# Quantifying the electron scattering by electrostatic fluctuations in the Earth's bow shock

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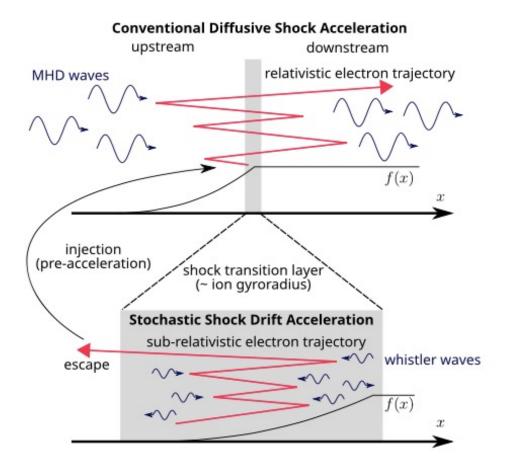
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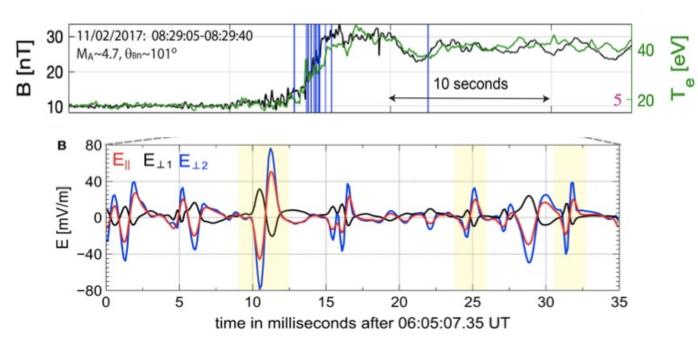
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#### T. Amano et al. 2020



Stochastic Shock Drift Acceleration(SSDA) – needs wave turbulence

#### I.Y. Vasko et al. 2020



Electrostatic component of the Bow-shock turbuelnce is dominated by ion-holes (IHs).

For what energies can IHs maitain SSDA?

Turbulence of whistlers is sufficient for W > 1 keV

## Estimations along unperturbed orbits

#### Electron equations of motion

$$egin{aligned} \dot{p}_{||} &= \partial_z \Phi \quad oldsymbol{\dot{p}}_{\perp} &= 
abla_{\perp} \Phi - \omega_{ce} \; (oldsymbol{p}_{\perp} imes oldsymbol{z}), \ \Phi &= \Phi_0 exp \left[ -(z cos heta + y sin heta - V_s t)^2 / 2 l^2 
ight] \end{aligned}$$



Integrate it over unperturbed trajectories

$$z = z_0 + V_{||}t$$

$$y = y_0 + \rho_L \cos(\omega_{ce} t - \varphi),$$

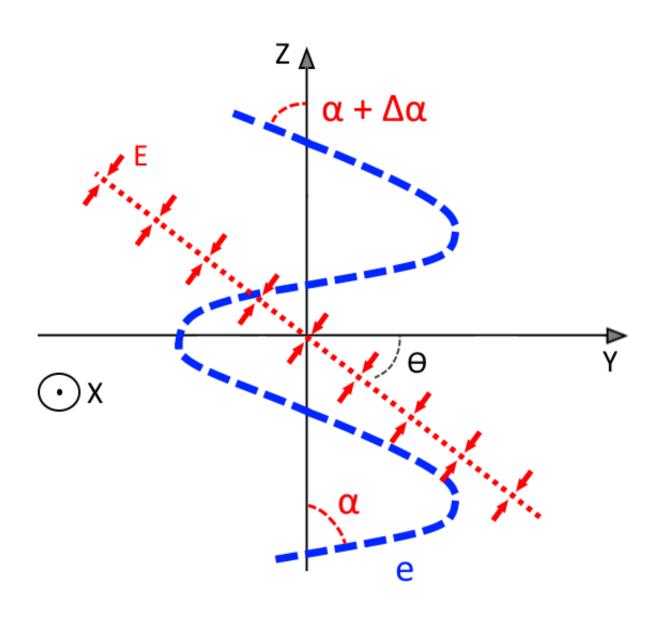
$$x = x_0 - \rho_L \sin(\omega_{ce} t - \varphi)$$

