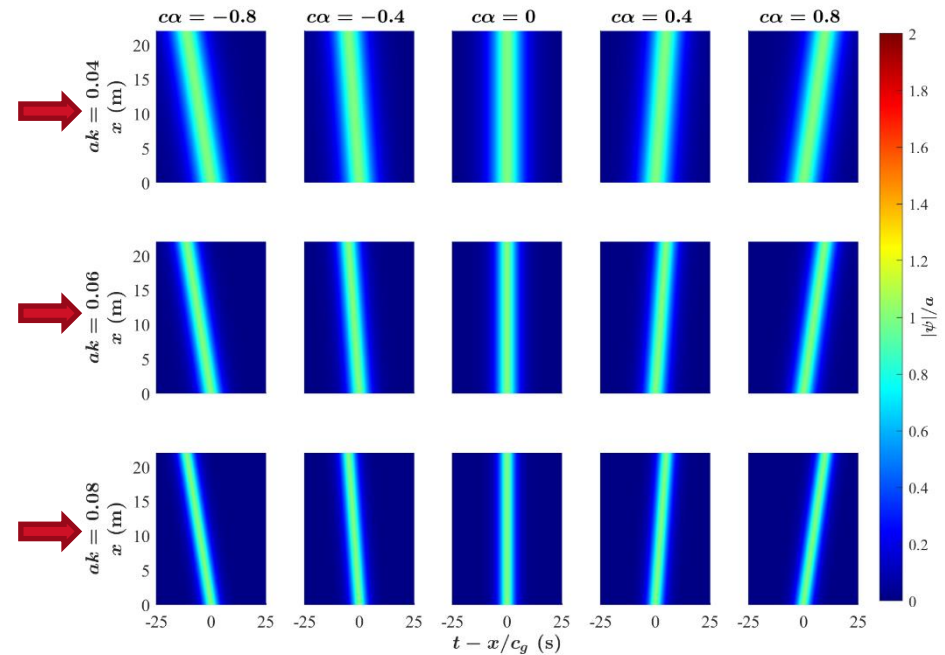
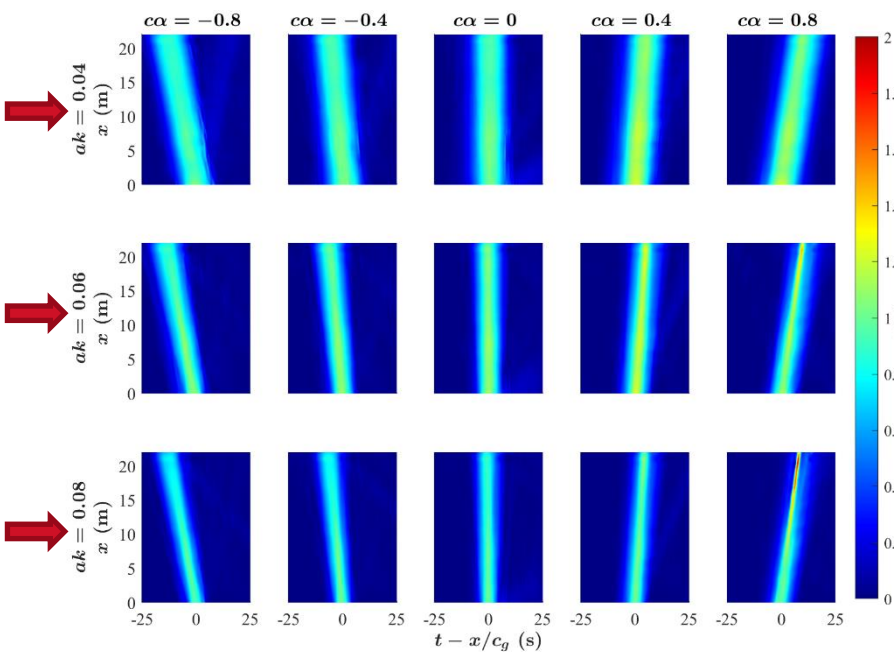
Absorptive beach
Reflectivity ~ 0 if $\alpha \approx 8^\circ$



