Modelling Transverse Dispersion in Heterogenoues Porous Media

Aronne Dell'Oca, Marco Dentz

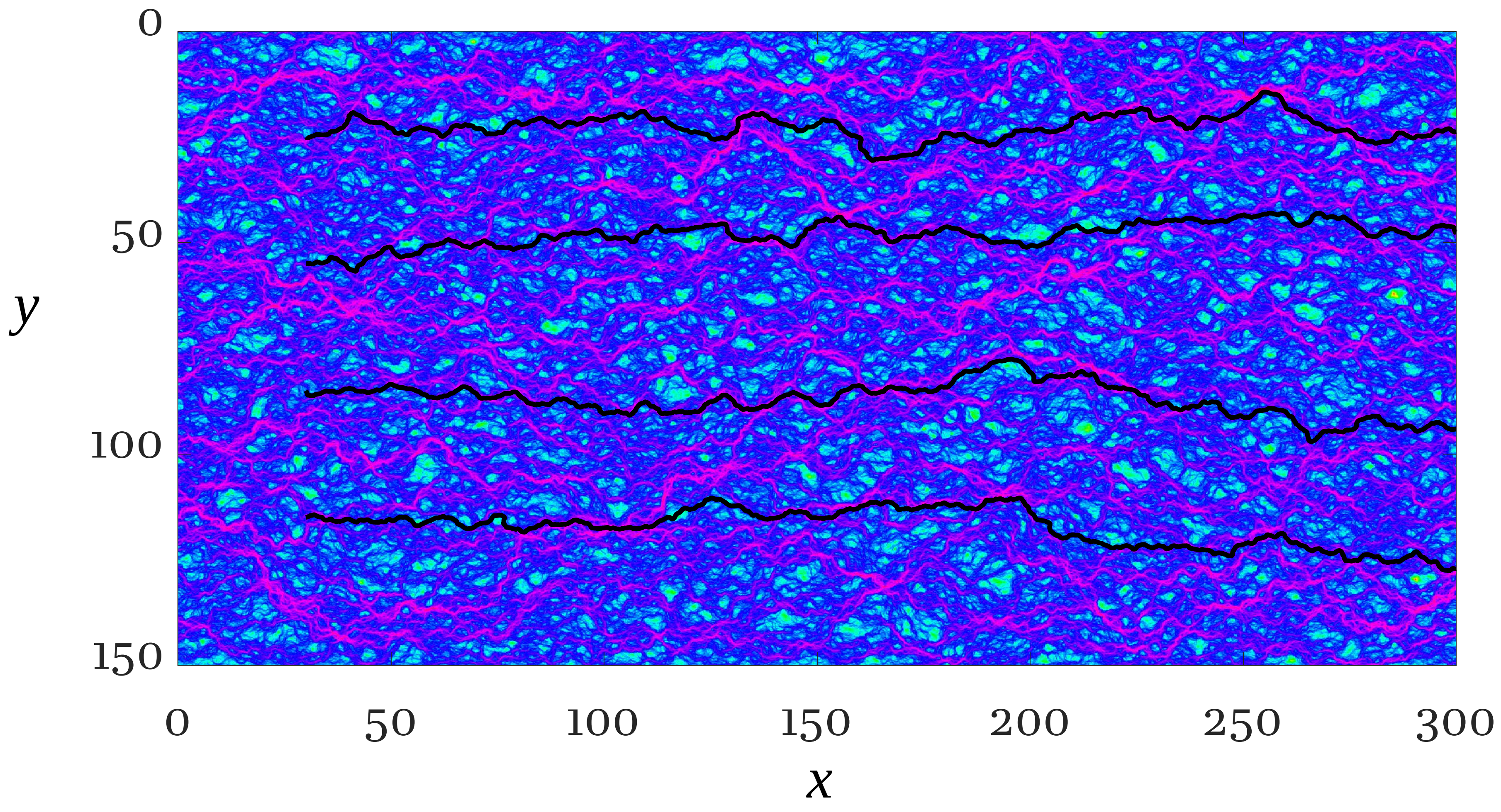
IDAEA-CSIC

European Geophysical Union

