# Bayesian Inference of Climate Parameters Using Multibox EBMs

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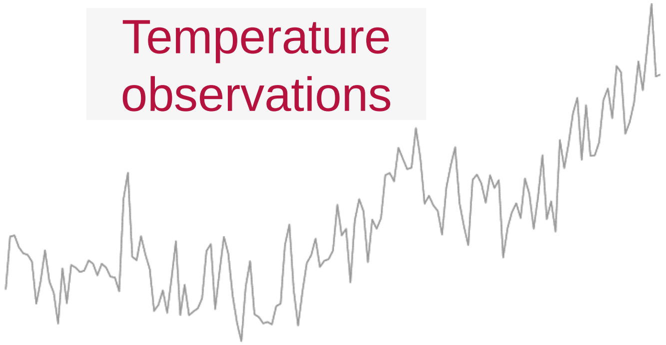
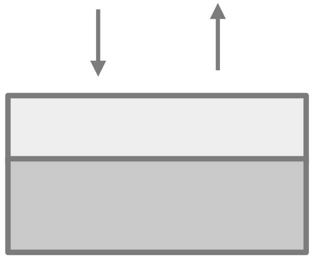
Temperature observations



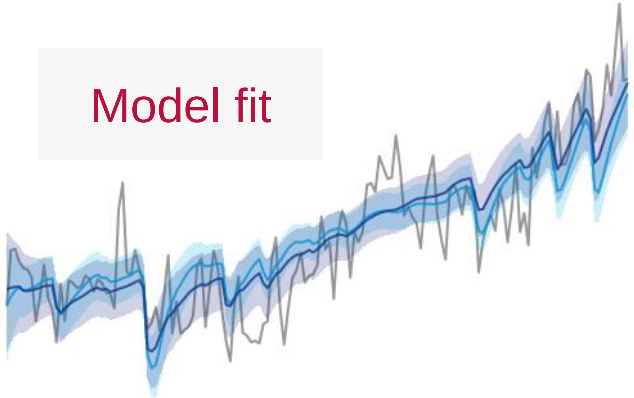


Multibox EBM

Bayesian Inference

ClimBayes

Model fit



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EGU | Session CL5.1.2 | Fri, 27 May 2022

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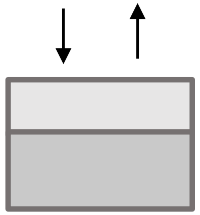
# Workflow of the ClimBayes package

## Linear stochastic two-box EBM

$$\boldsymbol{C}\,\frac{d\boldsymbol{T}}{dt}(t) = \boldsymbol{K}\,\boldsymbol{T}(t) + \boldsymbol{F}(t) + \boldsymbol{\xi}(t)$$



$$T(t) = \int_{-\infty}^{t} R(t-s)\,\Big(\underbrace{\frac{1}{C_1}F(s)ds}_{\text{forced}} + \underbrace{dW(s)}_{\text{internal}}\Big)$$