Working frequency (for original envelope soliton waves): 1.5 Hz

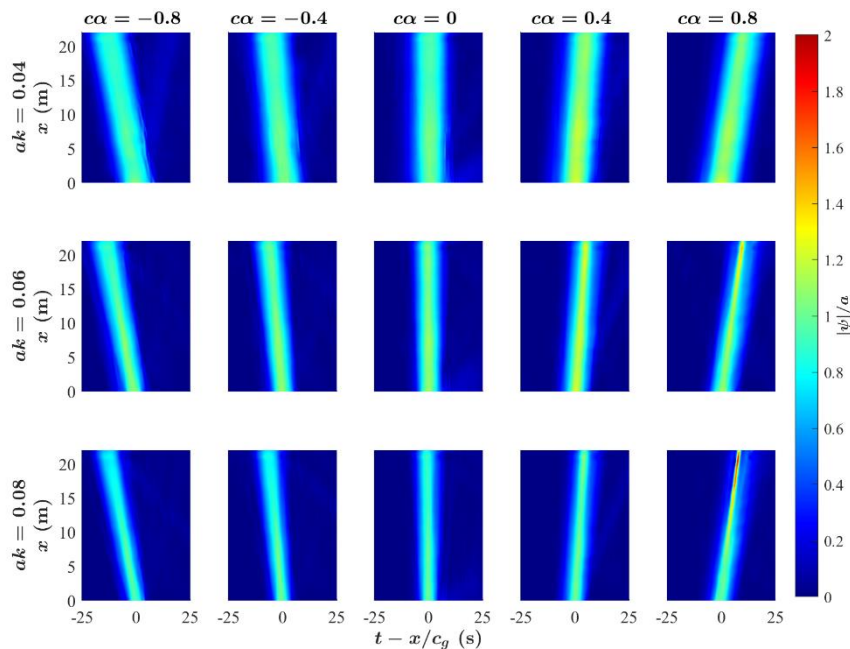
Experimental results

Theory (NLSE)

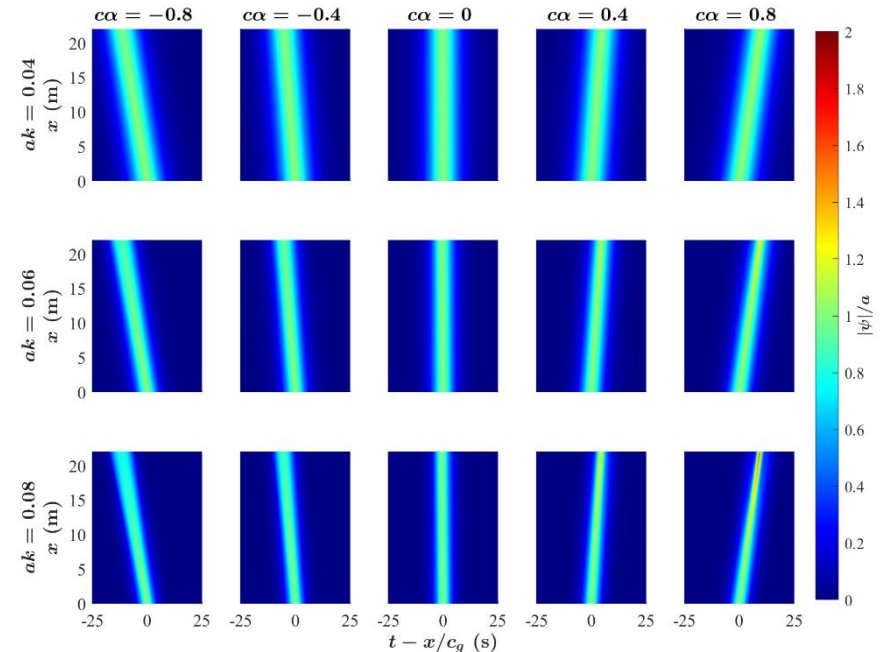


1. Higher-order NLSE modelling (MNLSE)

Experimental results



Higher-order model



E. Kit and L. Shemer, Spatial versions of the Zakharov and Dysthe evolution equations for deep-water gravity waves. *Journal of Fluid Mechanics*, 450, 201-205 (2002).

A. Goullet and W. Choi, A numerical and experimental study on the nonlinear evolution of long-crested irregular waves. *Physics of Fluids* 23, 016601 (2011).

