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Eikonal approximation for landscapes dominated by threshold hillslopes

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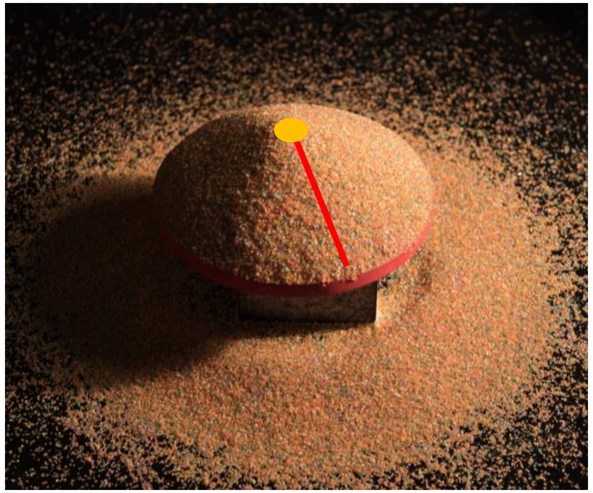

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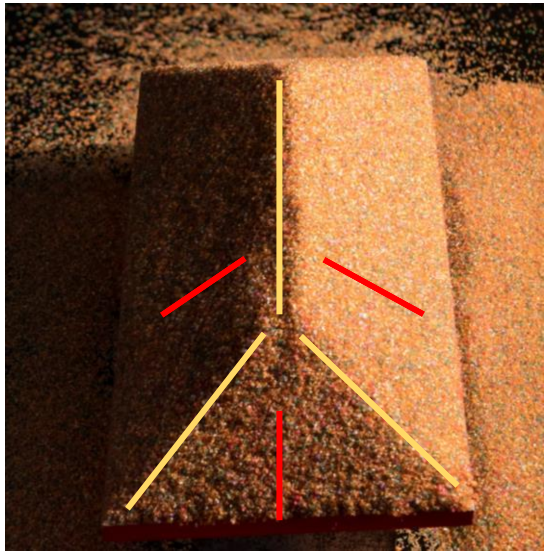
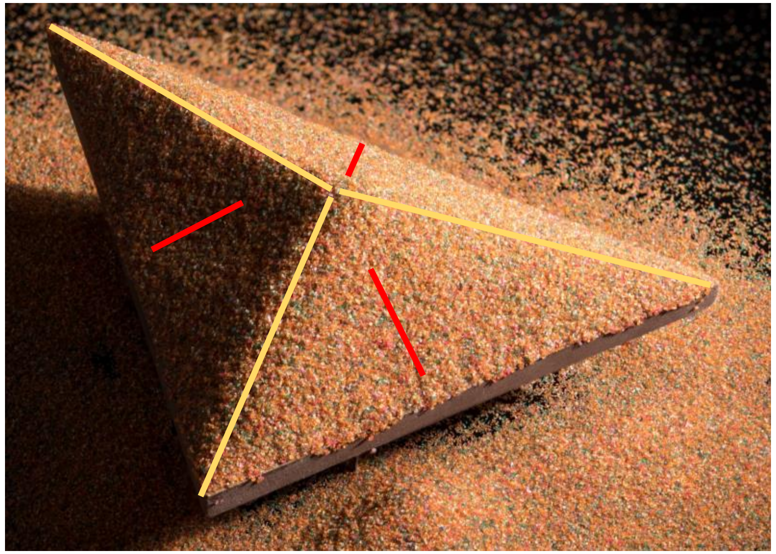
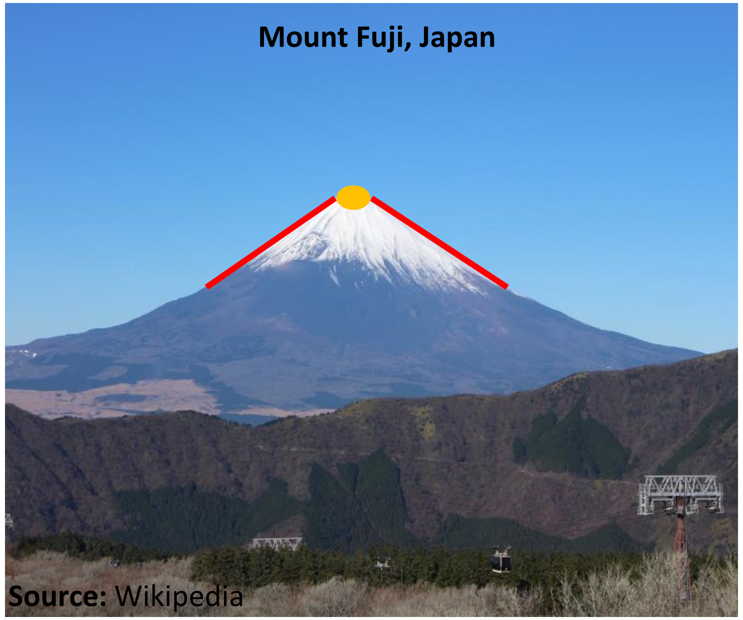
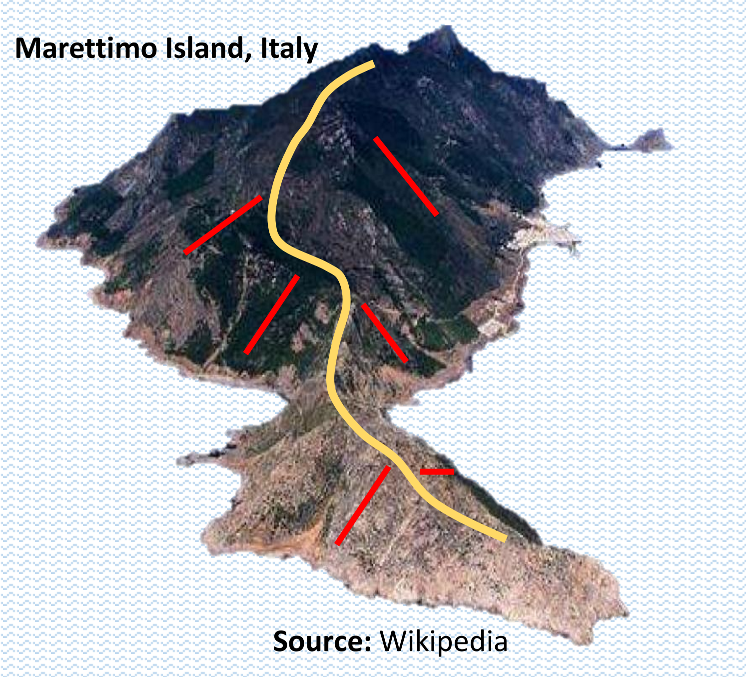
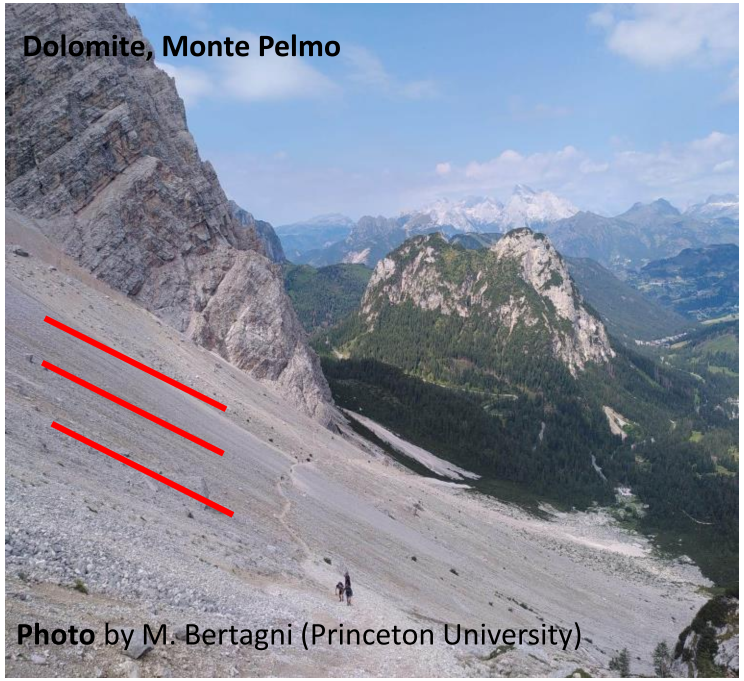










