Global scale numerical geodynamic modelling with a free surface using a volume of fluid method

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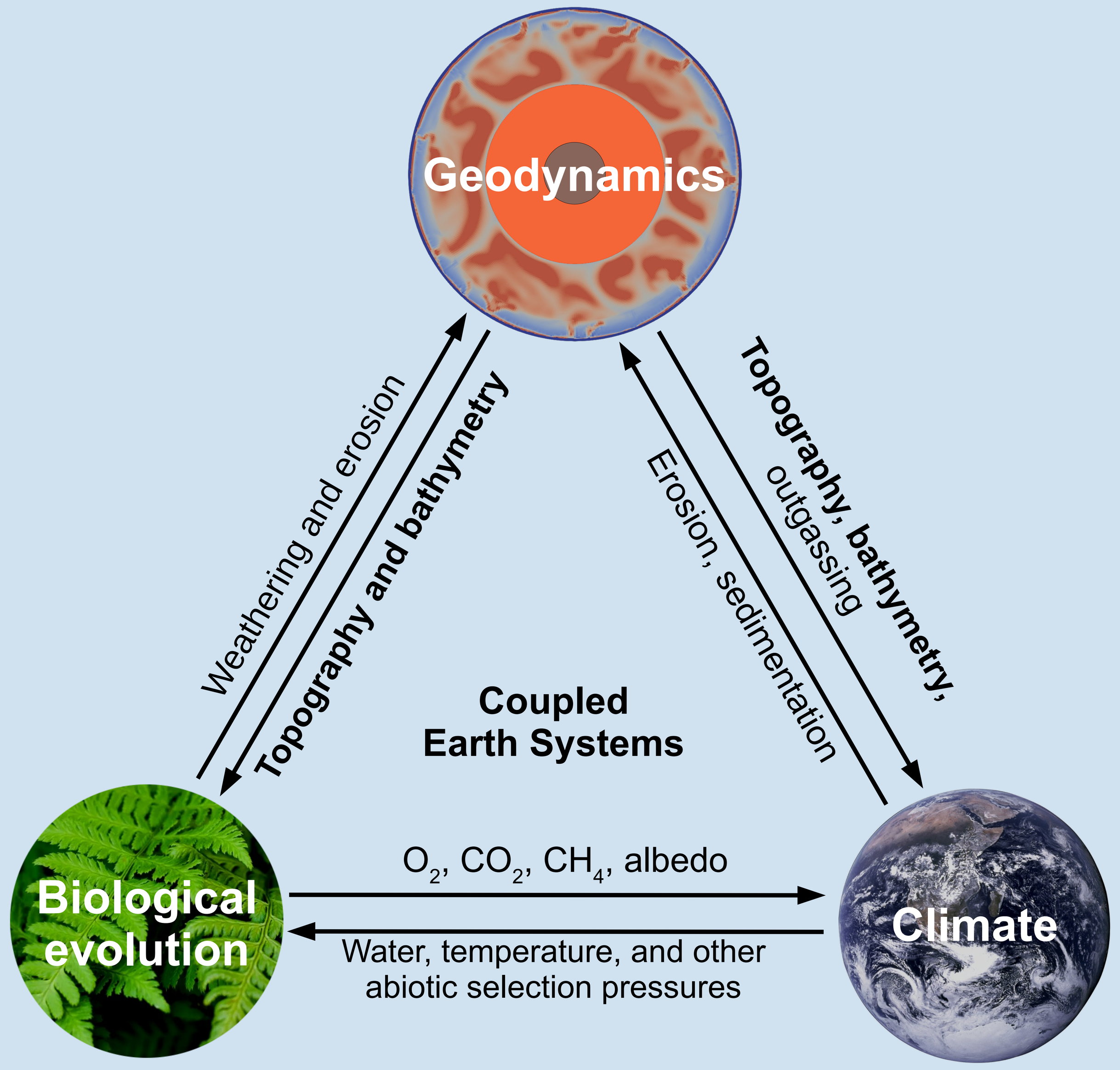
Introduction and Motivation

The study of coupled Earth systems, and in particular the coupled interactions between the lithosphere, atmosphere, and biosphere, have received greater attention in recent years (Gerya et al. 2020). Interactions between these systems occur primarily at the surface, and are driven on the large scale by topographic and bathymetric evolution controlled by deep mantle processes. However, due to the large difference in length scales between the mantle and the surface, it is difficult to capture topographic evolution to a high degree of accuracy in existing global mantle convection models including a free surface boundary condition.



