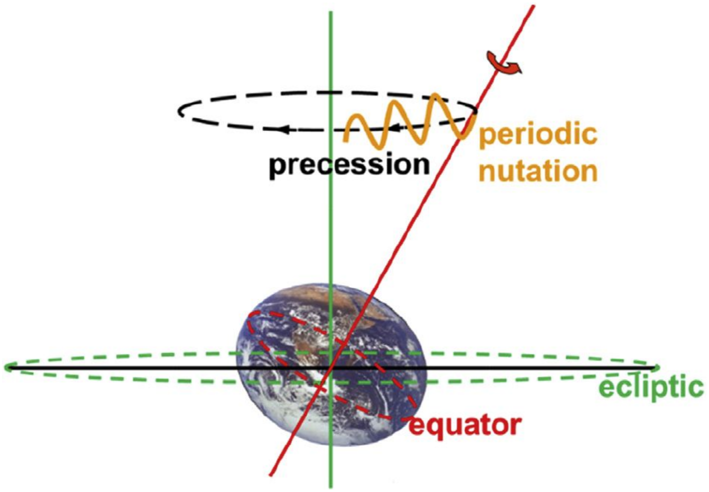
Effective Viscosity at the Earth's Core-Mantle Boundary

Liquid Iron Viscosity

- **Free Core Nutation:** $10^{-1} \sim 10^{-2}\ m^2\ s^{-1}$ (Gwinn et al. 1986; Mathews et al. 2002; Koot et al. 2011)
- **Ab inito calculations:** $10^{-6}\ m^2\ s^{-1}$ (Pozzo et al., 2013; Ichikawa and Tsuchiya, 2015)

Coupling Mechanisms

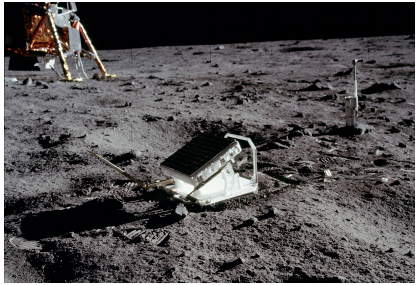
- **Topographic coupling** (Wu and Wahr 1997; Dehant et al. 2014)
- **Electromagneitc coupling** (Buffett, 1992; Mathews and Guo, 2005; Deleplace and Cardin, 2006; Buffett and Christensen, 2007)
- **Viscous coupling with turbulent effect** →



Dehant et al. (2017)

Precession as the source of turbulence
(Triana et al. 2019; Buffett 2021)

Lunar Dissipation

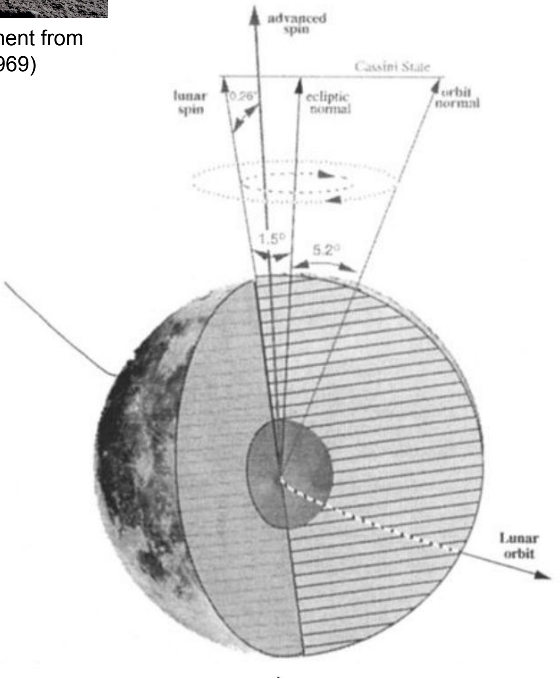


Lunar Laser Ranging Experiment from the Apollo 11 mission (1969)