forced internal

$$R(t) = w_1 e^{-\lambda_1 t} + w_2 e^{-\lambda_2 t}$$

## Data

Temperature observations $Y$

Prior information on uncertain parameters

$$X = (\lambda_1, \lambda_2, w_1, T_0, F_0)$$

Deterministic forcing $F(t)$

## Bayesian Inference

Approximate posterior $p_{X|Y=y}(x)$
Via MCMC algorithm

ClimBayes

## Output

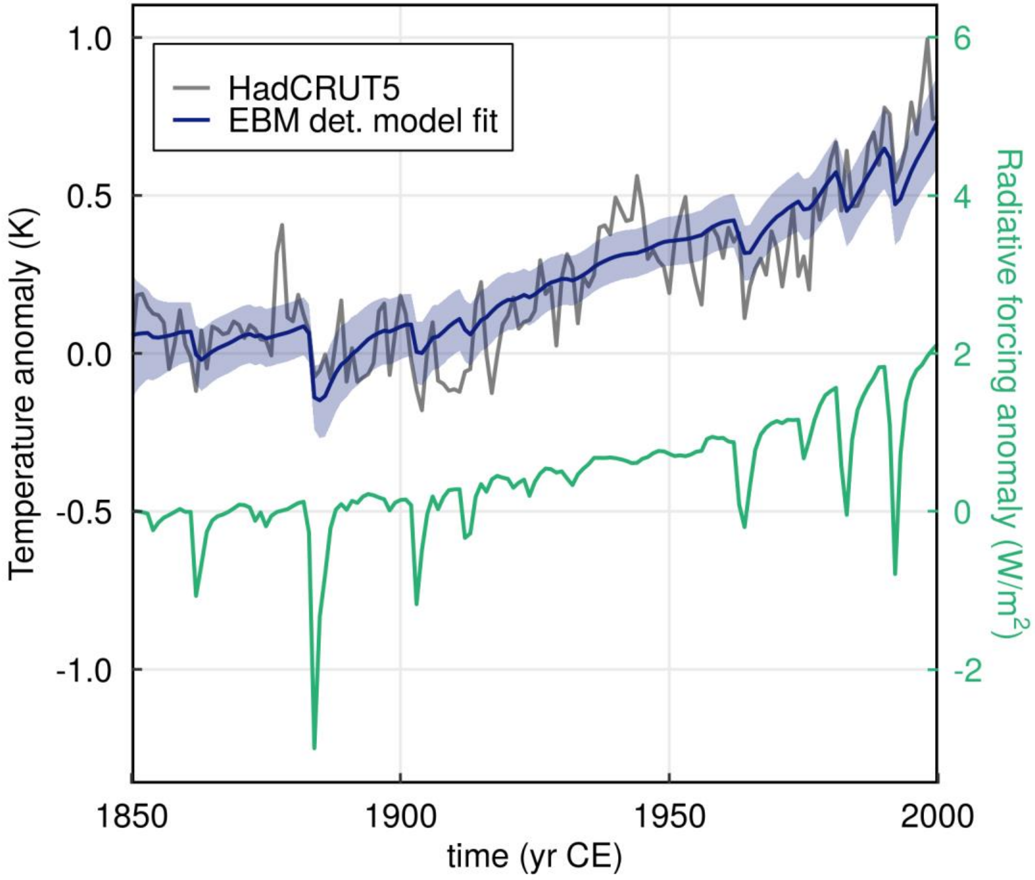
Posterior distributions for $X$

Best estimate of forced and internal temperature variations

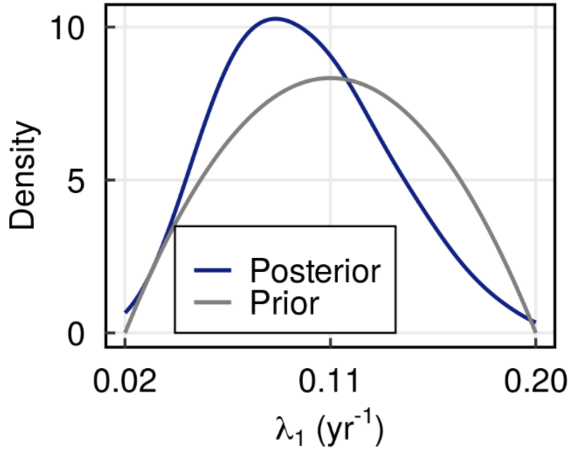
Figure adapted from Schillinger, M. et al. (to be submitted, 2022); Ref.: Geoffroy et al. 2013, Fredriksen and Rypdal 2017, Myrvoll-Nilsen et al. 2020