2. Soliton order effect

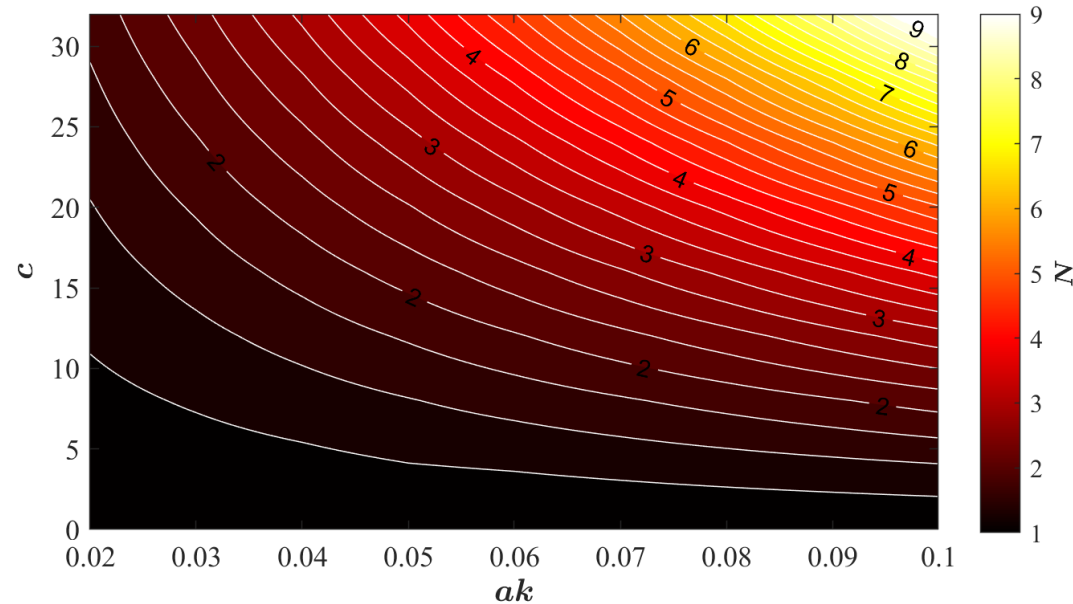
- In terms of initial condition, we let

$$\psi_{G,\text{sech}}(a, k, c, x = 0, t) = R\psi_{\text{sech}}(a', k', x = 0, t),$$

where R is the order factor, $a' = \frac{a}{R}$.

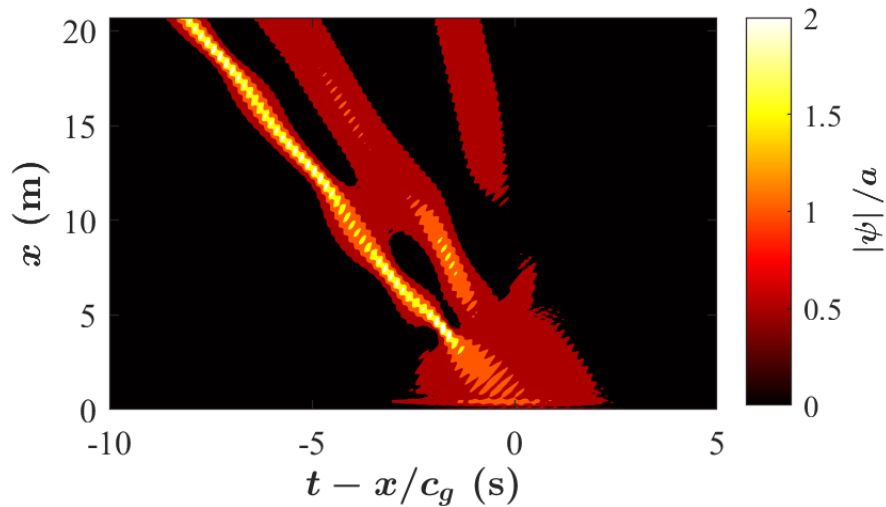
- It was derived analytically that

$$R = \left(1 + \frac{\sqrt{2}}{4} cak\right)^3$$

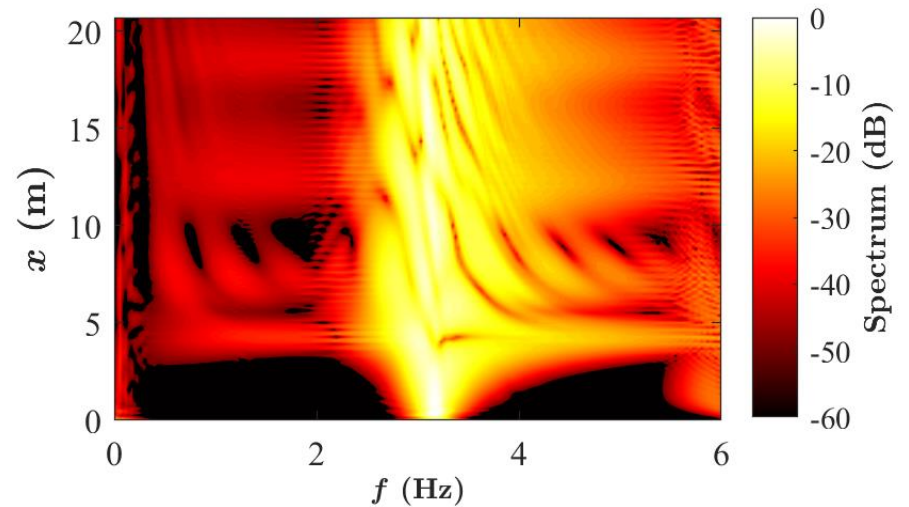


HOSM simulation of Galilean envelope soliton (Multi-soliton of order 4)

Envelope evolution



Spectrum evolution





› Summary

- At $\mathcal{O}(\epsilon^3)$ level the Galilean envelope solitons are steady;
- Experiments do not support the steadiness of the solution;
- The MNLSE shows good agreements with the experiments;
- Connection between Galilean-transformed soliton and multi-soliton dynamics;
- Soliton fission and supercontinuum as a result.

Thanks for voting! 😊

› Results are currently in press:



Physica D: Nonlinear Phenomena

Available online 19 May 2022, 133342

In Press, Journal Pre-proof



Galilean-transformed solitons and supercontinuum generation in dispersive media

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