<u>Solid-body tidal dissipation</u>

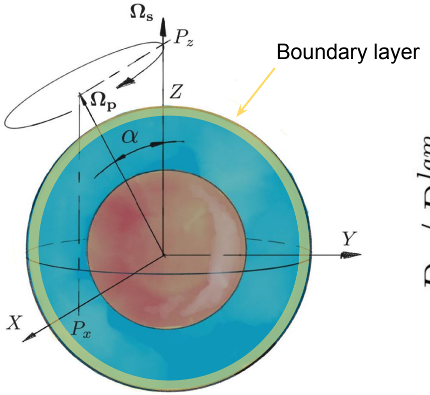
<u>Dissipation at liquid-core/solid-body boundary</u>

- **Analysis of Lunar Laser Ranges:** *58 - 84 MW* (Williams et al. 2001; Williams et al. 2014; Williams and Boggs, 2015)
- **Turbulence Model:** *126 MW* (Sous et al. 2013)

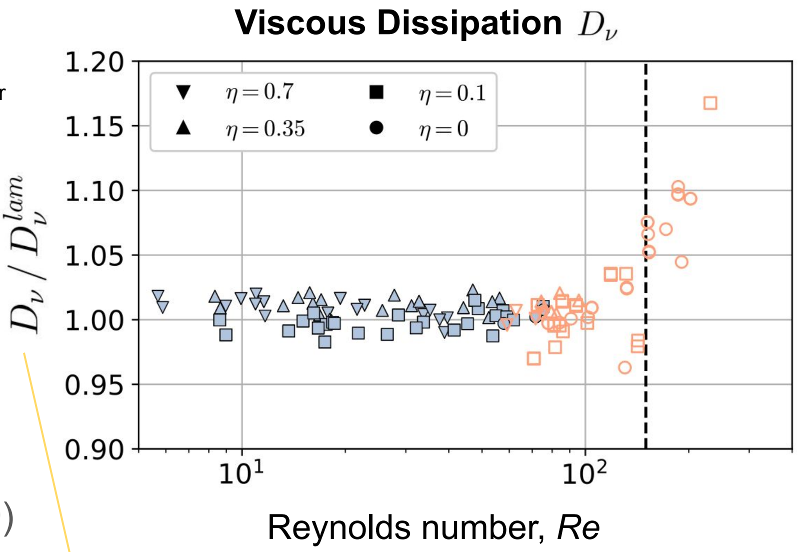


Williams et al. (2001)

Limitation of Global Model



Cébron et al. (2019)



$$\mathcal{D}_\nu^{\mathrm{lam}} = -2.62\, I \Delta\Omega^2 \sqrt{\nu\Omega}/R$$

(Greenspan, 1968)

Features

- Global model
- Computationally demanding

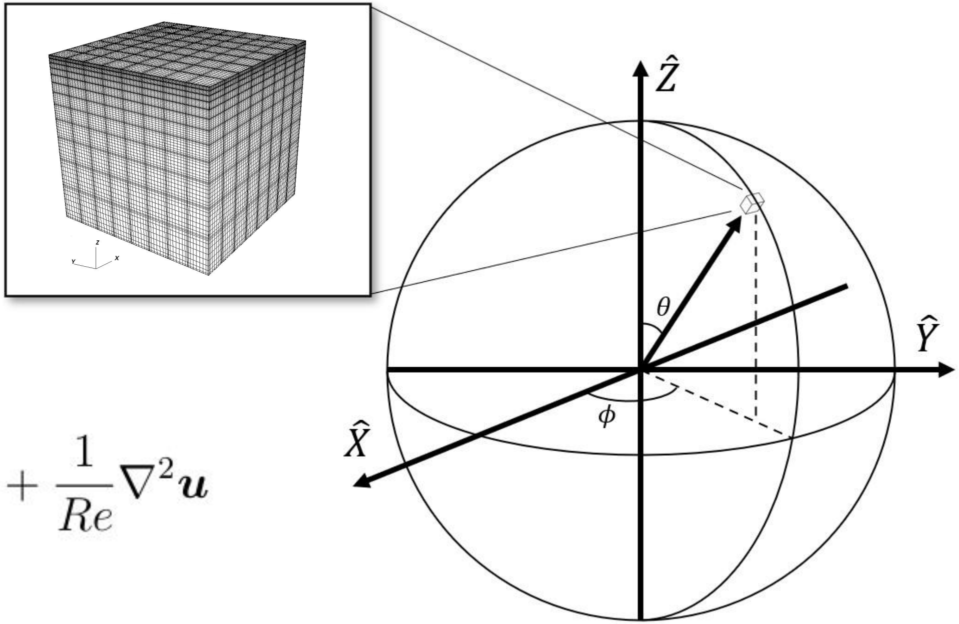
Applications

- Earth: *Re = 500*
- Moon: *Re = 16,700*

Local Model (e.g., Buffett 2021)