Research goal

Interactions between these processes occur primarily at the surface of our planet. Hence, the primary goal is to investigate the evolution of topography and bathymetry, which may then be coupled to models of climate and biological evolution to study the coupled effect of various geodynamic regimes on the the entire planet.

Tools

We use the global mantle convection code StagYY (Tackley, 2008). StagYY uses a staggered grid finite volume marker-in-cell approach, and can model mantle convection on the global scale in both 2D and 3D using cartesian or spherical coordinates.

Existing surface modelling methods

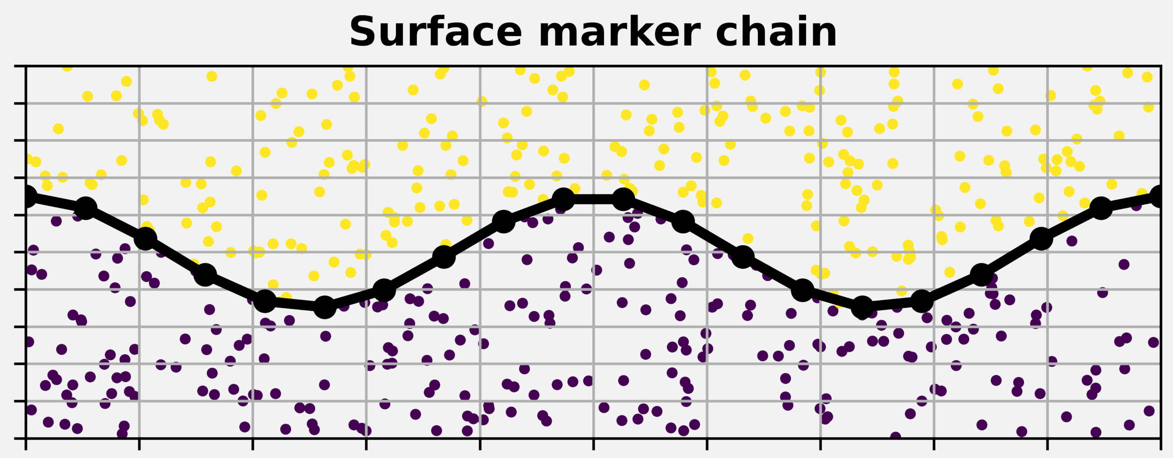
StagYY has two existing methods for extracting topography and bathymetry:

- **Free slip boundary** conditions provide smooth long wavelength topography based on stresses at the surface, but are unsuitable for short wavelength features such as subduction zones.
- **A free surface with sticky air** or alternative boundary conditions (Duretz et al. 2016), produces topography that is rough due to being estimated by cell averaging of Lagrangian markers.

Methods

Surface modelling using markers

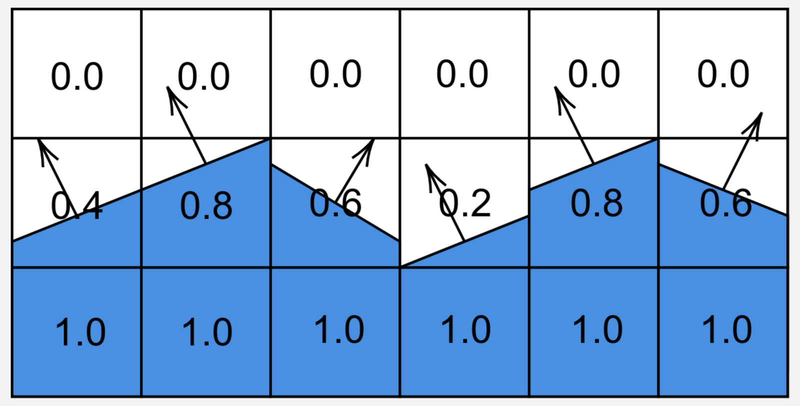
A simple way of modelling a surface in 2D is to use a marker chain (see below). However, a marker chain can be diffusive as reinitialisation is required as the chain distorts over time.



