Using complex networks to predict abrupt changes in the dynamics of coupled oscillators

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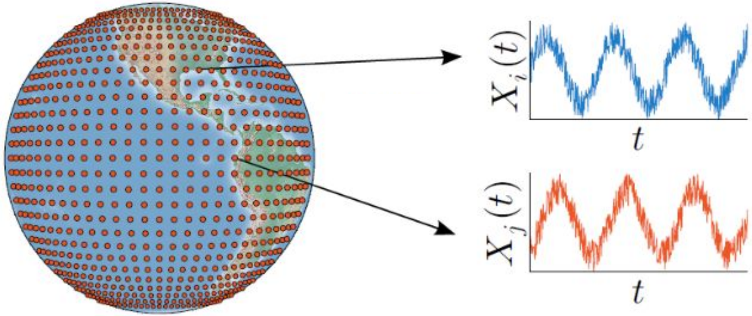
UNIT OF EXCELLENCE MARÍA DE MAEZTU

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 813844.

CAFE Climate Advanced Forecasting of sub-seasonal Extremes

Background: the climate network idea



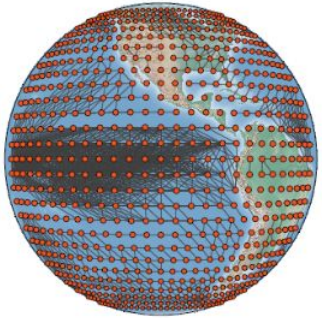
Similarity measure

Eg. Pearson correlation

$$C = \begin{pmatrix} C_{11} & C_{12} & \dots & C_{1N} \\ C_{21} & C_{22} & \dots & C2N \\ \vdots & \vdots & C_{ij} & \vdots \\ C_{N1} & C_{N2} & \dots & C_{NN} \end{pmatrix}$$

Thresholding

$$\begin{pmatrix} 1 & 1 & \dots & 0 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & 1 & \vdots \\ 0 & 1 & \dots & 1 \end{pmatrix}$$



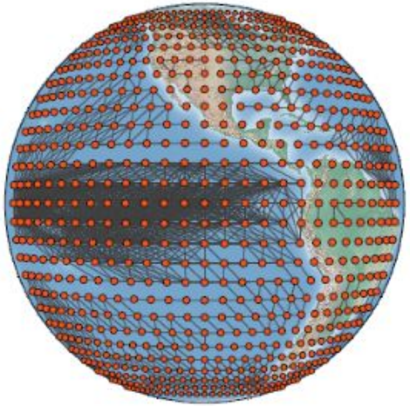
X is a spatio-temporal field, corresponding to observations of a geophysical variable (eg SST).

Network **nodes** ↔ spatial locations

Nodes are **linked** based on the degree of statistical interdependence between the time series of X at the corresponding sites.

Figure parts from [1]

Background: the climate network idea



Changes in the network's topology
⟺
changes in the system's dynamics.

Figure parts from [1]

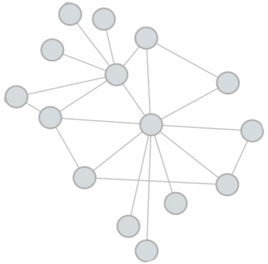
Motivations

Recently several studies have successfully used network- based frameworks to describe ENSO[2,3,4].

In particular, *V. Rodríguez-Méndez et al.*[2] constructed a network describing the ***correlation*** of ***SSTs*** for the area shown on the right (top panel).

Percolation metrics, e.g. the ***size of the network's largest connected component*** (S_1), were used to monitor changes in the network's structure.

They showed that S_1 (bottom panel) increases before every El Niño/La Niña event.