Parameters

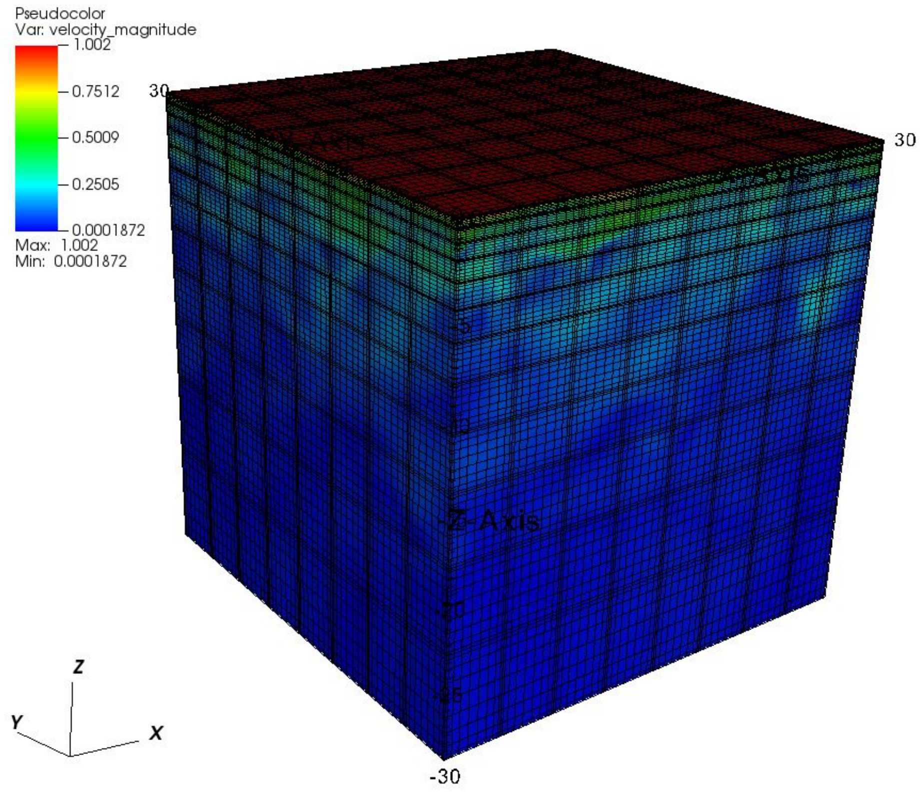
- Re = 5 ~ 700

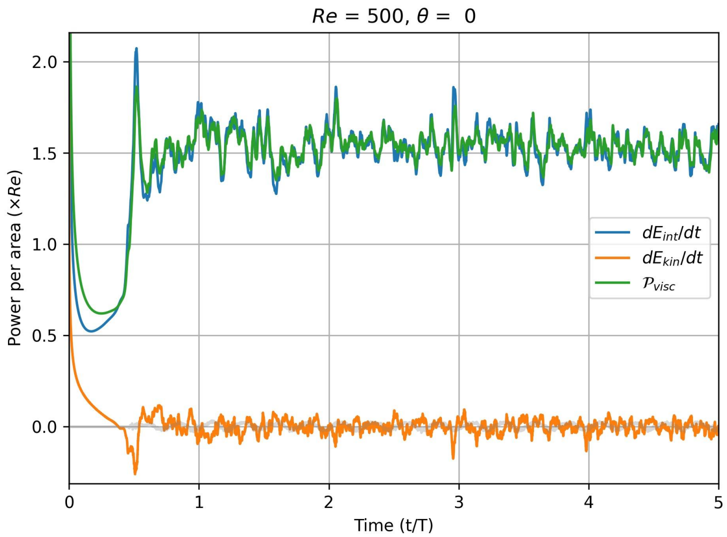


Navier Stokes equation:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \frac{1}{Re}\left(2\hat{\boldsymbol{k}}_f \times \boldsymbol{u}\right) = -\nabla P + \frac{1}{Re}\nabla^2 \boldsymbol{u}$$