A surface marker chain is suitable for 2D problems, but the complexities of 3D require alternative methods to be used.

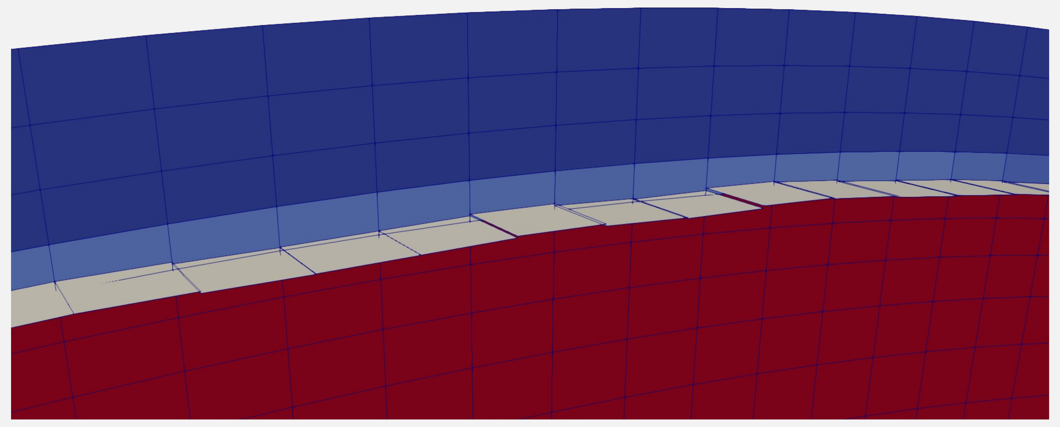
Volume of fluid method

A volume of fluid, (0 if cell is empty, 1 if rock) within each Eulerian cell is used to reconstruct the surface. The volume is advected with the velocity field, and a new surface is reconstructed.



Piecewise Linear Interface Reconstruction

A piecewise linear reconstruction is used in each surface cell. Hence, the surface is modelled as a series of piecewise lines (or planes in 3D), with a certain normal vector. This information is then used to advect the fluid in a way that ensures mass conservation.



Benefits of a VOF implementation:

Requirements for a successful surface tracking method include:

- Provides sub-grid level topography resolution
- Stable over many time steps
- Low computational cost

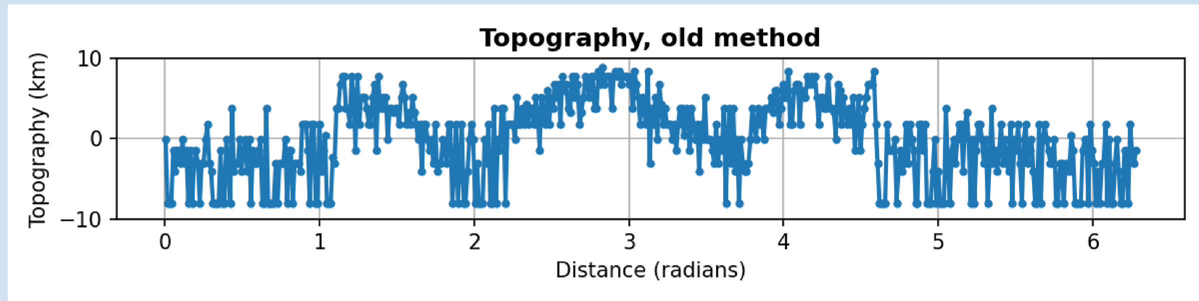
gVOF

The implementation of the Volume of Fluid method in StagYY is taken from the open source Fortran 77 code gVOF (Lopez & Hernndez, 2022). The code is applicable to 2D and 3D, and also works for non-Cartesian geometries.