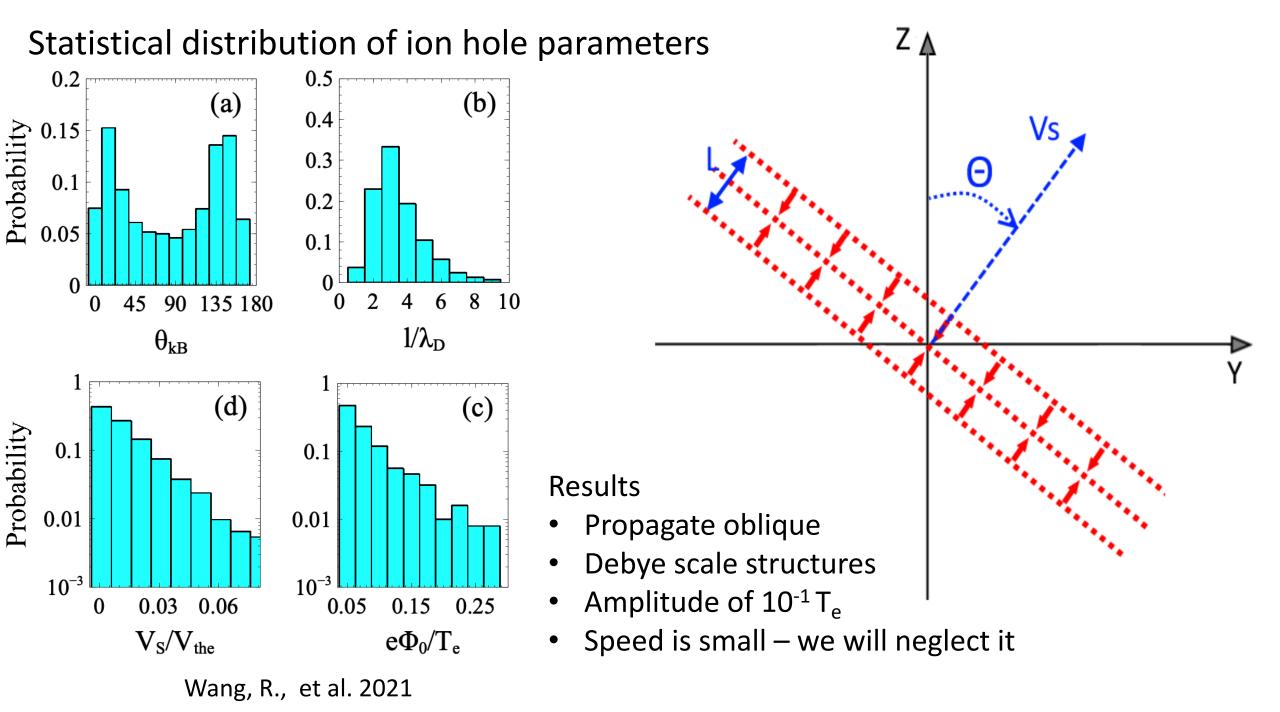


Assuming uniformly distributed phases  $0 < \phi < 2\pi$ 

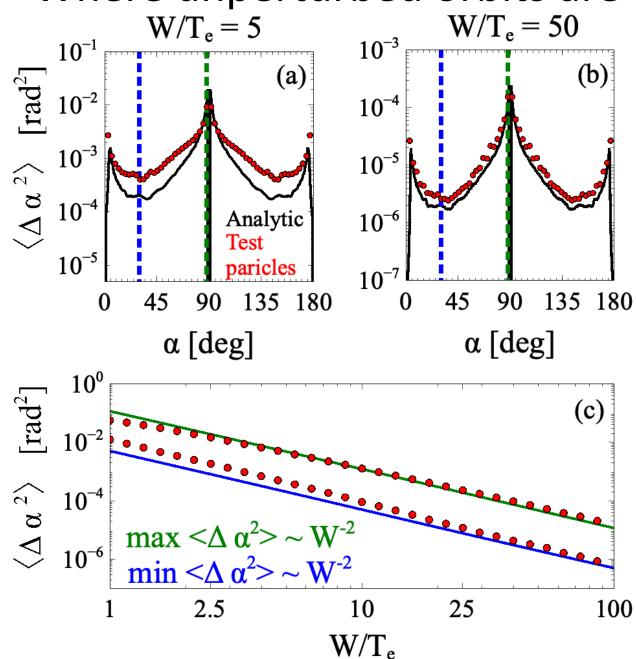
$$\langle \Delta \alpha^2 \rangle_{\varphi} = \frac{4\pi \Phi_0^2}{W^2 \sin^2(2\alpha)} \frac{\omega_{ce}^2 \ l^2 \ (V_s \cos^2 \alpha - V_{||} \cos \theta)^2}{(V_s - V_{||} \cos \theta)^4} \times$$

$$\times \sum_{n=1}^{\infty} n^2 J_n^2 \left( \frac{n V_{\perp} \sin \theta}{V_s - V_{||} \cos \theta} \right) \exp \left[ -\frac{n^2 \omega_{ce}^2 \ l^2}{(V_s - V_{||} \cos \theta)^2} \right]$$