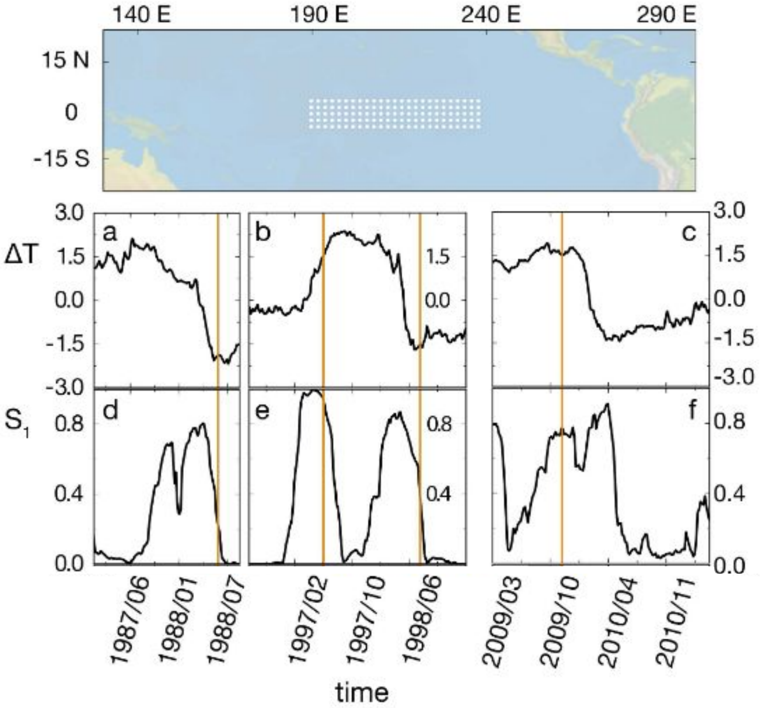


Figure from Rodríguez-Méndez et al.[2]

This result is not only interesting from a forecasting perspective but also from methodological perspective. So far, the study of percolation in correlation networks has mainly focused on the case of ***critical transitions***[1]. (And it has been found that the percolations metrics provide - sometimes very - early warning signals of upcoming transitions.)
The success of the applications of the percolation measures to the prediction of ENSO, motivates us to “go beyond” the case of critical transitions.

Objective

Better understand the behavior of the percolation metrics of correlation networks in systems presenting ***irregular oscillatory behaviors***.

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Objective

Better understand the behavior of the percolation metrics of correlation networks in systems presenting ***irregular oscillatory behaviors***.

We study the percolation properties of correlation networks in two systems presenting oscillations

- System 1: N coupled Fitzhugh-Nagumo oscillators
- System 2: N coupled recharge-oscillator models for ENSO (Jin 1997)

System 1: coupled Fitzhugh-Nagumo oscillators

N=500 coupled oscillators,

the state (u_i, v_i) at node *i* evolves according to:

$$\epsilon \dot{u}_i = u_i - u_i^3 - v_i + (\Delta u)_i + \xi_i^{(u)},$$
$$\dot{v}_i = u_i - \alpha v_i + (\Delta v)_i + \xi_i^{(v)},$$