Analogous setting in tabletop granular pile and natural landscapes with threshold hillslopes

Eikonal model for spatial organization of threshold hillslopes:

$$|\nabla z| = \theta,$$

$$z(x, y) = z_b \ at \ (x, y) \in \mathcal{B}$$

where θ is the threshold angle, with z_b elevation at the boundary $\mathcal{B}$.

Sandpile Experiment (From Pauli and Gioia (2007))

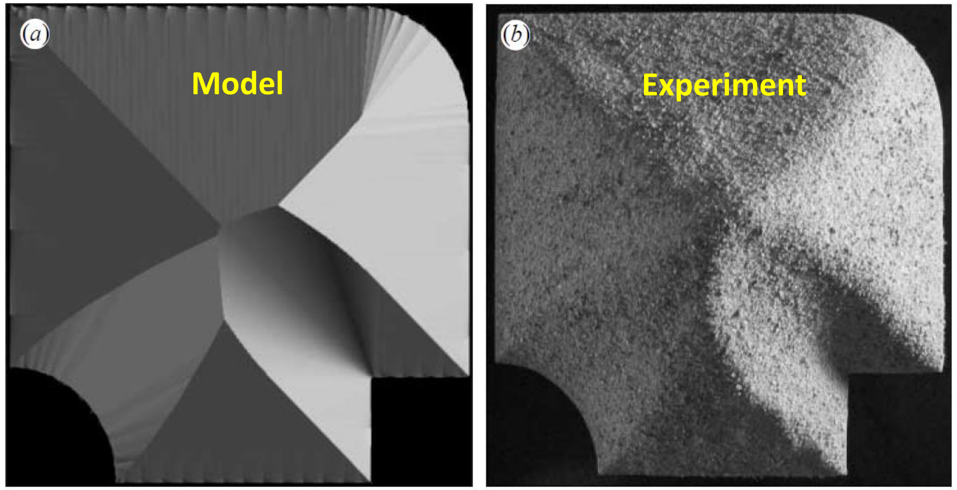
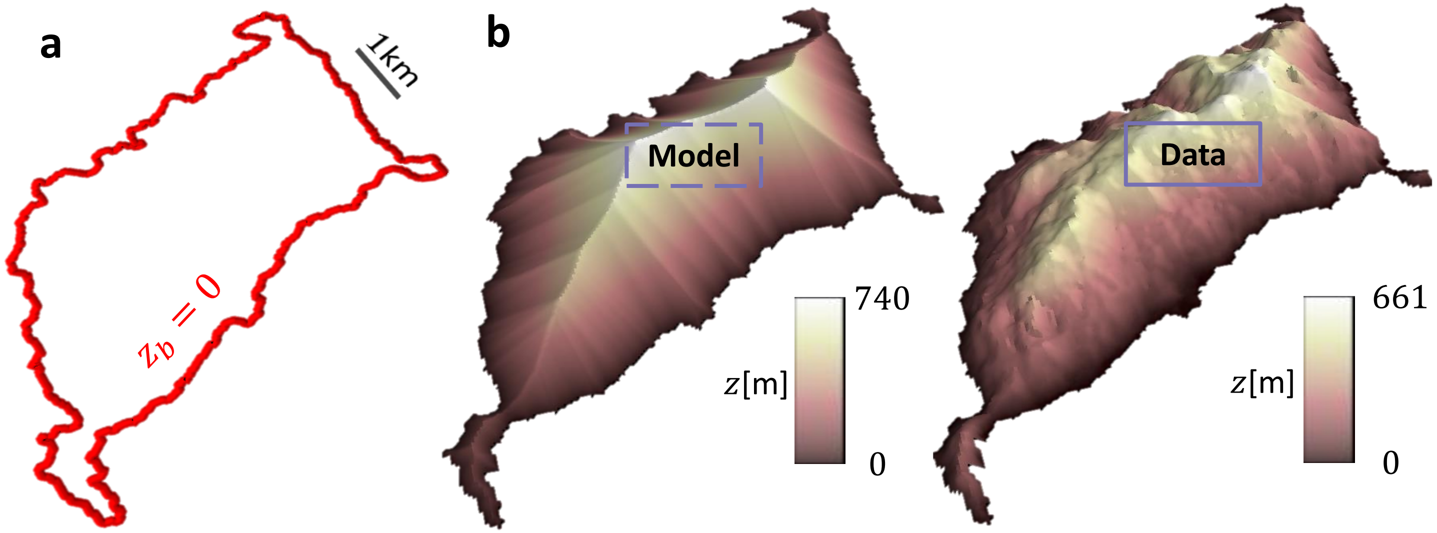


Figure 7. Comparison of (a) theoretical and (b) experimental pile for a simple domain.

Our Result (Marettimo Island, Italy)



The proposed model captures hillslopes organization close to the threshold angle in natural landscapes

Derivation from a Simple Landscape Evolution Model

$$\partial z / \partial t = D \, \nabla^2 z - K a^m |\nabla z| + U$$

Howard et al. (1994), Perron et al. (2008), Bonetti et al. (2020)