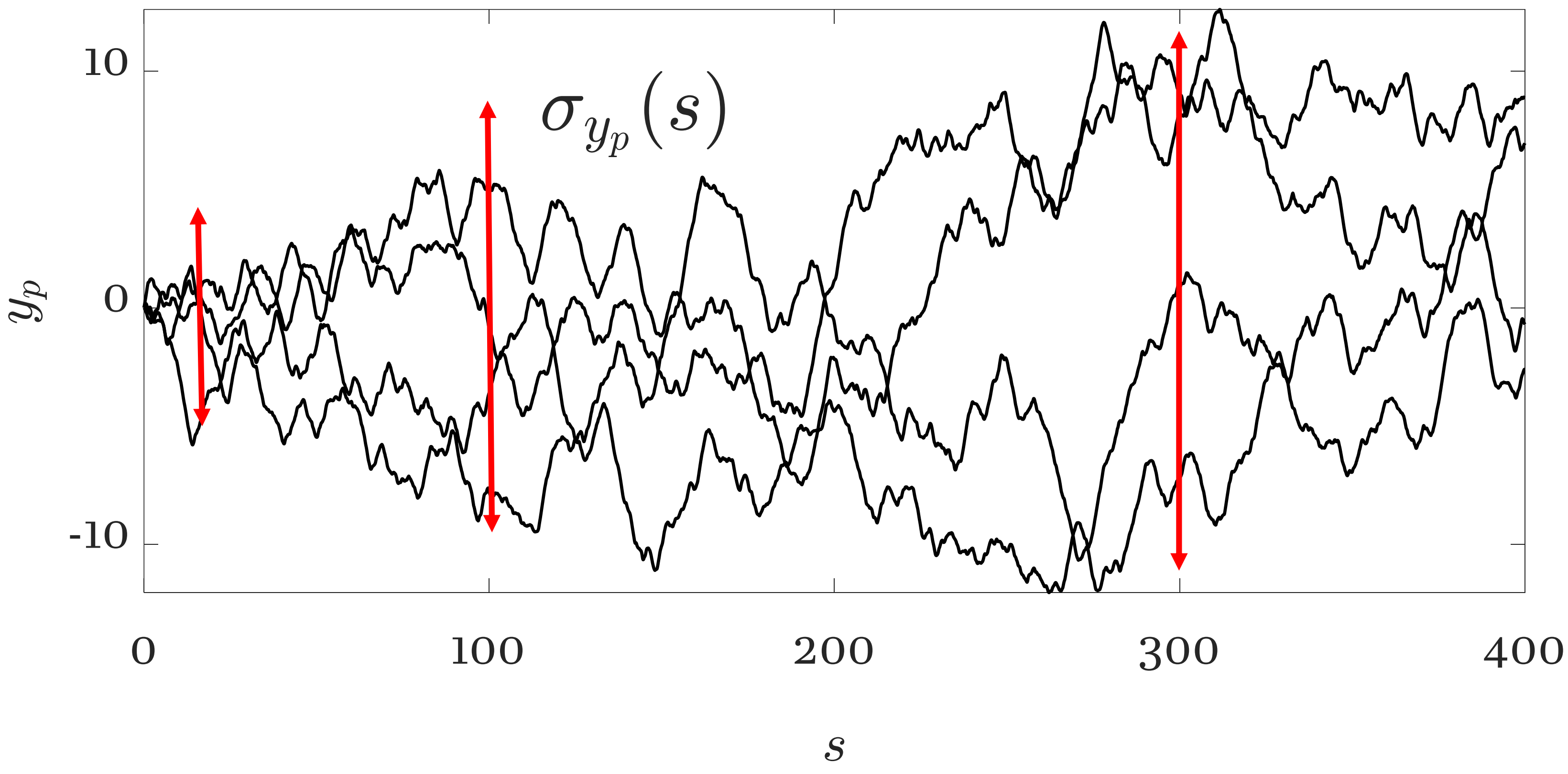
General Assembley, 2022

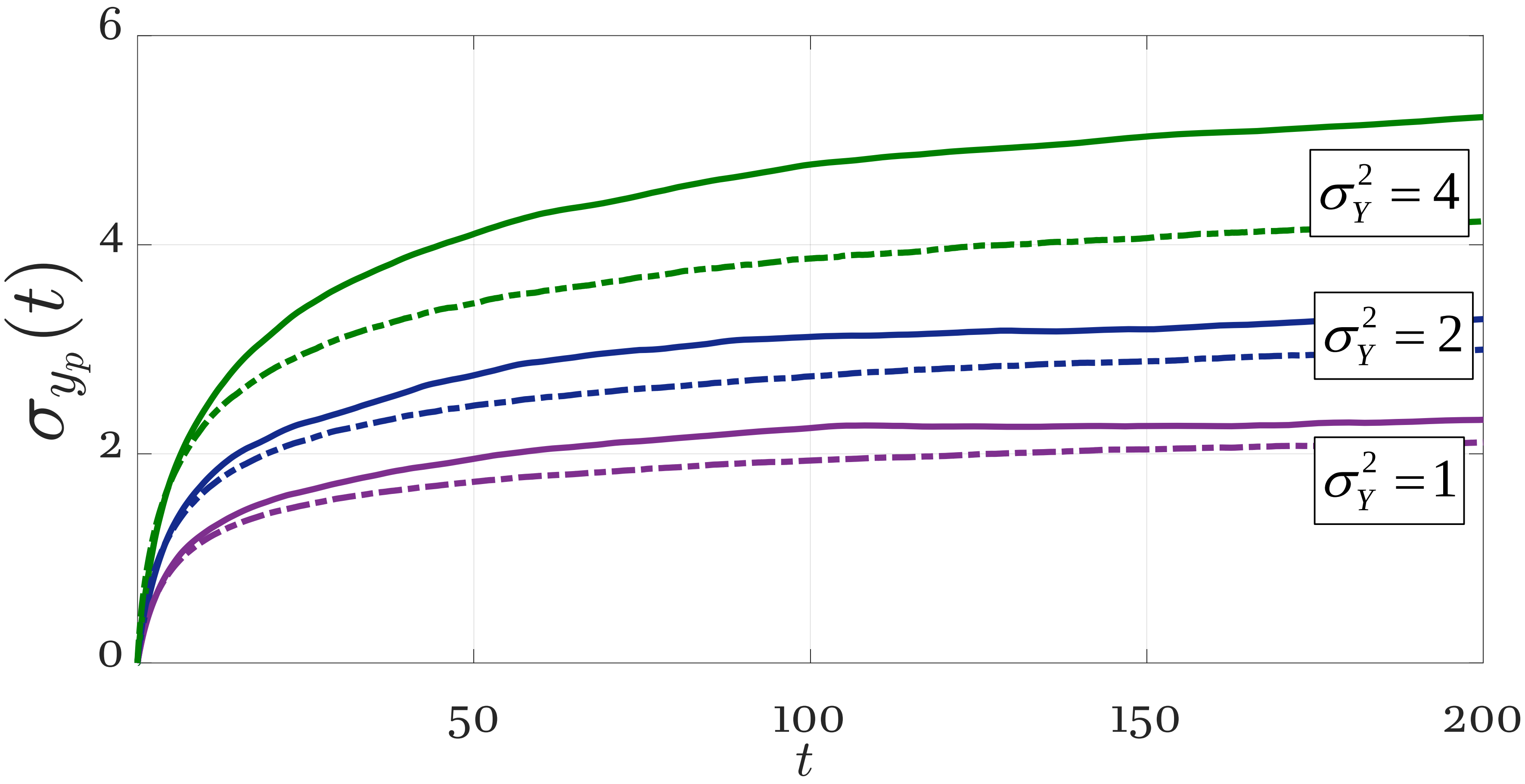
Vienna, Austria & Online

Lagrangian Particles Trajectories



Transverse Ensemble Dispersion