$$(\Delta u)_i = (u_{i+1} + u_{i-1} - 2u_i)$$

$$(\Delta v)_i = (v_{i+1} + v_{i-1} - 2v_i)$$

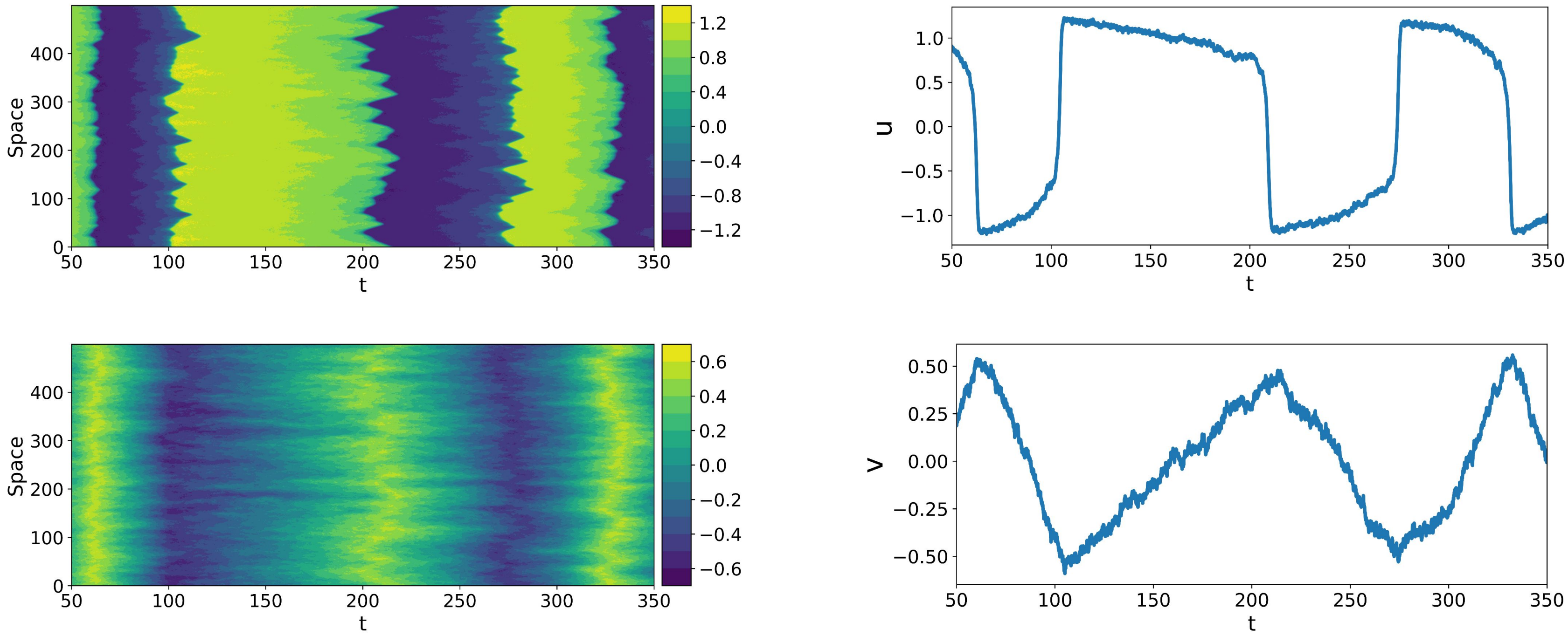
α = -0.83, ϵ randomly selected in (0,0.035) every 100 time units

$\xi^{(u,v)}$ gaussian white noise

1D lattice, with periodic BC

System’s solution

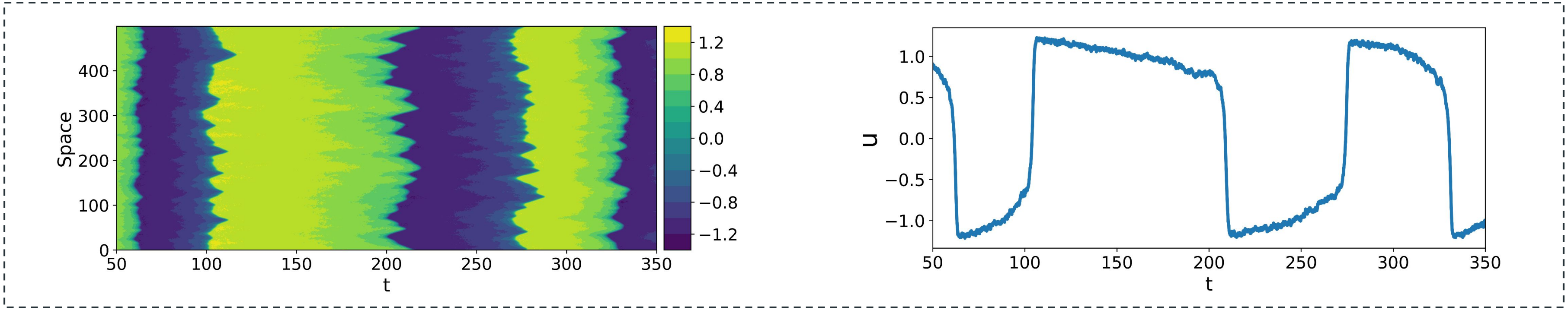
The oscillations are close to homogeneous in space and irregular in time (non periodic).