Flow in the Boundary Layer

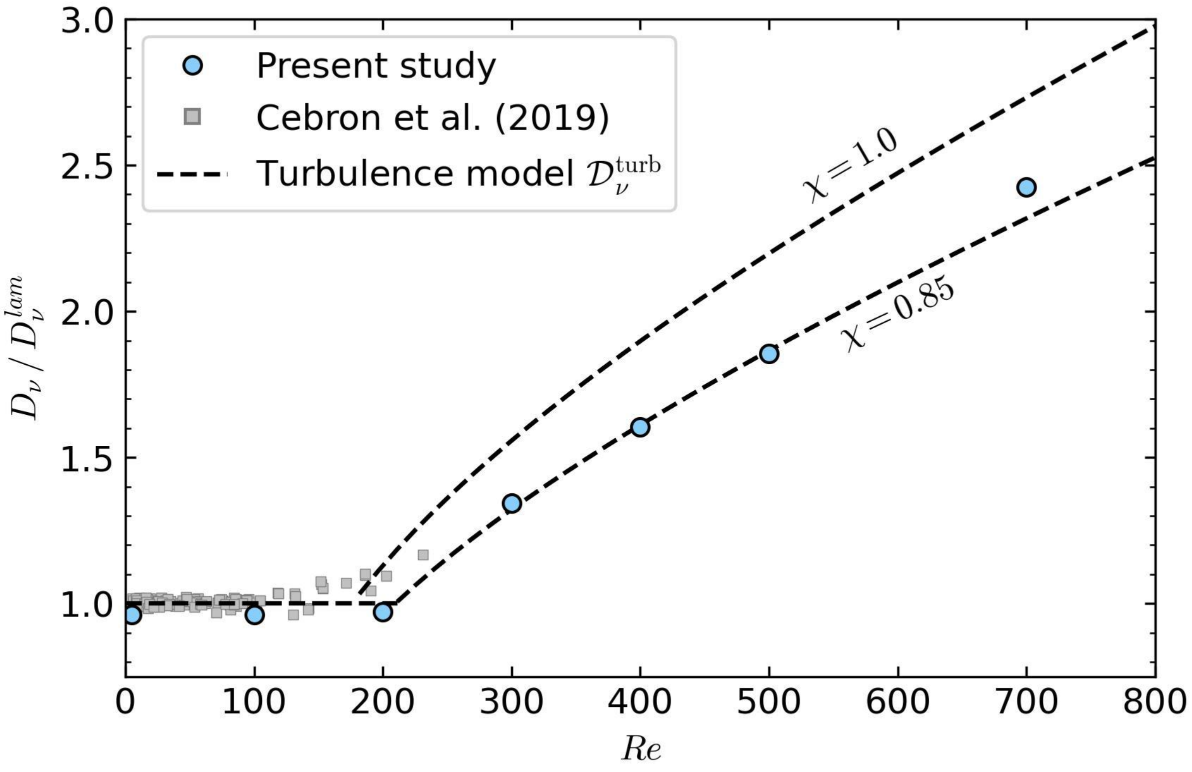




Validation

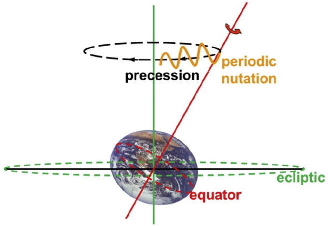
- Resolution test
- Domain height test
- Law of the wall (Tennekes et al. 1972)

Total Dissipation



Implications

Liquid Iron Viscosity

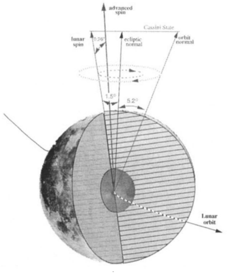


- **Free Core Nutation:** *10^{-1} ~ 10^{-2} m^2 s^{-1}* (Gwinn et al. 1986; Mathews et al. 2002; Koot et al. 2011)
- **Ab inito calculations:** *10^{-6} m^2 s^{-1}* (Pozzo et al., 2013; Ichikawa and Tsuchiya, 2015)
- **This study:** *3.5 x 10^{-6} m^2 s^{-1}*