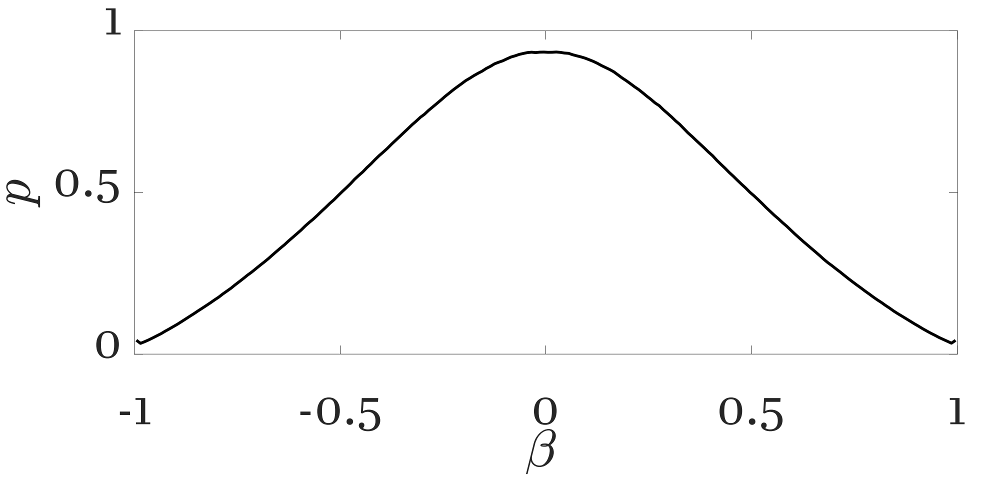
$$\sigma_{y_p}^2 = l_Y^2 \sigma_Y^2 \left[\ln(t) - \frac{3}{2} + E - Ei(-t) + 3\frac{1-(1+t)e^{-t}}{t^2} \right] \sim \ln(t)$$

'Ultra-slow' diffusion
