Modelling Pore Size Distribution, Water Retention & Hydraulic Conductivity of Granular substrates using a Universal Multifractal-based approach for Nature Based Solutions

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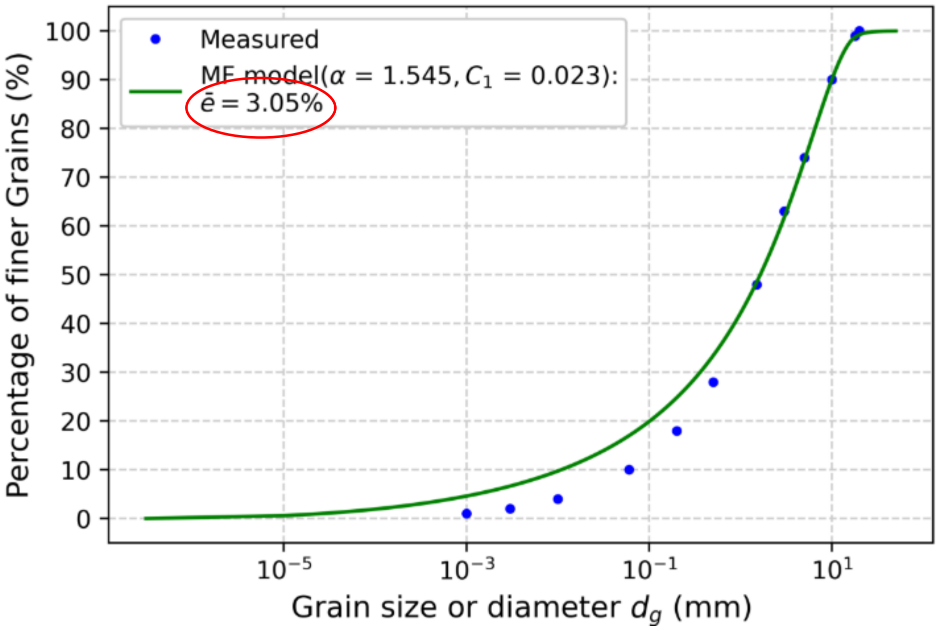
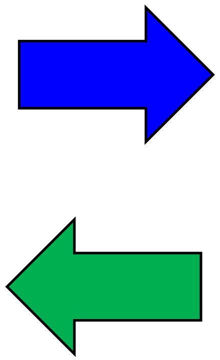
Research gap addressed

- Grain Size Distribution (GSD) in porous media (used in Nature Based Solutions - NBS) → highly heterogeneous
- Few recent studies (Stanic et al., 2021) use Universal Multifractal Framework (UMF) for GSD
- GSD and Pore Size Distribution (PSD) are similar
- **But still PSD is modeled in simplistic manner (Fractal-based model)**
- **Further ramifications for Water Retention (WR) and Hydraulic conductivity (HC)**
- **Even when GSD model is more accurate → finally simulated hydrological properties are not!**
- **New UM-based approach to model PSD, WR & HC is proposed**
- **And tested on 4 different Green Roof Substrates**

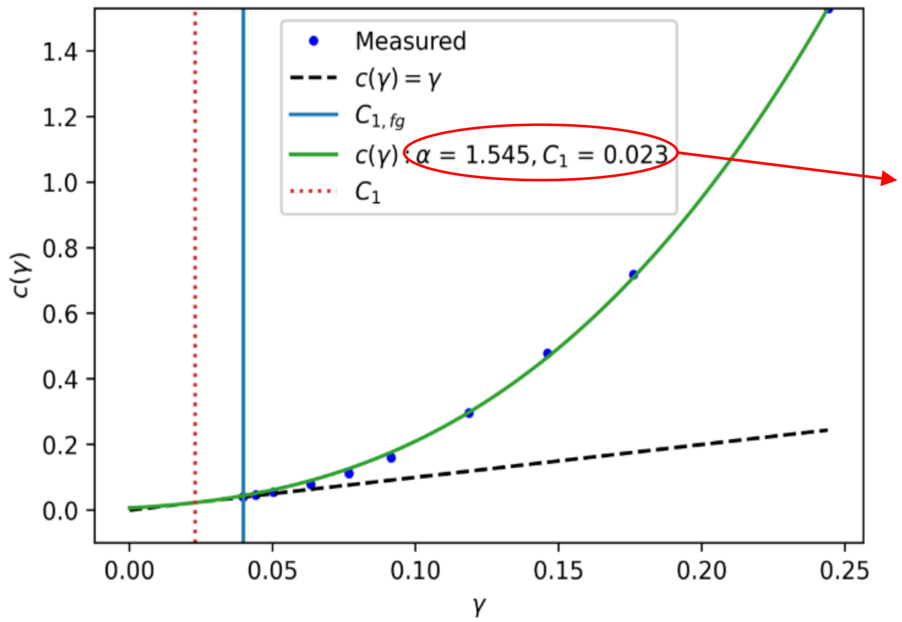
UM parameter estimation from measured GSD