Coupling Mechanisms

- **Topographic coupling**
- **Electromagneitc coupling**
- **Viscous coupling with turbulent effect**

Dissipation at liquid-core/ solid-body boundary



- **Analysis of Lunar Laser Ranges:** *58 - 84 MW* (Williams et al. 2001; Williams et al. 2014; Williams and Boggs, 2015)
- **Turbulence Model:** *126 MW* (Sous et al. 2013)
- **Turbulence Model:** *72 - 84 MW* (this study)

More details …

Method

Turbulence Model (cf. Yoder 1995, Williams et al. 2001; Cébron et al. 2019)

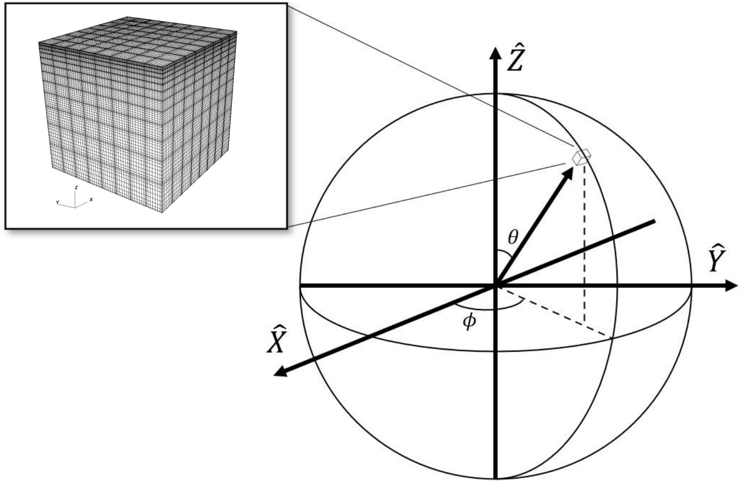
$$\mathcal{D}_{\nu}^{\mathrm{turb}} = -2.62\,\sqrt{2}kI\Delta\Omega^3$$

$$k = (u_*/U)^2 \cos\beta$$

Friction velocity Veering angle

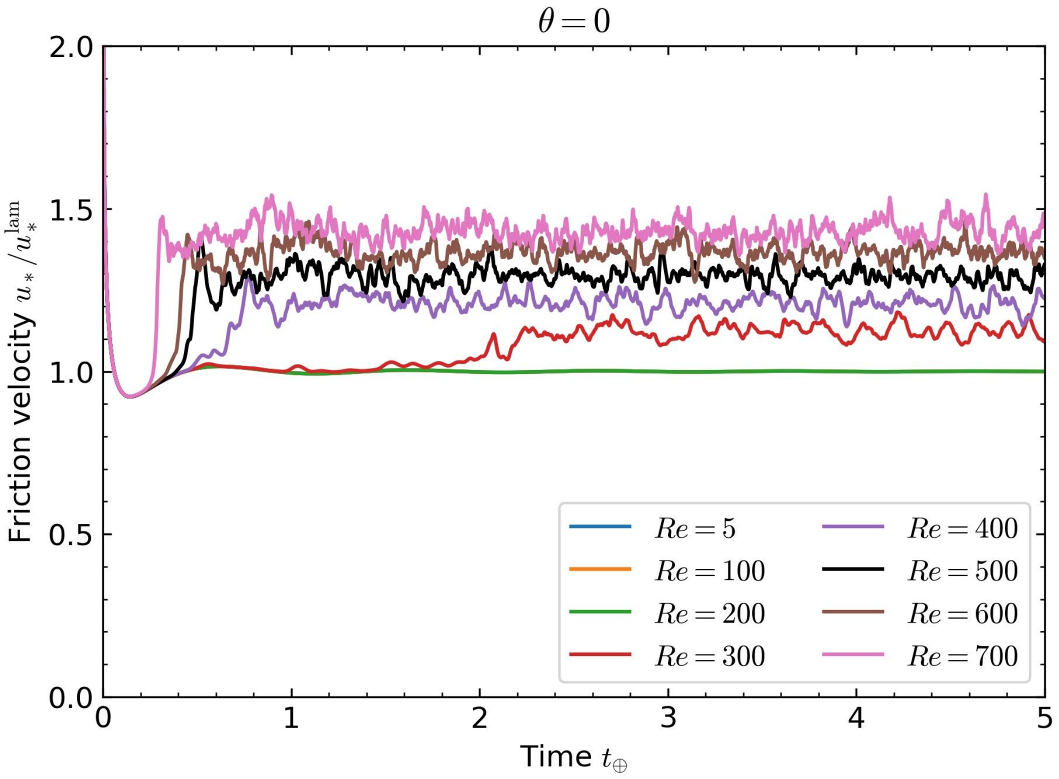
Numerical Simulation (e.g., Buffett 2021)