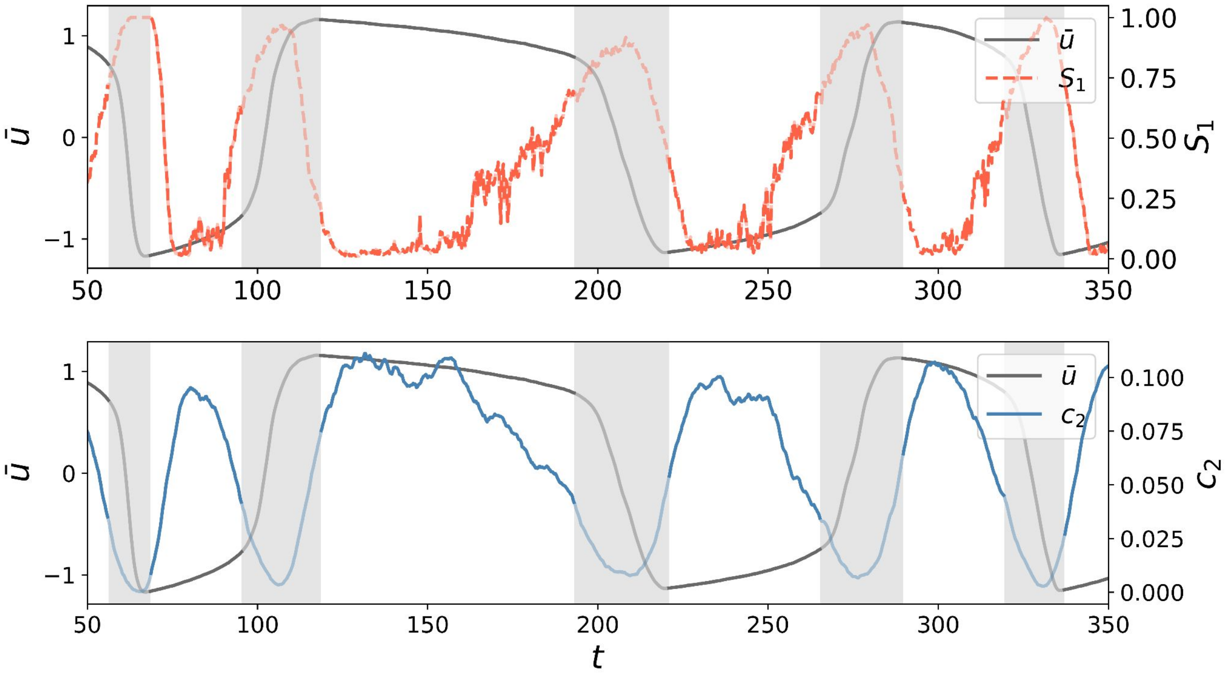
Left, spatio-temporal evolution of the state variables *u (top) and v (bottom)*
Right, temporal evolution of *u* and *v* at node *i=50*

System's solution

The oscillations are close to homogeneous in space and irregular in time (non periodic).