# Example: historical data

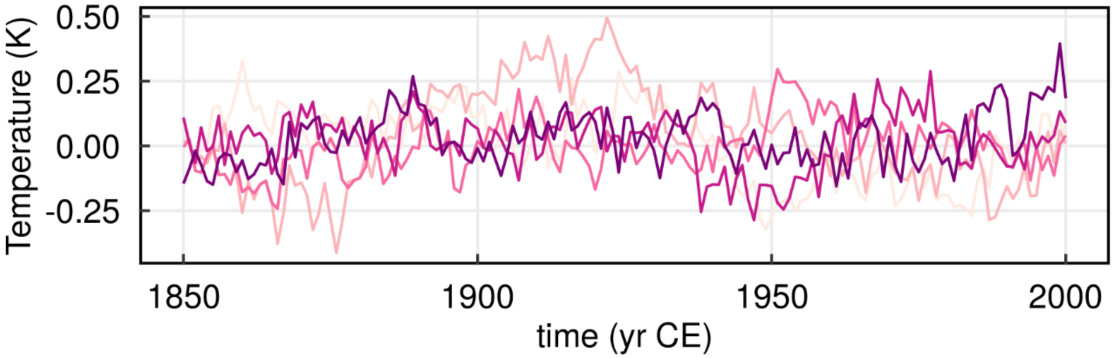
## Estimate of forced response




## Posteriors for $X$, e.g.




## Estimate of internal variability



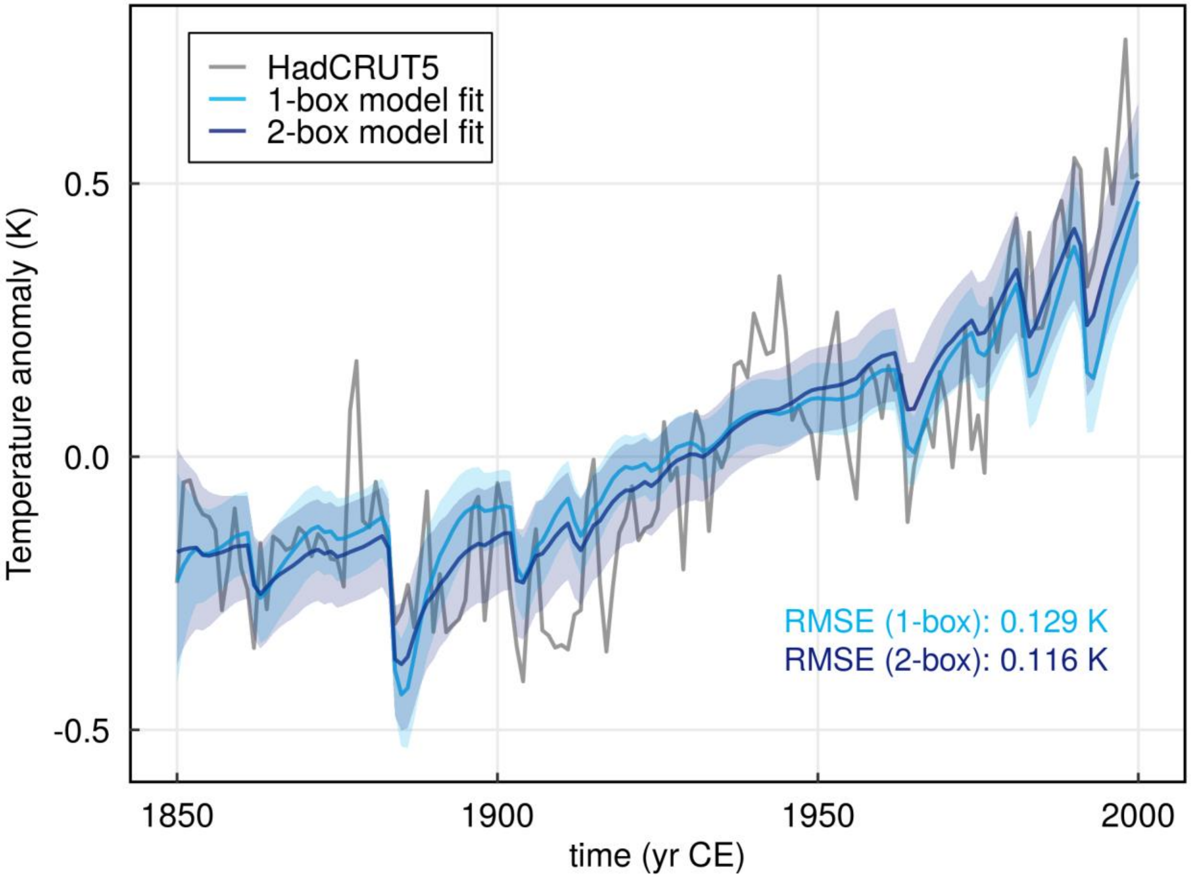

Data: Morice et al. 2021