Applications

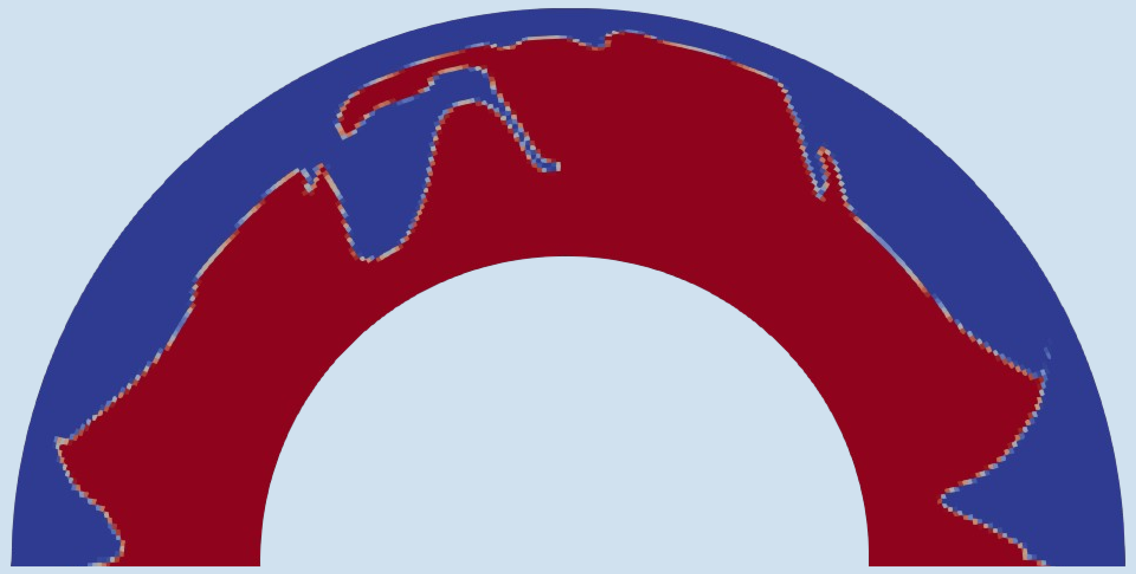
Tracking free surfaces

Compared to the existing method of tracking topography and bathymetry using marker interpolation, our method provides much smoother results and is independent of marker density due to being based on the Eulerian grid.