Classical Sieving on Green Wave Substrate (GWS) sample size L→ Measurements





UMP estimation (avoiding multifractal phase transitions)



Close to density field's UMPs

$C_1 = \gamma_{min}$

Grain Size Distribution (GSD) model

$$\Pr\left(d_g < d_{g,s}\right) = 1 - \frac{\left(\frac{L}{d_{g,s}}\right)^{-c\left(\frac{\log\left(\frac{1}{1-\phi}\right)}{\log\left(\frac{L}{d_{g,s}}\right)}\right)}}{1-\phi};$$

Grain diameter/size

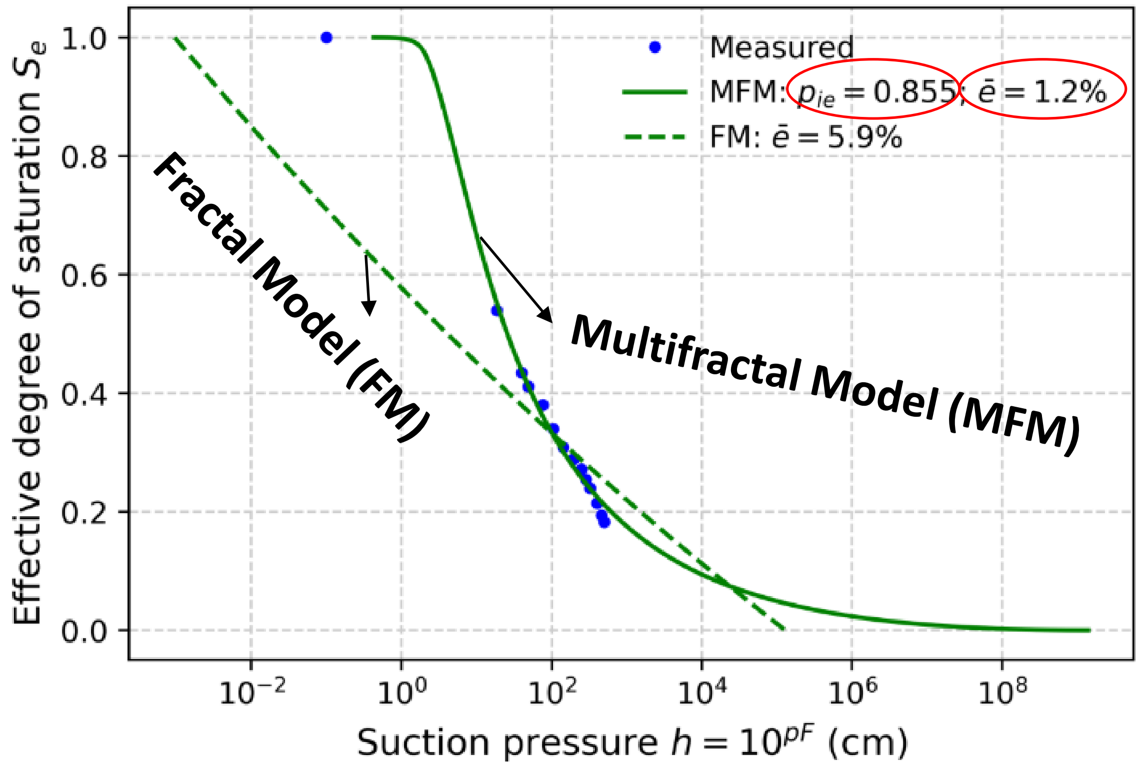
porosity

Codimension of singularities

$$c(\gamma) = \begin{cases} C_1\left(\frac{\gamma(\alpha-1)}{C_1\alpha} + \frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}; \alpha \neq 1 \\ C_1 e^{\left(\frac{\gamma}{C_1}-1\right)};\ \alpha = 1 \end{cases}$$

Pore Size Distribution (PSD) & Water Retention (WR)

Grain Size Distribution ↔ Pore Size Distribution → Capillary pores → Water Retention



WR model

$$\Pr(d_p < d_{p,c,s}) = 1 - \frac{\left(\frac{d_{p,c,max}}{d_{p,c,s}}\right)^{-c\left(\frac{\log\left(\frac{1}{1-(\theta_s-\theta_r)}\right)}{\log\left(\frac{d_{p,c,max}}{d_{p,c,s}}\right)}\right)}}{1-(\theta_s-\theta_r)}$$

Pore size

Capillary relevant Pore size

Saturated water content

Residual water content

$$S_e(h) = 1 - \frac{\left(\frac{h}{h_a}\right)^{-c\left(\frac{\log\left(\frac{1}{1-(\theta_s-\theta_r)}\right)}{\log\left(\left(\frac{h}{h_a}\right)^{p_{ie}}\right)}\right)}}{1-(\theta_s-\theta_r)}$$

(p_{ie})

Better water retention (at smaller suctions) ← **Steepness of GSD/PSD curve (smaller range of sizes)** ← **Ink-bottle effect (smaller p_{ie})**