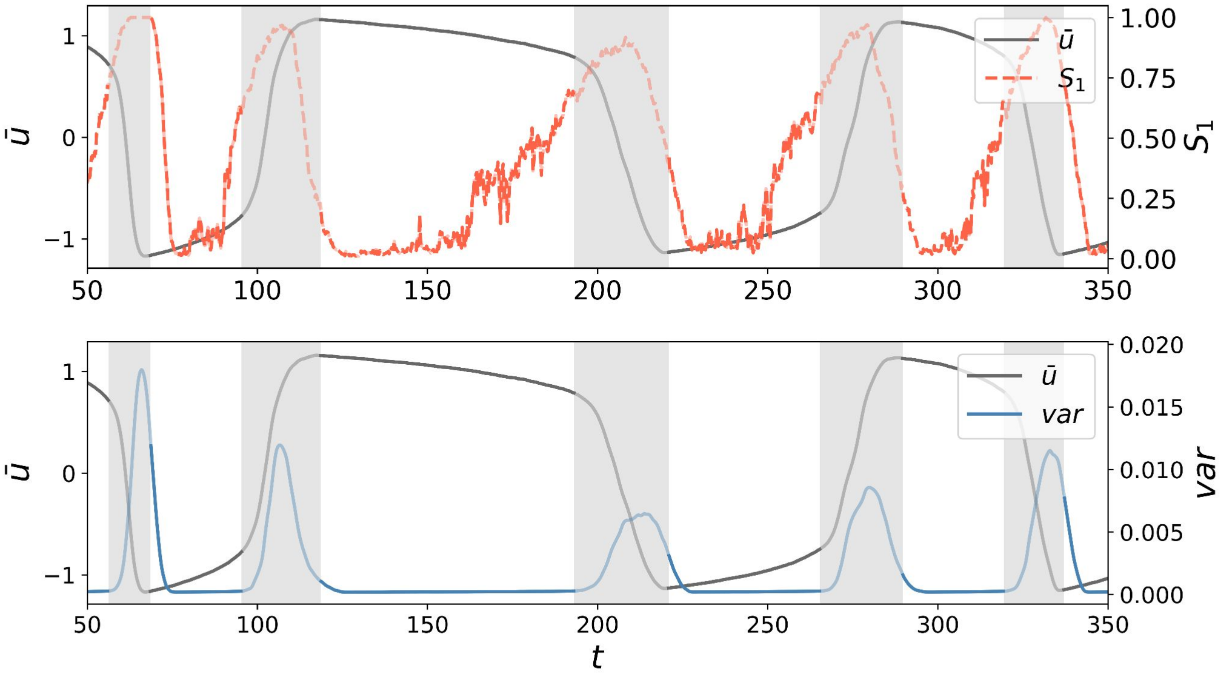


The correlation network is computed using the methodology described on slide 2, with the (detrended) time series from the *u variable only*.



S_1 (red), the relative size of the largest connected component in the network.
c_2 (blue), the probability that a randomly chosen node belongs to a component of size 2.
The grey lines show the spatial average of the solution u.

S_1, c_2 anticipate abrupt changes in *u*.



S_1 (red), the relative size of the largest connected component in the network.
var (blue), the temporal variance of each subseries used for the network computation, spatially averaged.
The grey lines show the spatial average of the solution u.

S_1 is able to anticipate the abrupt changes in *u* whereas the *variance* is not able to.

System 2: coupled recharge-oscillator models for ENSO

N= 20x60 coupled oscillators,

the state ($u_{i,j}$, $v_{i,j}$) at node (i,j) evolves according to:

$$\dot{u}_{i,j} = \gamma b u_{i,j} - c u_{i,j} - \gamma v_{i,j} + (v \quad + b u_{i,j})^3 + (\Delta u)_{i,j} + \xi_{i,j}^{(u)},$$
$$\dot{v} \quad = -(r v \quad + \alpha b \bar{u}),$$