Rooted in the anti-correlation

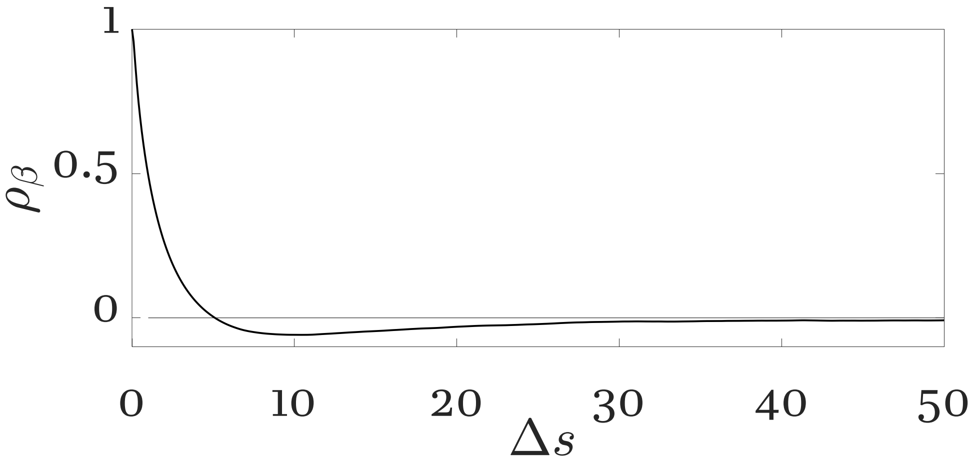
Transvere Motion

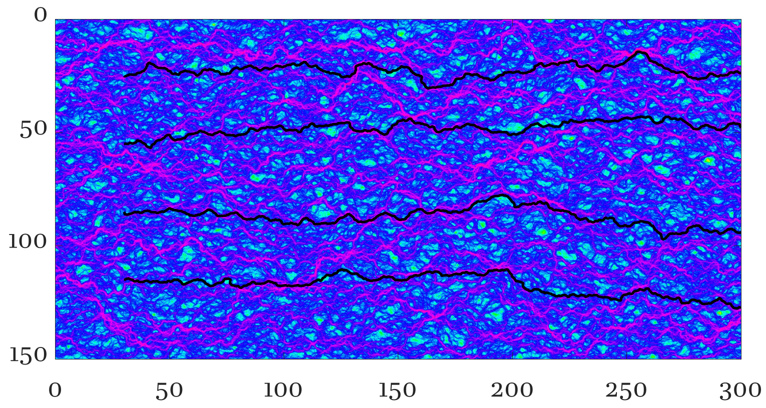
$$\frac{dy_p(s)}{ds} = \frac{v_{y_p}(s)}{v_p(s)} = \beta(s)$$