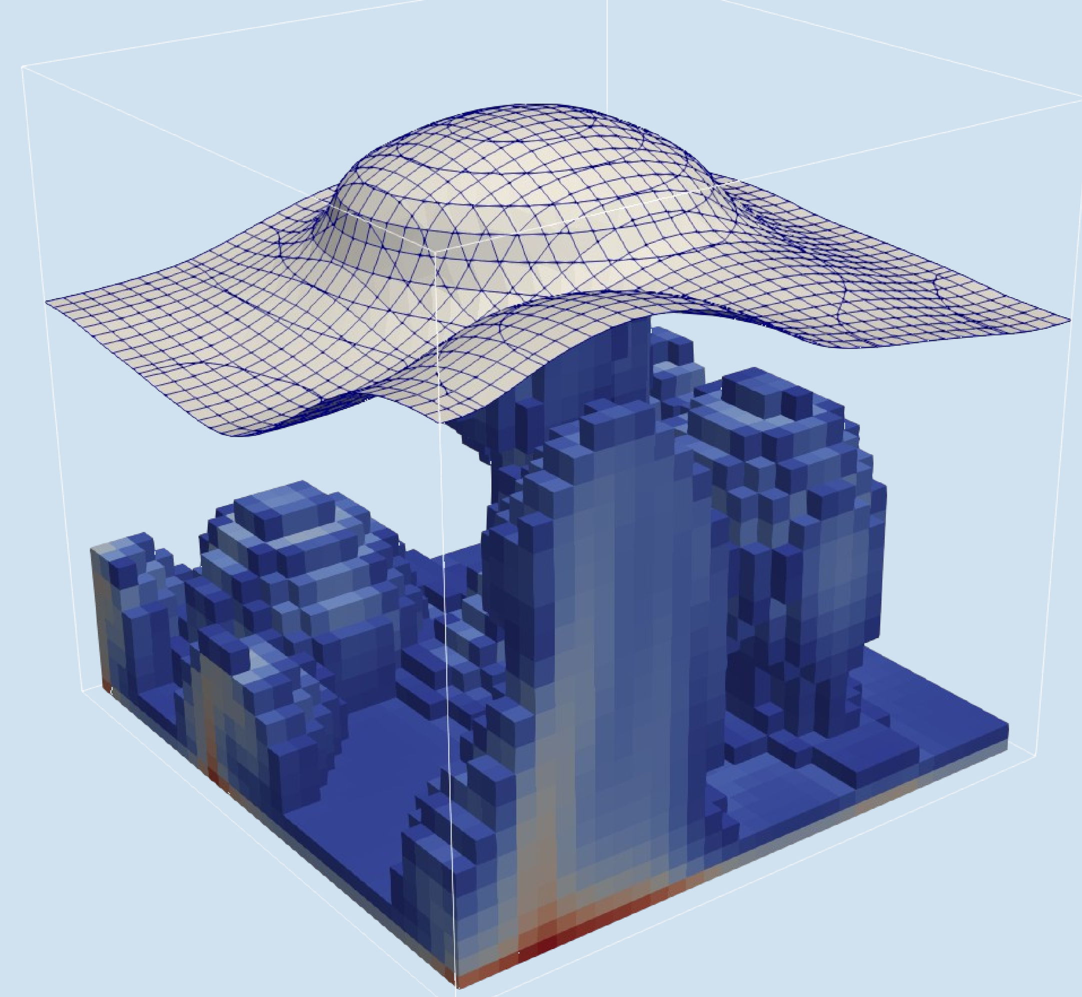
Tracking other interfaces

Our method is also extensible to other interfaces that we might be interested in, such as the lithosphere-asthenosphere boundary, core-mantle boundary, or boundaries between rock types.



3D, non-cartesian geometry

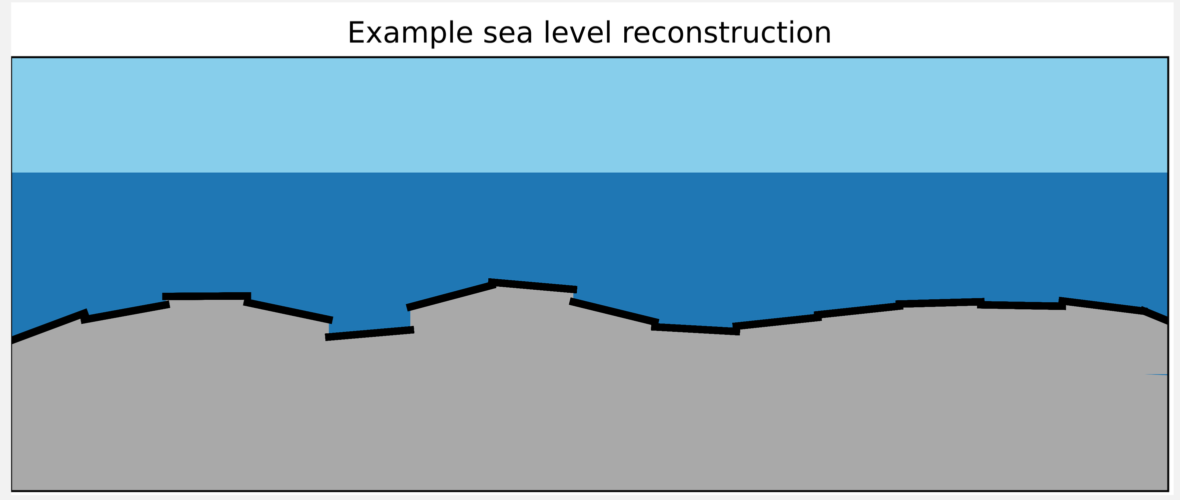
The method is applicable to 3D, and non-cartesian geometry, and the advection method remains mass conservative under these conditions. An example of isosurface advection in 3D cartesian coordinates is shown here.