# Possible applications

**Compare models with different number of boxes**

**Fit EBM to various data sources and time periods**

**Characterise sensitivity of estimators to noise**




Data: Morice et al. 2021

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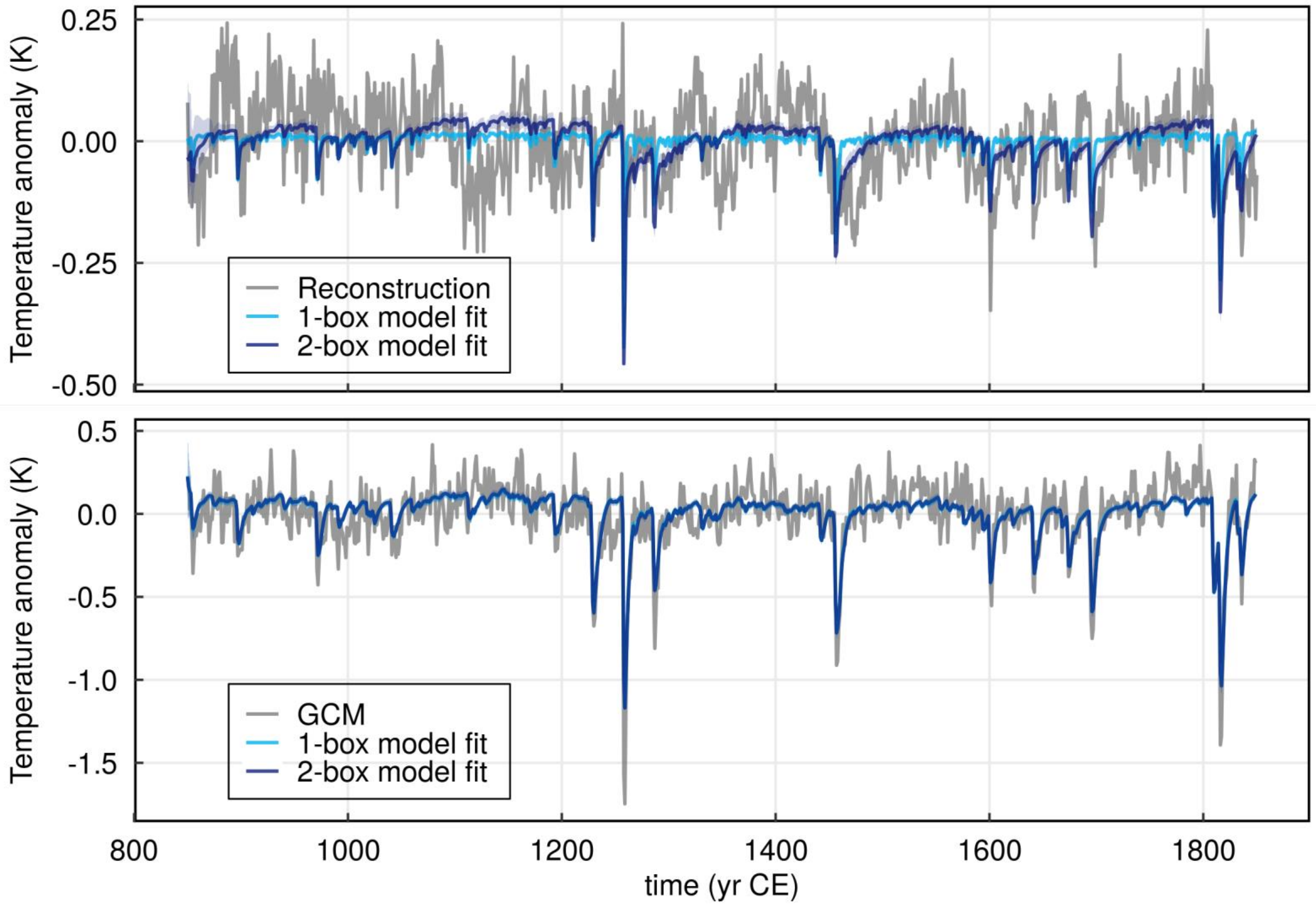