Distribution of Transverse Tortuosities



Long Range Anti-correlated

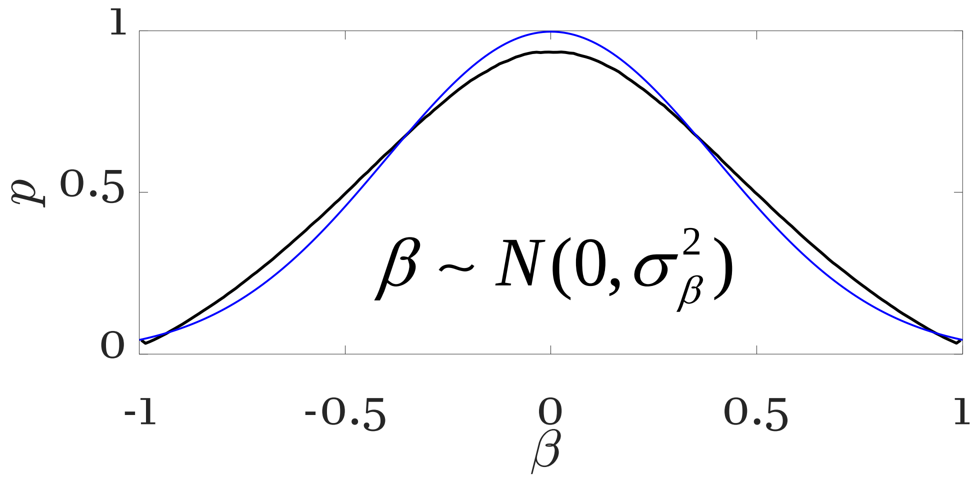




Modelling

$$\frac{dy_p(s)}{ds} = \frac{v_{y_p}(s)}{v_p(s)} = \beta(s)$$

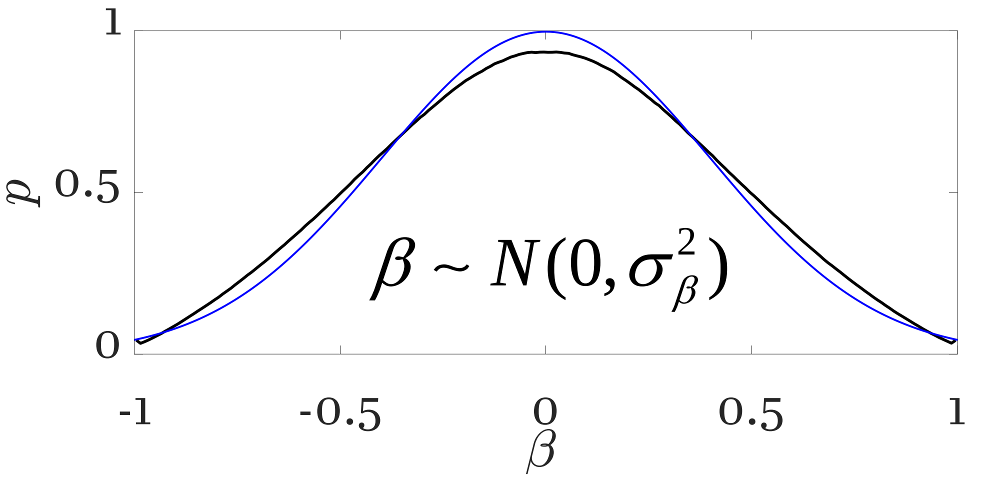
Distribution of Transverse Tortuosities



Modelling

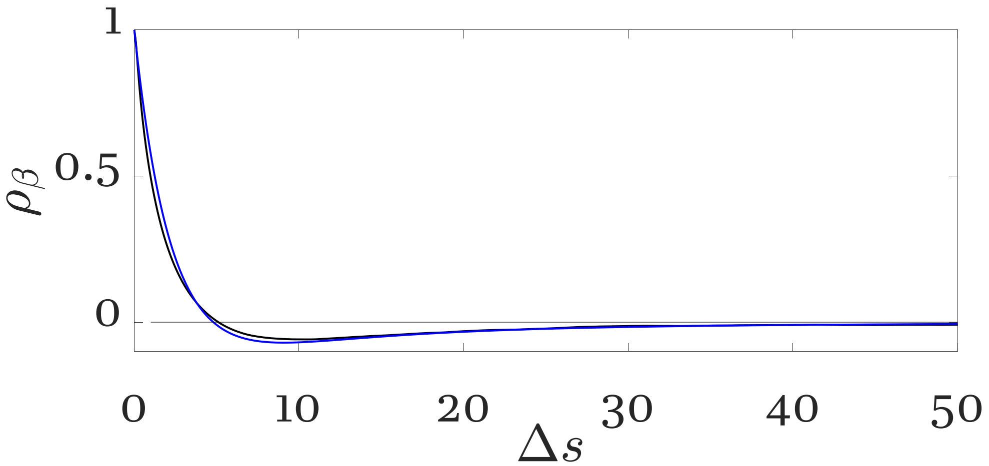
$$\frac{dy_p(s)}{ds} = \frac{v_{y_p}(s)}{v_p(s)} = \beta(s)$$