$$(\Delta u)_{i,j} = u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j}$$

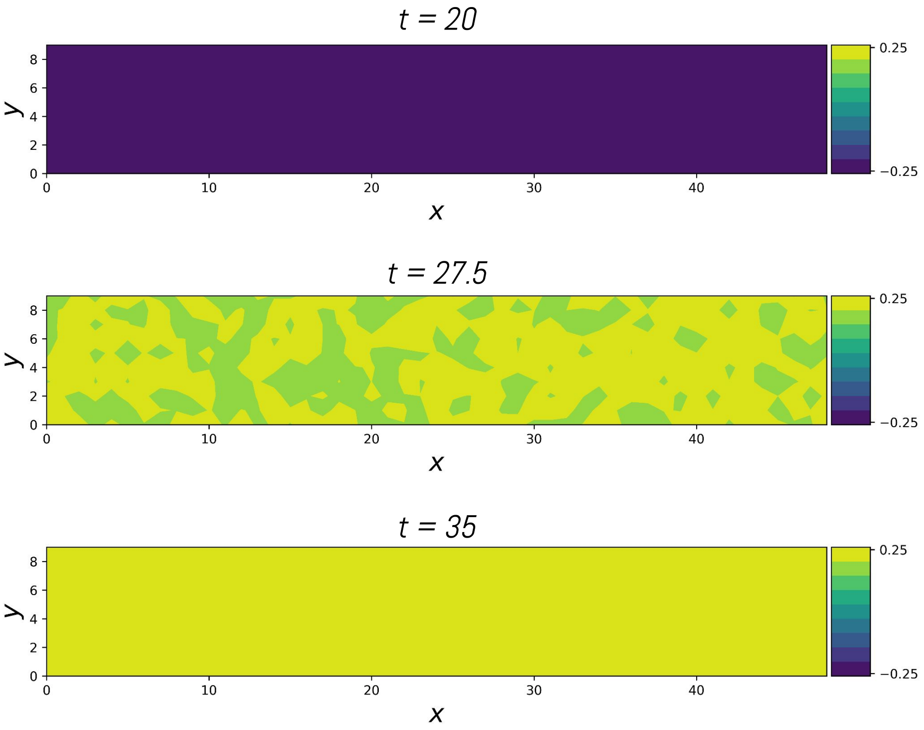
With u ↔T_e (temperature anomaly, eastern equatorial Pacific) and v ↔ h_w (thermocline depth, western Pacific) in Jin 1997. We allow some stochasticity in the parameters values, selecting them randomly every 50 time units from a chosen range whose bounds are within reasonable distance from the parameters chosen in Jin 1997.

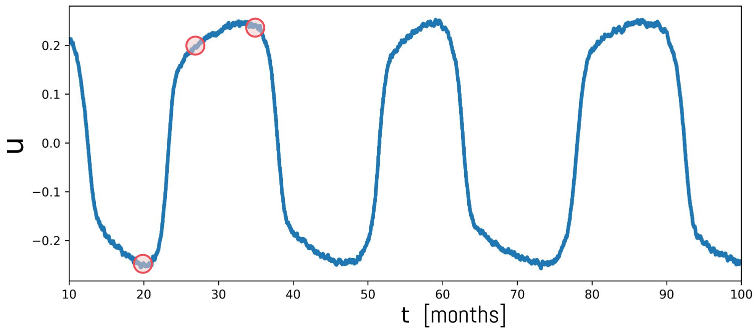
b in (2.25, 2.75), ***c*** = 1, γ =0.75, ***r*** in (0.056, 0.074), **α** in (0.0625, 0.125)