# Where unperturbed orbits are valid?



$$<(\Delta\alpha)^2>=\int f(x)<(\Delta\alpha)^2>^{(x)}dx$$

- Analytic expression
- Can be taken from
  - Test-particles

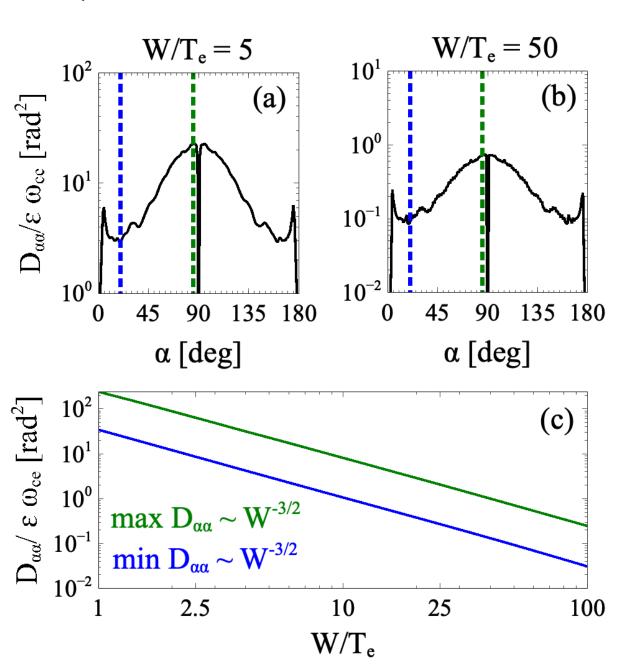
Integration variable is  $x = (L, \phi, \Theta)$ , probability function f(x) is chosen according to the data-driven distributions

#### Results

- Underestimates for W < 10 T<sub>e</sub>
- Works well for W > 10 T<sub>e</sub>
- Although generally yields a correct estimations on all energies

## Quasi-linear Diffusion coefficient

### Quasi-linear results



$$D_{\alpha\alpha} = \varepsilon \omega_{ce} \frac{l^3}{w^2 sin^2(\alpha)} \frac{\sqrt{\pi}}{2} \frac{\omega_{ce}}{\omega_{pe}} \frac{[v_s cos(\alpha) - \sqrt{w} cos(\theta)]^2}{[v_s - \sqrt{w} cos(\alpha) cos(\theta)]^3} \times \sum_{n=1}^{n=+\infty} n^2 J_n^2 \left( \frac{n\sqrt{w} sin(\alpha) sin(\theta)}{v_s - \sqrt{w} cos(\alpha) cos(\theta)} \right) \times \exp \left( -\frac{\omega_{ce}^2}{2\omega_{pe}^2} \frac{n^2 l^2}{[v_s - \sqrt{w} cos(\alpha) cos(\theta)]^2} \right)$$

$$\varepsilon = \frac{E_w^2}{4\pi N_e T_e} \approx 10^{-3}$$

$$\frac{\omega_{ce}}{\omega_{pe}} \approx \frac{1}{100}$$

Dimensionless of turbulence intensity

#### Results

- Diffusion coefficient depends on two main parameters Turbulence Intensity, Freq. ratio
- $D_{\alpha\alpha}$  decreases with energy as  $D_{\alpha\alpha} \propto W^{-3/2}$

# Estimations of the maximum energy gain

According to Amano et al. the maximum energy gain is given as follows

$$W_{max} \approx 6\eta \cdot \frac{m_e V_n^2}{2\cos^2 \theta_{Bn}} \frac{D_{\mu\mu}}{\omega_{ci}},$$

 $V_{n}$  - the upstream plasma flow velocity  $\theta_{\text{Bn}}$  - the angle between the upstream magnetic field and shock normal

This equation should be resolved with respect to  $W_{\text{max}}$ , in our particular case

$$D_{\alpha\alpha} \approx K \cdot \varepsilon \omega_{ce} \cdot (W/T_e)^{-3/2},$$

where K  $\approx$  100 and  $<\epsilon\omega_{ce}>$   $\approx$  10<sup>-2</sup>

It yields the following estimation of the maximum energy gain provided by ion-holes turbulence

## Results

- Scattering by ion holes has been considered in the framework of the QLT.
- Test particle simulations were employed in order to verify the applicability of the QLT
- We provide quasi-linear diffusion coefficients averaged over the statistical ensemble of ion holes with data-driven parameters
- Effectiveness of pitch-angle scattering in application to SSDA mechanism is estimated

$$\frac{W_{\text{max}}}{T_e} \approx \frac{2}{|\cos \theta_{Bn}|^{4/5}}, \quad W_{max} \approx 0.1 - 0.5 keV$$