- **Local model**

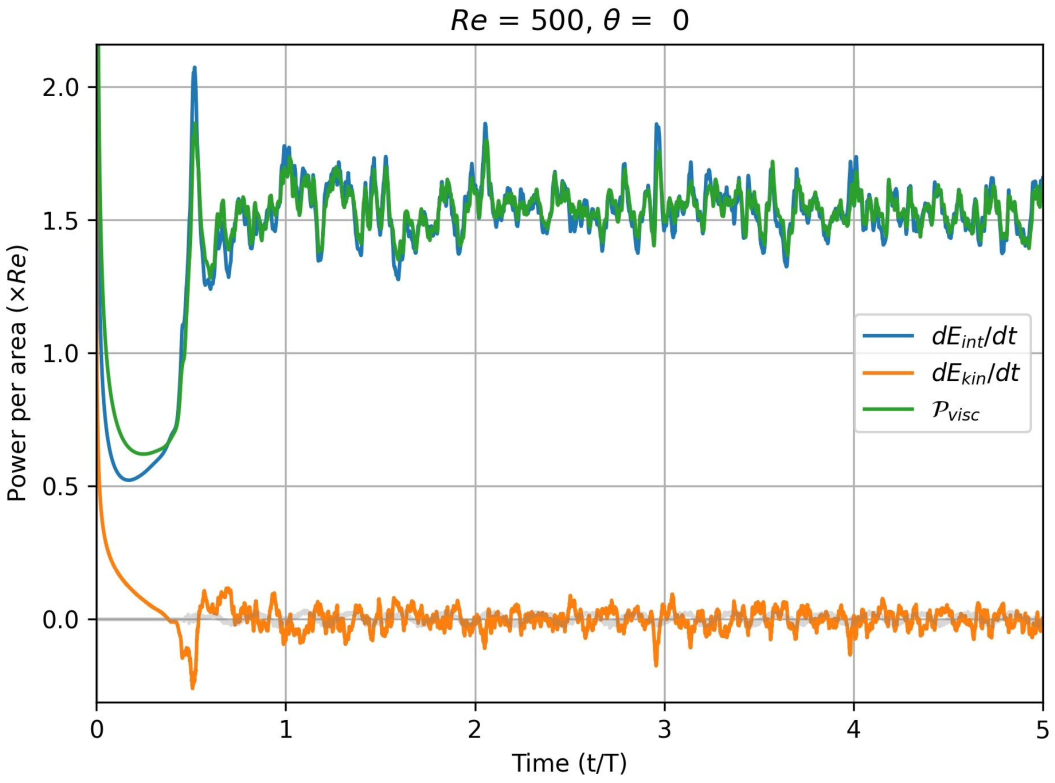


$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + \frac{1}{Re}\left(2\hat{\boldsymbol{k}}_f \times \boldsymbol{u}\right) = -\nabla P + \frac{1}{Re}\nabla^2\boldsymbol{u}$$