Assuming dynamic equilibrium $\partial z / \partial t \rightarrow 0$,

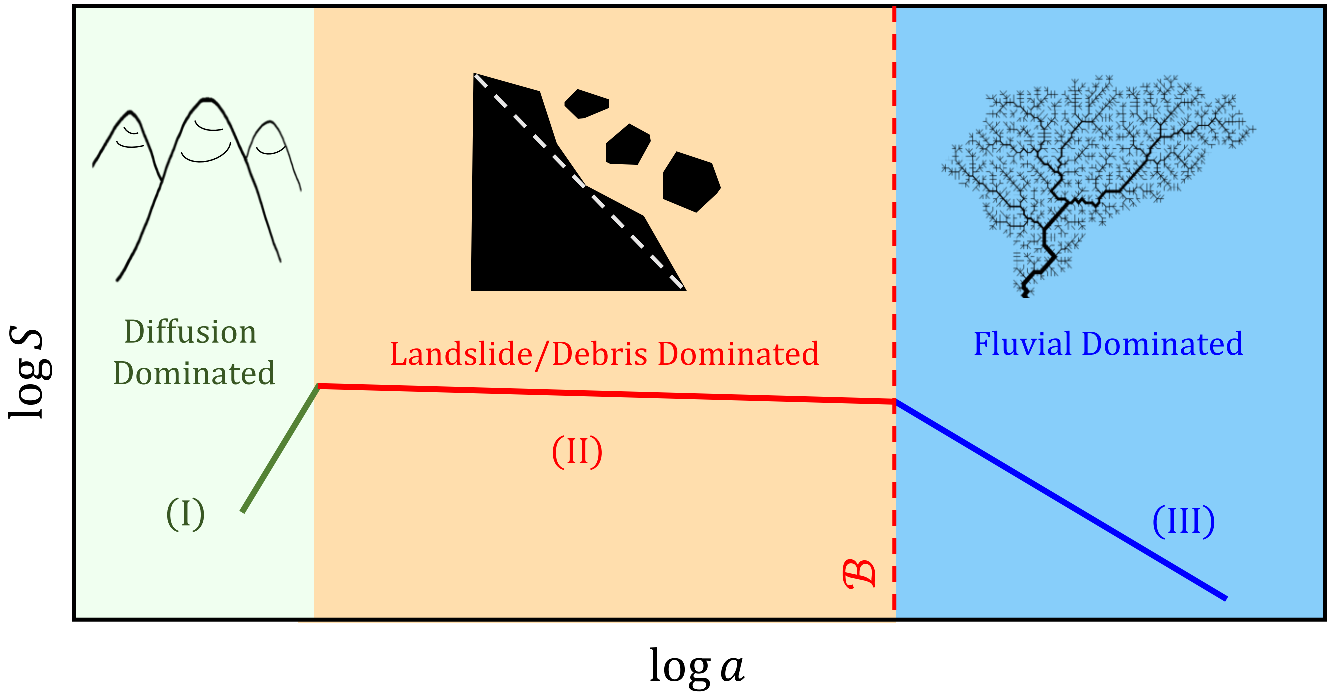
negligible soil creep $(D \rightarrow 0)$,

and decoupled water dynamics $m \rightarrow 0$,

one gets the Eikonal equation
$$|\nabla z| = \theta,$$

with boundary condition
$$z(x, y) = z_b \; at \; (x, y) \in \mathcal{B}$$

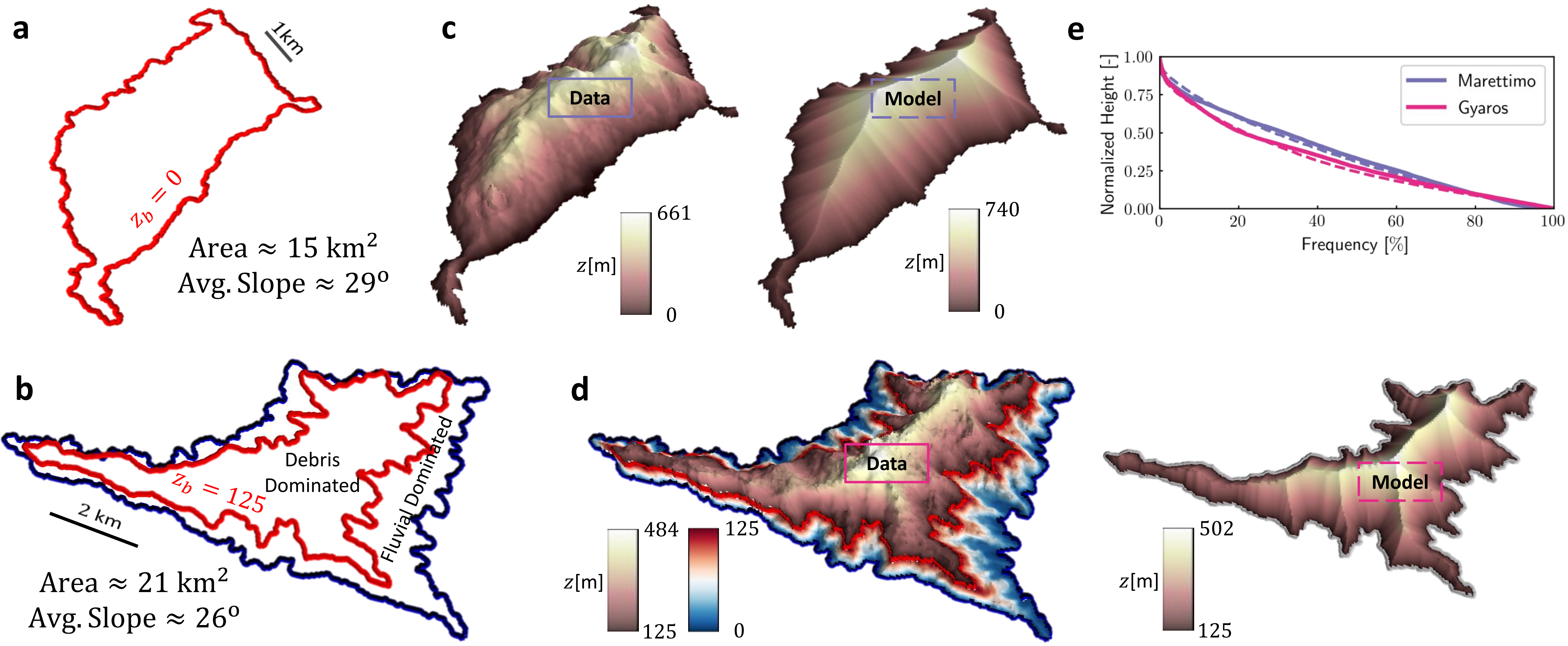
where θ is the threshold angle and z_b is the reference elevation at the downstream boundary $\mathcal{B}$.