Distribution of Transverse Tortuosities



$$\rho_\beta = \frac{e^{-\Delta s/l_\beta}}{8} - \left(\left(\frac{1}{8} + \frac{l_\beta}{\Delta s} + 4\frac{{l_\beta}^2}{\Delta s^2} + 9\frac{{l_\beta}^3}{\Delta s^3} + 9\frac{{l_\beta}^4}{\Delta s^4} \right) e^{-\Delta s/l_\beta} + \frac{{l_\beta}^2}{\Delta s^2} - 9\frac{{l_\beta}^4}{\Delta s^4} \right)$$

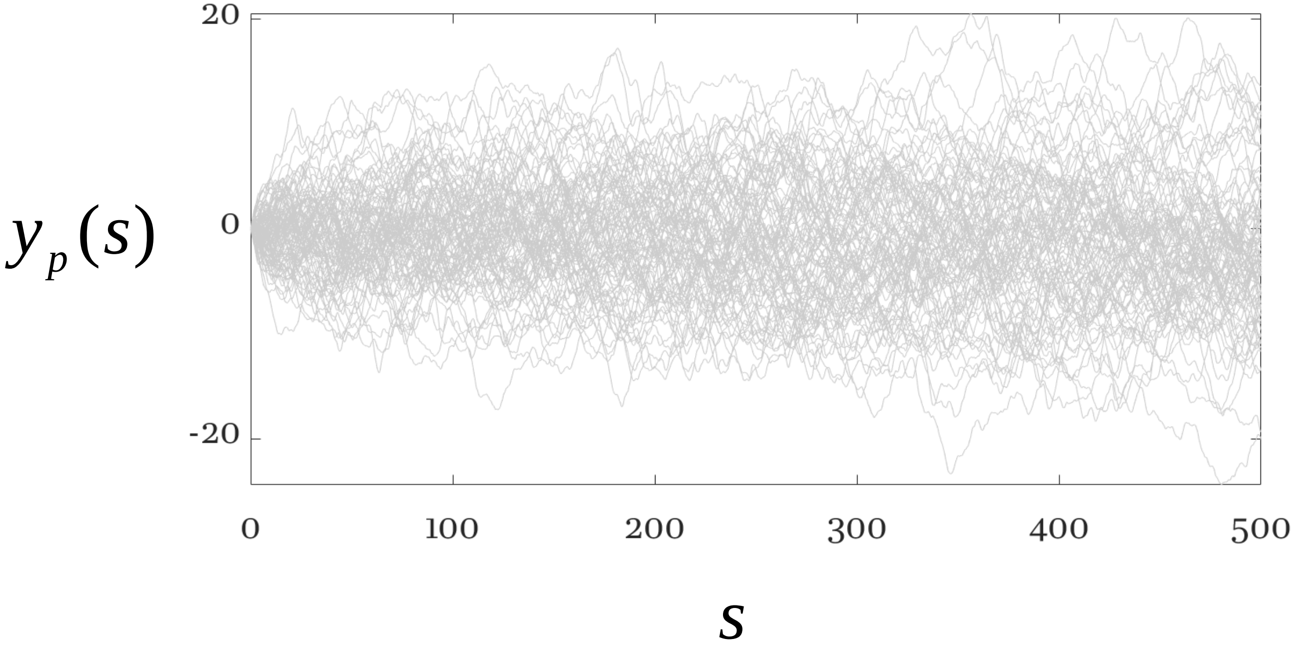