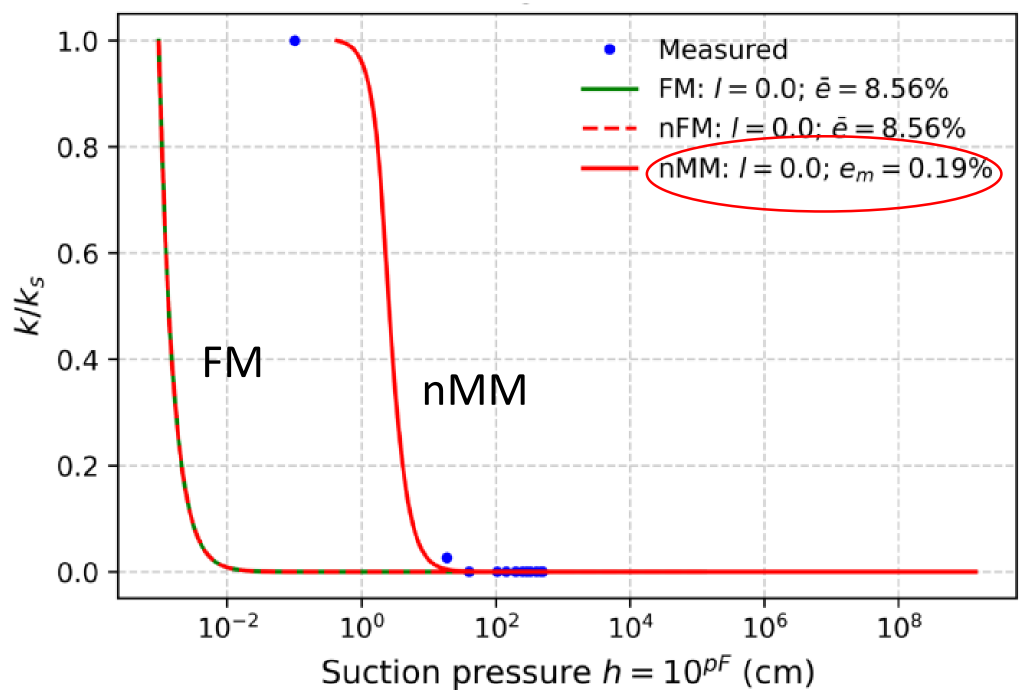
Hydraulic Conductivity

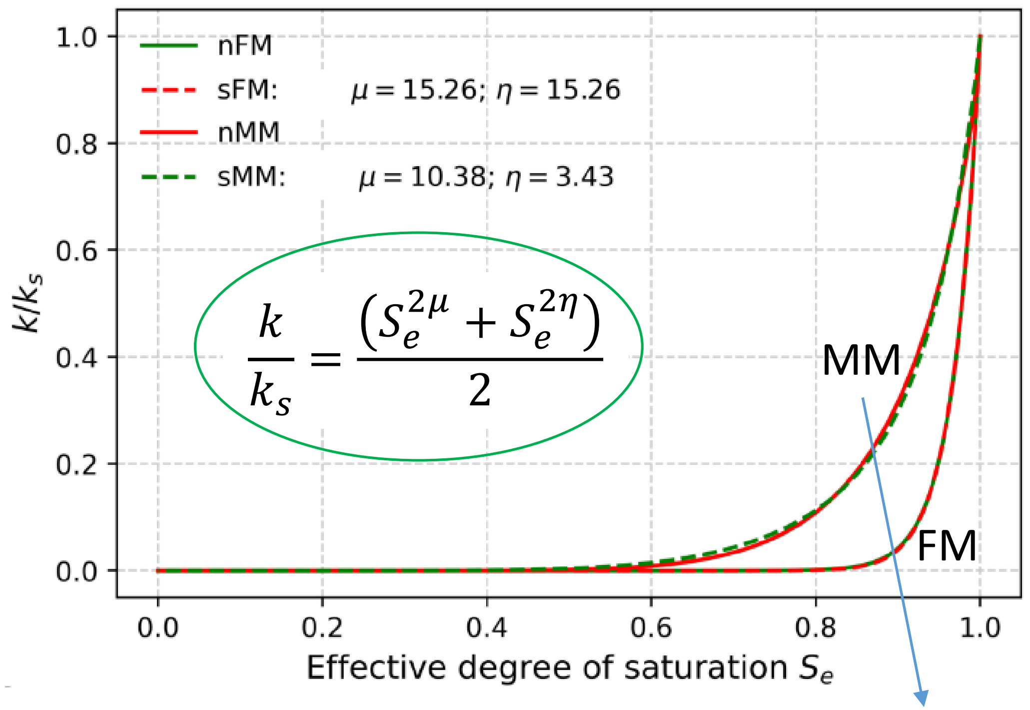
numerical Fractal Model (nFM)
numerical Multifractal Model (nMM)

l → pore tortuosity parameter

scaling law from Fractal Model (sFM)
scaling law from Multifractal Model (sMM)

More complex shape → one scaling exponent insufficient!