Conceptual sketch of the Slope vs Area relationship classically inferred from Digital Elevation Models (DEMs). Three regimes are identified, wherein the dominant sediment-transport mechanism is: diffusion for **Regime I**; landslides/debris flow for **Regime II**; fluvial erosion for **Regime III**.

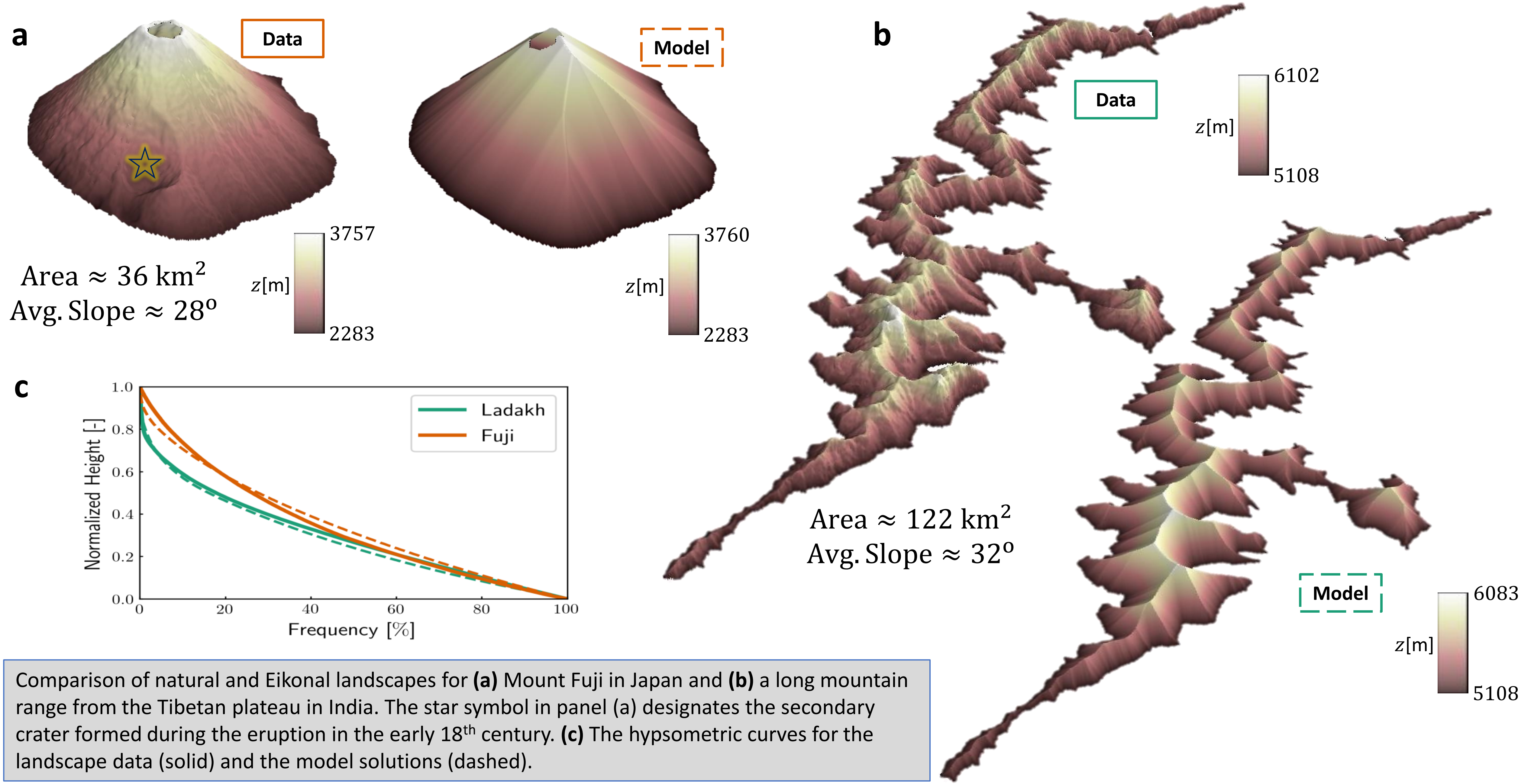
Modified from Montgomery, D. R., & Foufoula-Georgiou, E. (1993), Stock & Dietrich (2003).