Future Directions

Sea level determination

Determination of sea level and evolution of land masses is important for understanding couplings with climactic and biological processes. Assuming a constant ocean volume, tracking of sea level changes over time is easy using our new surface representation.

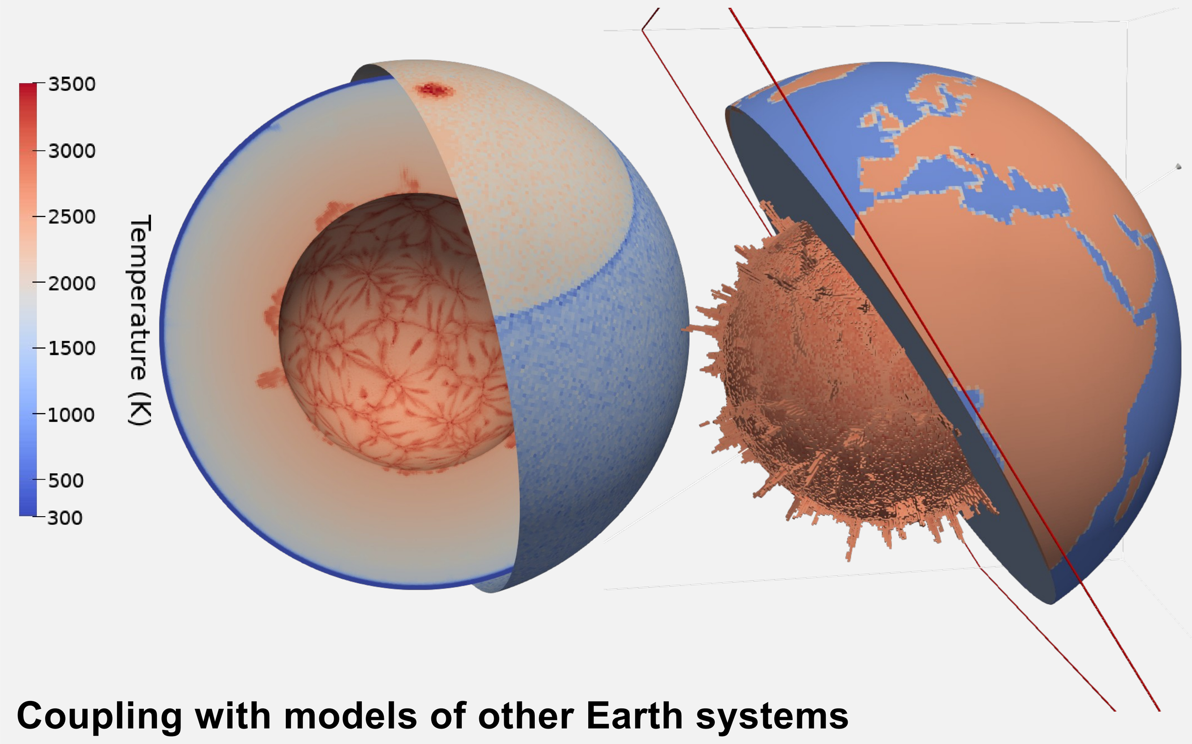


Incorporation of surface processes

Erosion and sedimentation are potentially very important to global scale events potentially involving the coupling between the surface and tectonic processes, such as the Cryogenian glaciations and their impact on global tectonic environment (Sobolev & Brown, 2019).

Global scale models with a free surface in 3D

With our method having a natural extension to 3D, we intend to move to full scale 3D spherical models using free surfaces, enabling accurate topographic reconstruction on the global scale.



Coupling with models of other Earth systems

Much of the work of the Biogeodynamics group at ETH is focused on the coupling between geodynamic models and models of climate and biological evolution. See http://jupiter.ethz.ch/~dstemmler/index.html

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