Higher conductivity at higher water contents

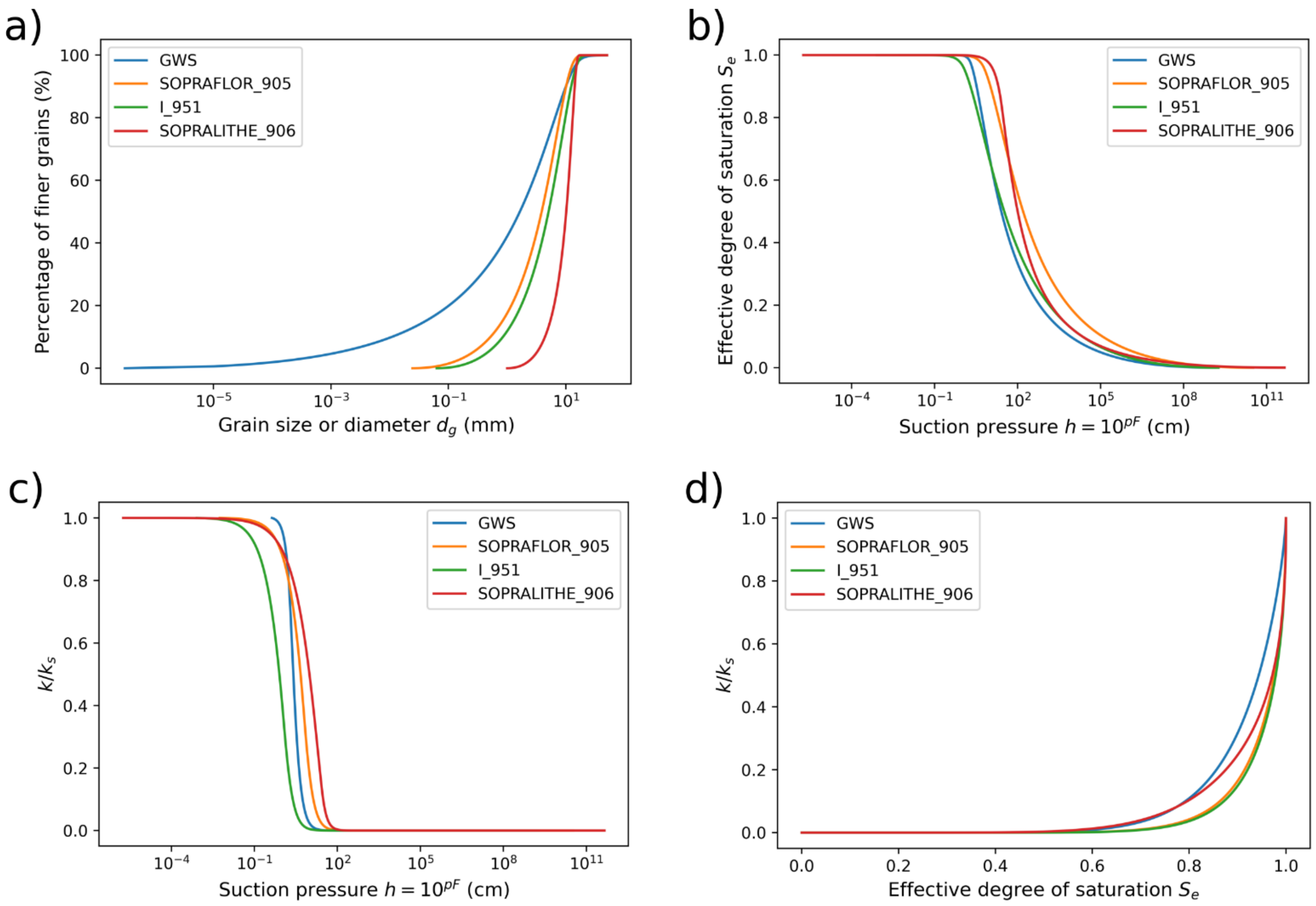
Mualem Model (numerical integration) →

$$k(u) = k_s S_e^l \left(\frac{\sum_{j=1}^{u} \left(\frac{h(j)^{-1} + h(j+1)^{-1}}{2} \right) |S_e(j) - S_e(j+1)|}{\sum_{j=1}^{N-1} \left(\frac{h(j)^{-1} + h(j+1)^{-1}}{2} \right) |S_e(j) - S_e(j+1)|} \right)^2 \quad \forall\, 1 \le u \le N-1$$

Conclusions

4 Different substrates

$\alpha > \alpha > \alpha > \alpha \,; C_1 < C_1 < C_1 < C_1 \;\Rightarrow p_{ie} > p_{ie} > p_{ie} > p_{ie}$



- Density field UMPs → GSD field UMPs
- UMP estimation from GSD (avoiding multifractal phase transitions)
- **PSD similar to GSD**
- **New UM-based approach to model PSD, WR & HC**
- **New parameter p_{ie} → Ink-bottle effect**
- **Total hydrological behavior →UMPs**
- **Improvement from earlier fractal-based approximations**
- **No explicit consideration → internal pores (complex grain shapes) & stratification → p_{ie} considers these implicitly**
- p_{ie} dependent on steepness of GSD → but still curve fit estimate → explicit function of α, C_1 → future scope…