Steep Island Landscapes



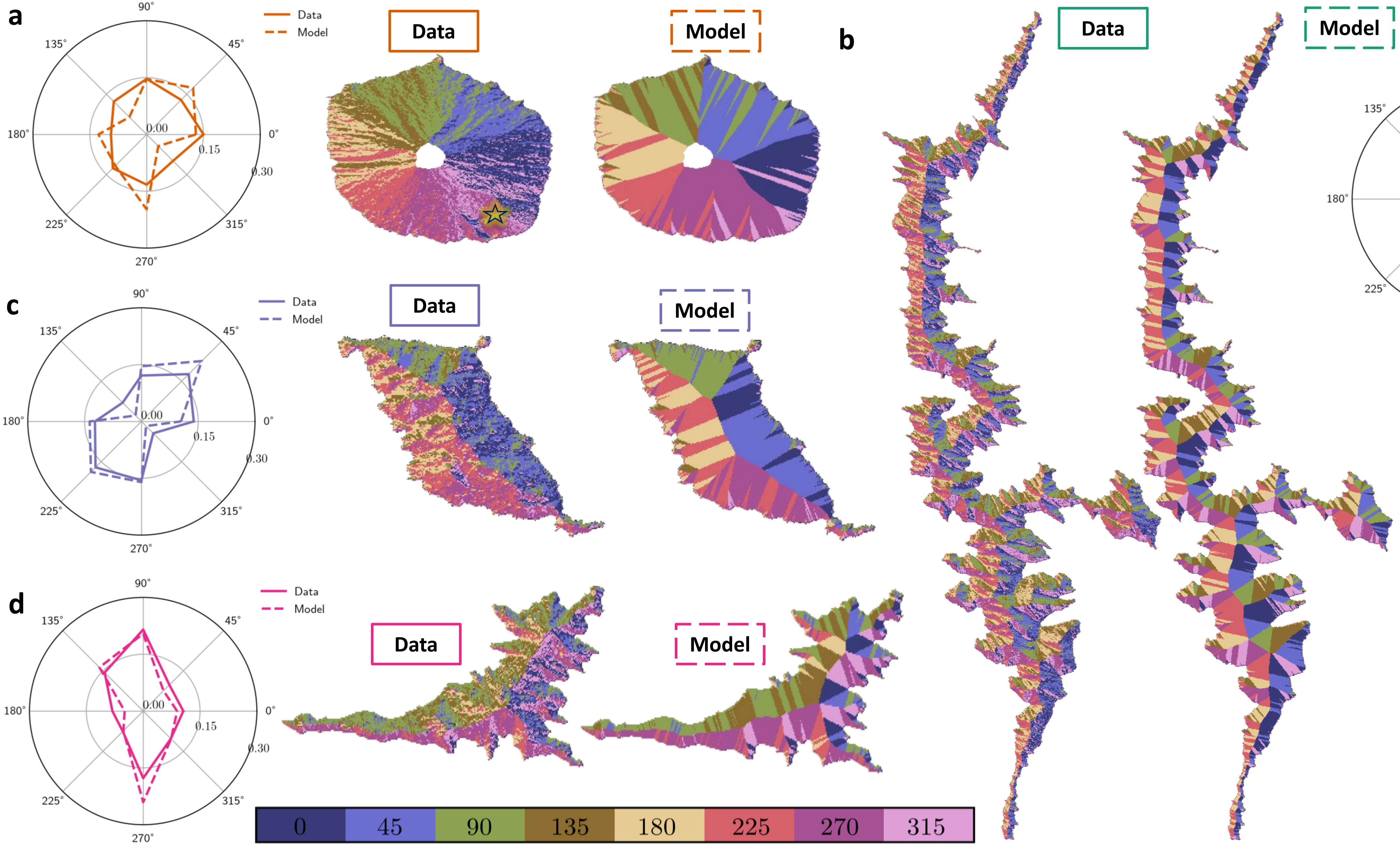
Downstream boundary used in the Eikonal reconstruction for **(a)** Marettimo Island in the Mediterranean Sea, **(b)** Gyaros island in the Aegean Sea, with a blue boundary at sea level and the red curve showing the elevation contour at 125 m that separates the downstream fluvial regime from the upstream debris-dominated area. Comparison of natural and Eikonal landscapes for **(c)** Marettimo Island and **(d)** Gyaros island. **(e)** The hypsometric curves for the landscape data (solid) and the model solutions (dashed).

Steep Inland Landscapes



Comparison of natural and Eikonal landscapes for **(a)** Mount Fuji in Japan and **(b)** a long mountain range from the Tibetan plateau in India. The star symbol in panel (a) designates the secondary crater formed during the eruption in the early 18th century. **(c)** The hypsometric curves for the landscape data (solid) and the model solutions (dashed).