Data: Neukom et al. 2019, Giorgetta et al. 2013

# Possible applications

Compare models with different number of boxes

Fit EBM to various data sources and time periods

**Characterise sensitivity of estimators to noise**

**Question:** How do the parameter estimates depend on **noise** in the data?

Synthetic data generation

Two-box EBM + Random Noise

⇩

Compute relative errors in parameter estimates

# Separating forced from internal variability

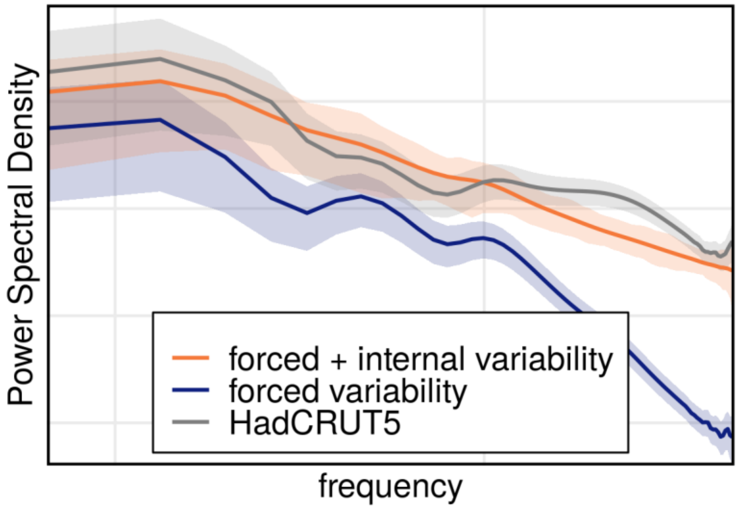
Linear stochastic EBM

$$T(t) = \int R(t-s)(\underbrace{F(s)ds}_{\text{forced}} + \underbrace{dW(s)}_{\text{internal}})$$

→ Bayesian Inference → 

Output

Best estimate of forced and internal temperature variations




→ Spectral analysis →

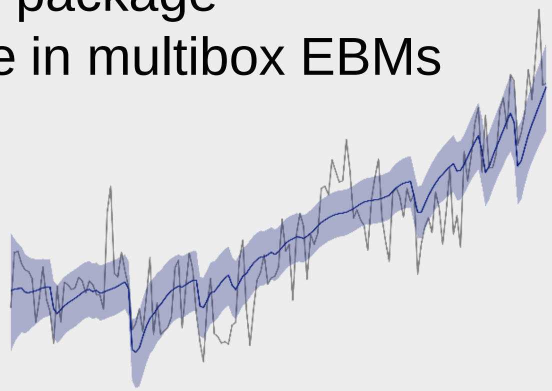
Separate forced from internal variability in temperature data on different timescales

Schillinger, M. et al. (to be submitted, 2022)

# Conclusions and Outlook

Easy-to-use & flexible package
for Bayesian inference in multibox EBMs

ClimBayes

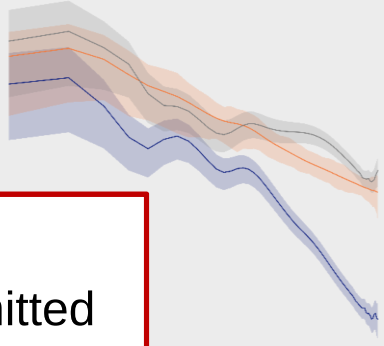


Release 01.07.2022
https://github.com/paleovar/ClimBayes

Separating forced from internal variability for…

… model simulations for last millennium
… along the model hierarchy

→ Variability on different time scales



Paper to be submitted

**Any Questions?**

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