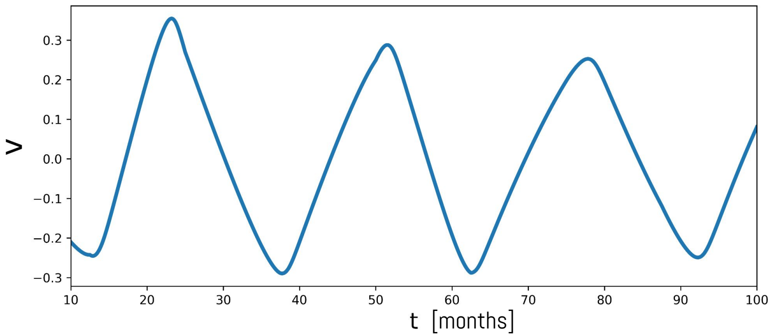
$\xi^{(u)}$ gaussian white noise

2D lattice with homogeneous Dirichlet BC

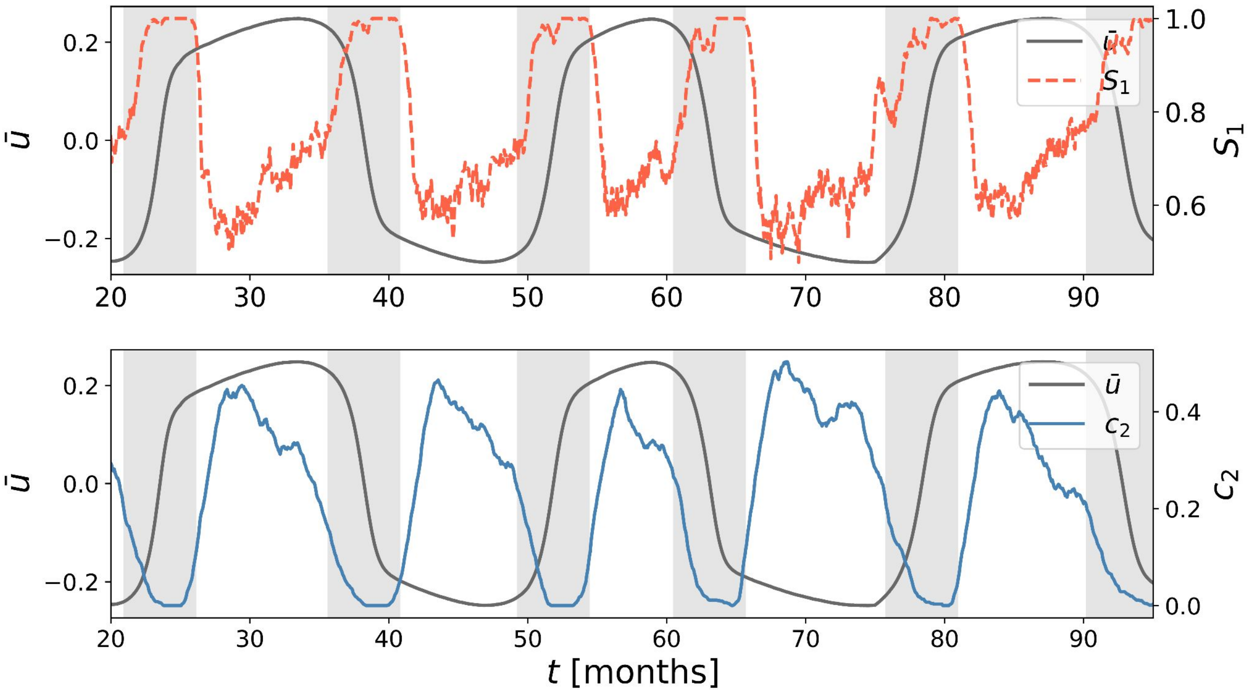
System’s solution







Left, the spatial field for the state variables *u* at the time of the 3 red dots on the right
Right, temporal evolution of *u* and *v* at node *(10,10)*



S_1 (red), the relative size of the largest connected component in the network.
c_2 (blue), the probability that a randomly chosen node belongs to a component of size 2.
The grey lines show the spatial average of the solution u.

S_1, c_2 anticipate abrupt changes in *u*.

Conclusions

The time-dependent functional network encodes the evolution of the system's spatial correlations, allowing to gain insights into dynamical processes that might not be detectable by other analysis techniques.

Percolation measures act **as precursors of abrupt changes** in extended systems, anticipating the global rises and falls of the state variable in

- the coupled (irregular) Fitzhugh-Nagumo systems
- the coupled recharge oscillator ENSO models

Significance

- a step towards a better characterization of the conditions under which percolation measures of correlation networks are informative of the proximity to abrupt changes.
- could eventually lead to a better understanding of the dynamical changes in the Niño 3.4 SST field which trigger the increase in correlations and hence a better understanding of ENSO mechanisms.

Part of the Outstanding Student & PhD candidate presentation contest.
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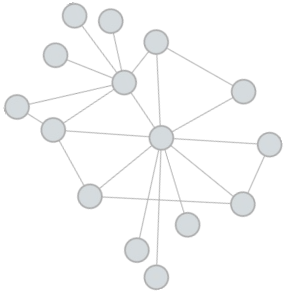
References



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