Distribution of the Hillslope Aspect



Hillslope aspect distribution (polar plot and the color plot of the area) for the case studies **(a)** Mount Fuji, **(b)** the Tibetan mountain range, **(c)** Marettimo Island, and **(d)** Gyaros island.

Main Takeaways

Many natural landscapes maintain steep near-planar hillslopes bounded at a typical angle, beyond which landslides/slope failures remove the excess material brought by tectonic forces.

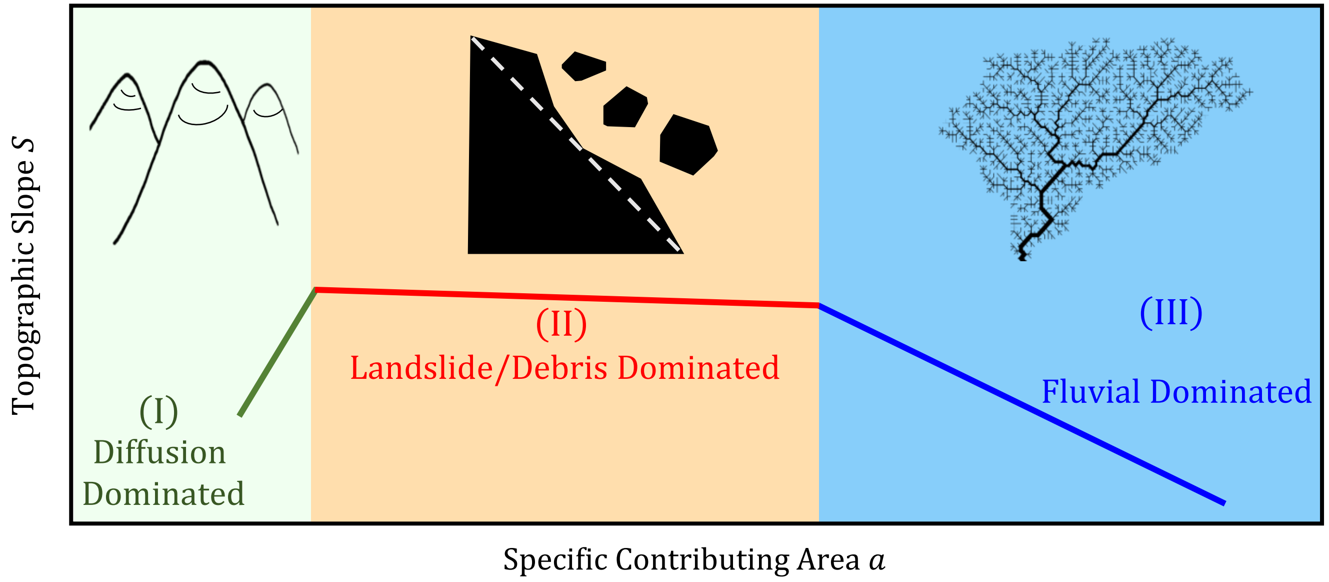
- Here we show that the resulting topographies are well captured by the famous eikonal equation, which is derived from a landscape evolution model in conditions of negligible soil creep and fluvial erosion.

- Eikonal landscapes reproduce various natural topographies, from small mountain islands to a stratovolcano and an extended mountain ridge.

  ✓ The uniqueness of the Eikonal equation lies in its simplified form of a nonlinear differential equation that can produce hillslope patterns based on majorly the complexity of the domain boundary.

  ✓ From a computational viewpoint, the time complexity for constructing an Eikonal landscape varies linearly with the number of nodes considered inside the domain. For comparison, this entire construction is on par with a single time-step computation of the fluvial erosion term in the efficient landscape evolution simulations.

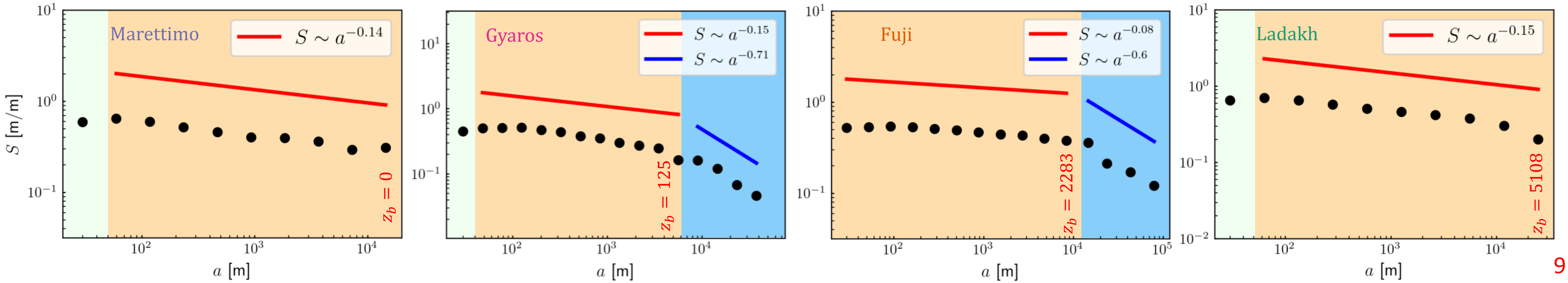
Slope Area Plots (Case Studies)