Long Range Anti-correlated



Modelling

$$\frac{dy_p(s)}{ds} = \frac{v_{y_p}(s)}{v_p(s)} = \beta(s)$$

Anti-correlated
Gaussian process



Speed space Markovian

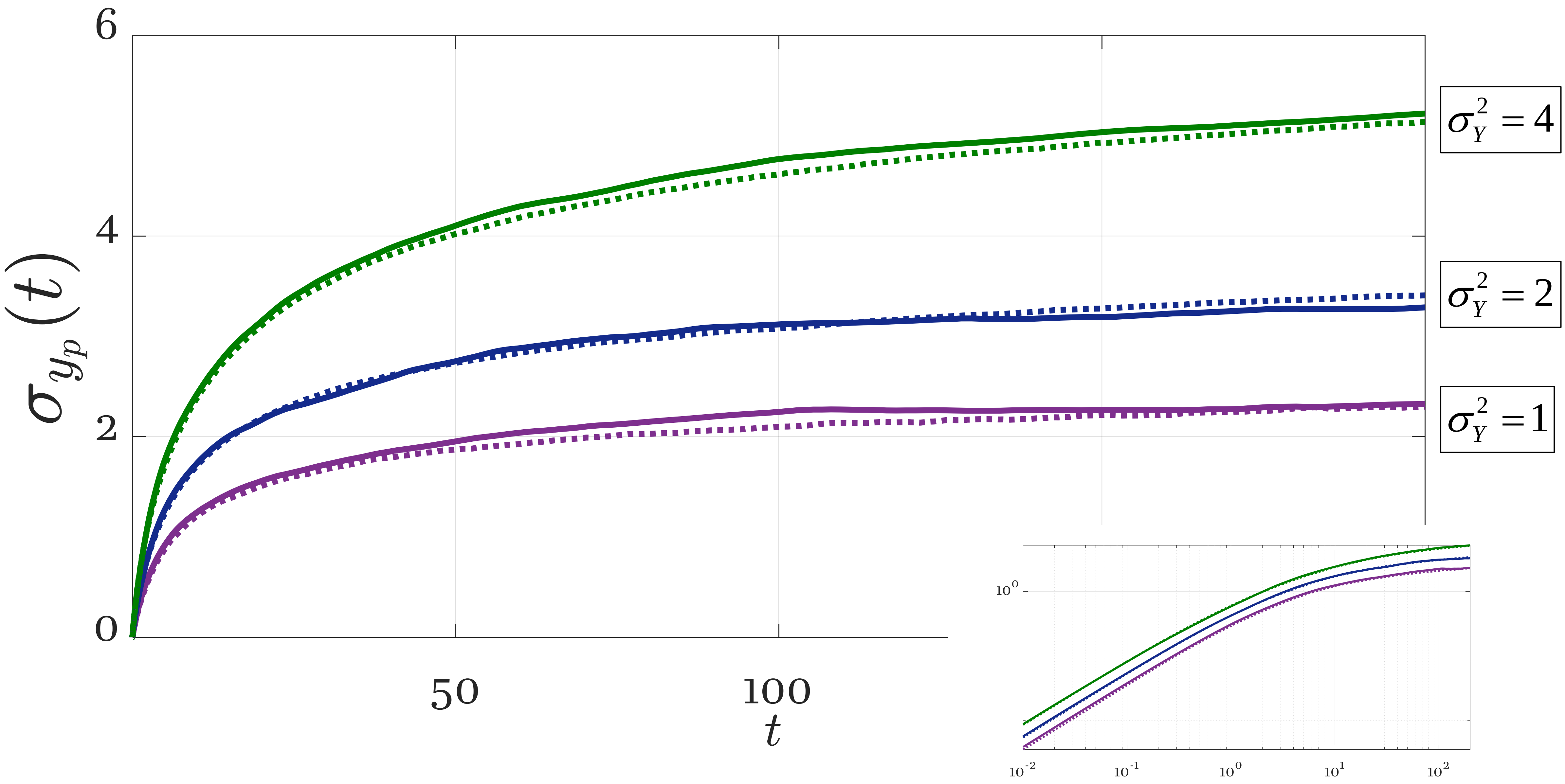
$$w = \Phi^{-1}\left[P(v_p)\right]$$

$$\frac{dw(s)}{ds} = -\frac{w(s)}{l_v} + \sqrt{\frac{2}{l_v}}\eta(s)$$

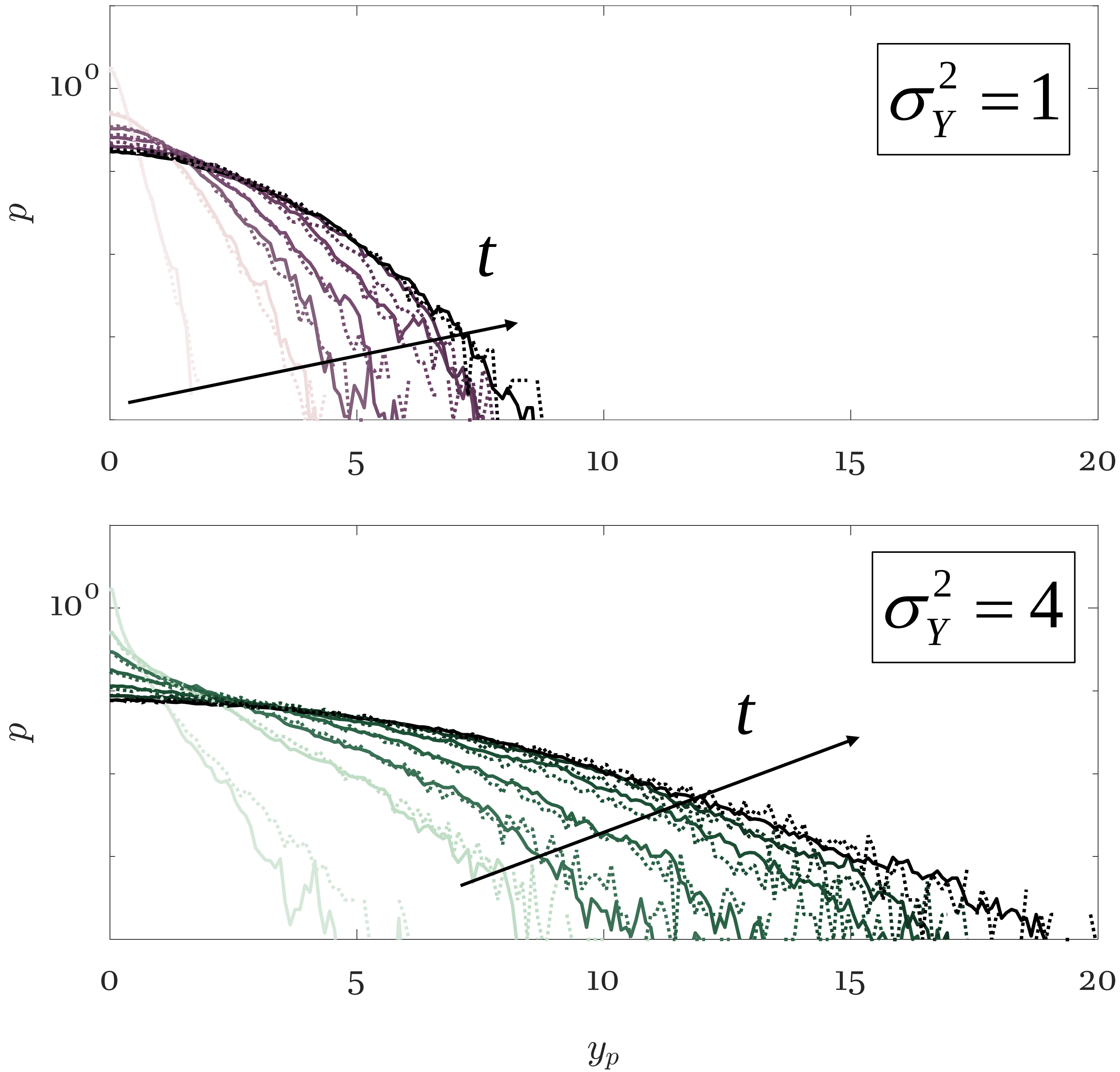
Time

$$dt(s) = \frac{ds}{v_p(s)}$$

$$t(s) = \int_0^s \frac{ds'}{v_p(s')}$$



$\sigma_{y_p}(t)$
6
4
2
0
50
100
t
$\sigma_Y^2 = 4$
$\sigma_Y^2 = 2$
$\sigma_Y^2 = 1$
10^0
10^{-2}
10^{-1}
10^0
10^1
10^2



$\sigma_Y^2 = 1$
t
10^0
p
0
5
10
15
20
$\sigma_Y^2 = 4$
t
10^0
p
0
5
10
15
20
y_p

Conclusion

Transverse transport dynamics (β) can be treated as an anti-correlated Gaussian noise (in space)

Acknowledgement: Marie-Curie fellowship MixUQ grant 895152