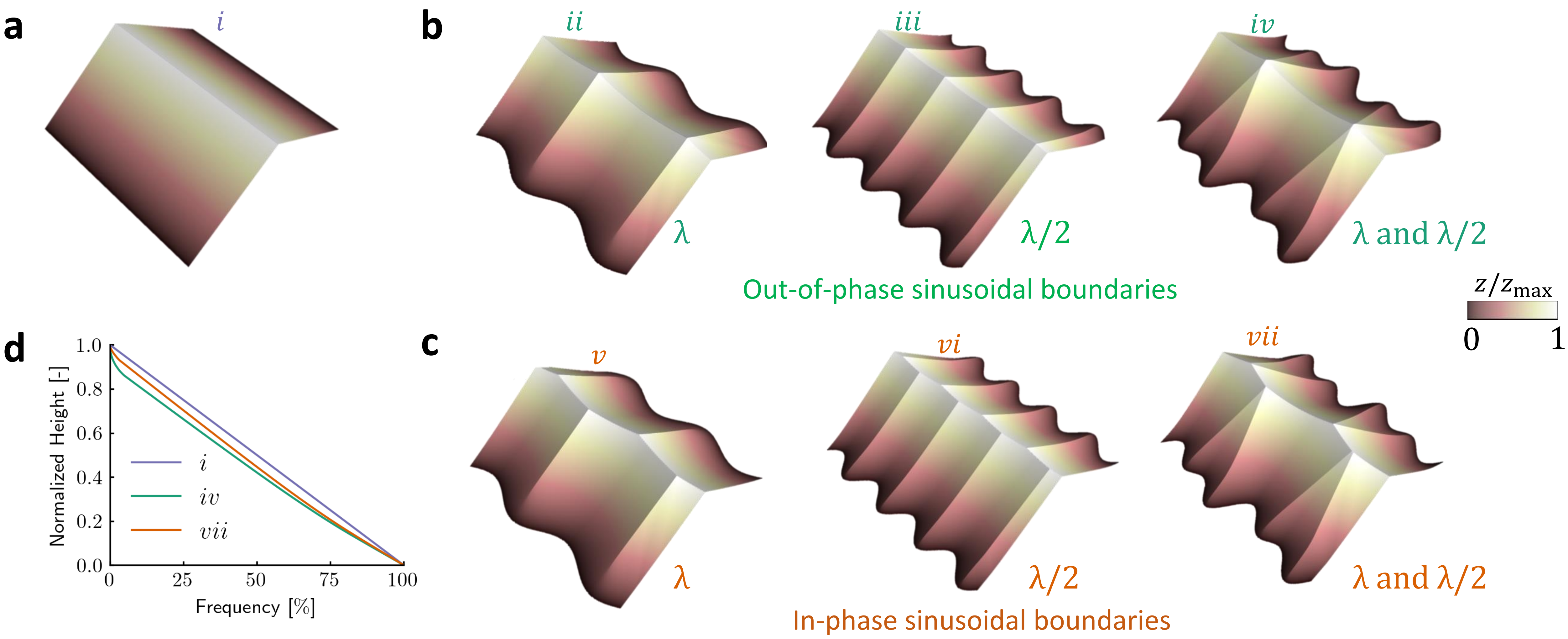
Average topographic slope S as a function of the contributing area per unit contour width is used for identifying the surface processes that dominantly regulate the steady landscape organization at a given scale. **Regime I** is dominated by soil diffusion with $S \propto a$ at small scales. **Regime III** occurs at very high a values, where fluvial erosion dominates the landscape organization, and the topographic slope decreases as a strong power-law function of the contributing area. $S \propto a^{-m}$, as $m \in (0.3 - 0.8)$.

Regime II at intermediate spatial scales is likely dominated by threshold hillslopes, with slope failures generating debris flow. The average value of the topographic slope remains high and shows a weak dependency on the contributing area.

Slope area curves for the case studies are shown in the bottom row, all displaying high spatial range of **Regime II**.



Role of Boundary Conditions



Boundary-condition effect on Eikonal landscapes. **(a)** Parallel boundaries result in a regular tent-type topography with straight contour lines and a divide in the middle. **(b)** Out-of-phase sinusoidal boundaries result in the convergent and divergent regions with the main sharp ridgeline in the center versus **(c)** Opposite boundaries as in-phase sine waves that change the spatial pattern of the main ridgeline and the hillslopes junctions. **(d)** Normalized hypsometric curves of the three steady landscapes indicate the impact of changes in the boundary conditions on the distribution of the elevation field z.