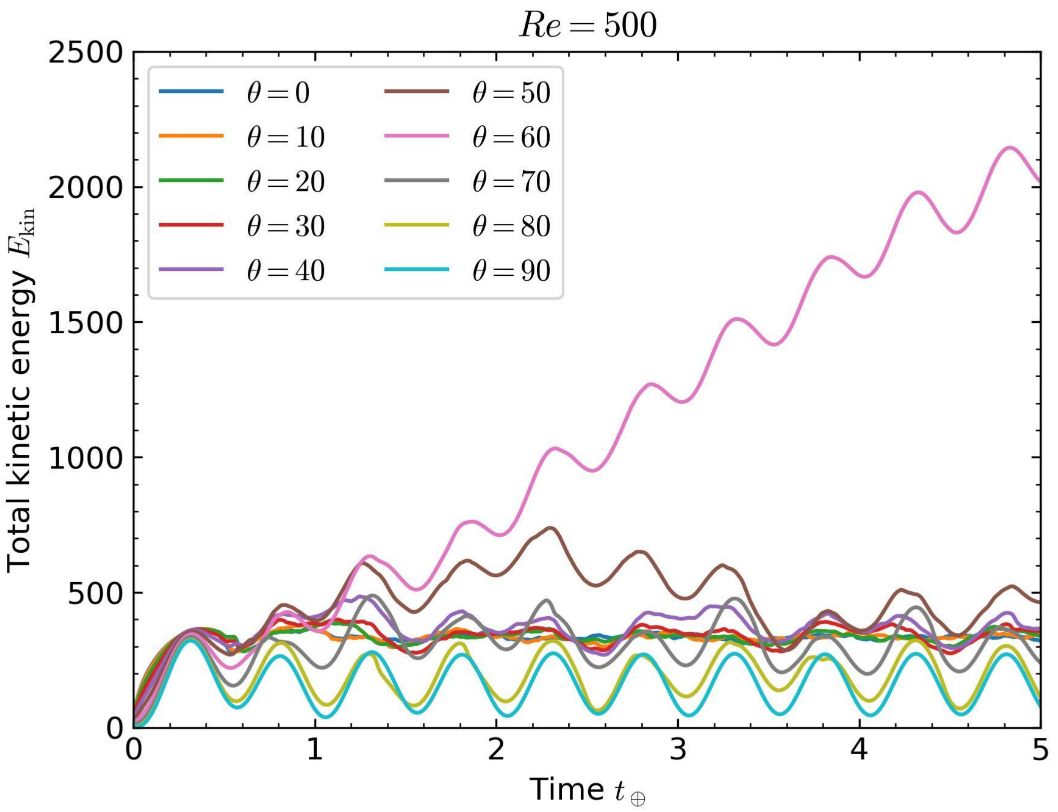
Friction Velocity



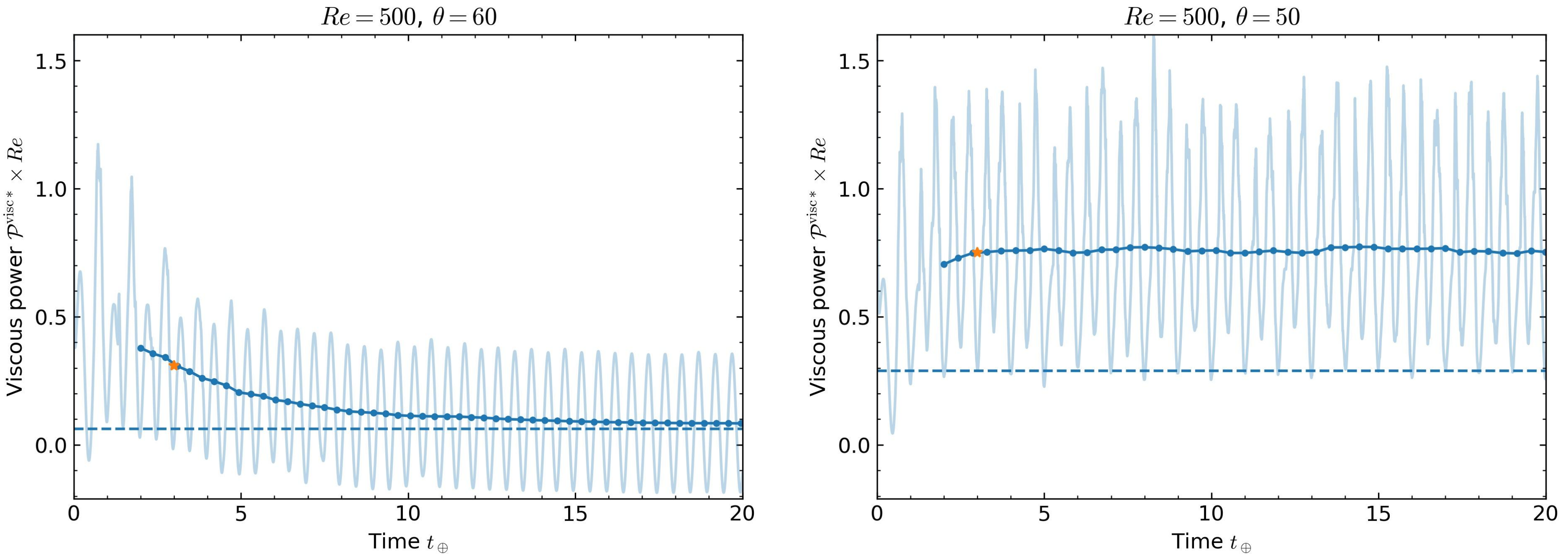
Time evolution



Kinetic Energy

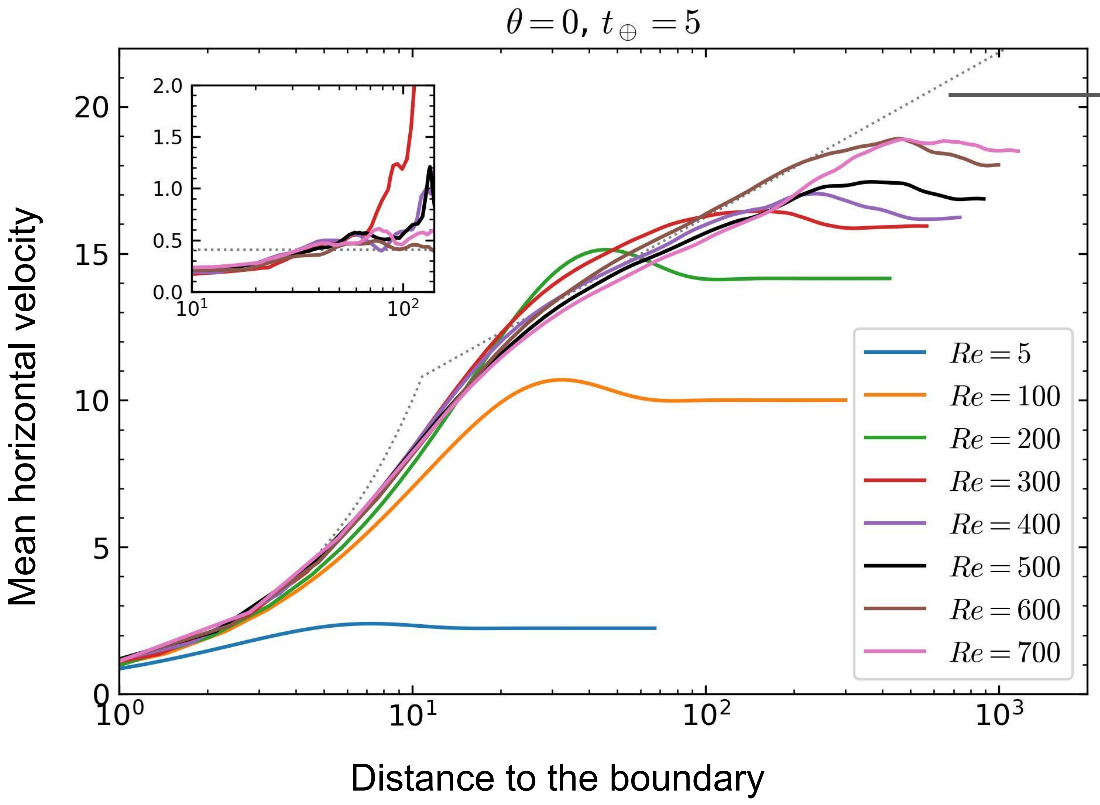


Two cases



$Re = 500,\ \theta = 60$
$Re = 500,\ \theta = 50$
Viscous power $\mathcal{P}^{\mathrm{visc}*} \times Re$
Time $t_\oplus$
1.5
1.0
0.5
0.0
0
5
10
15
20

Velocity Profiles



Law of the wall

(Tennekes et al. 1972)

Similarity Theory (Csanady, 1967; Spalart, 1989)

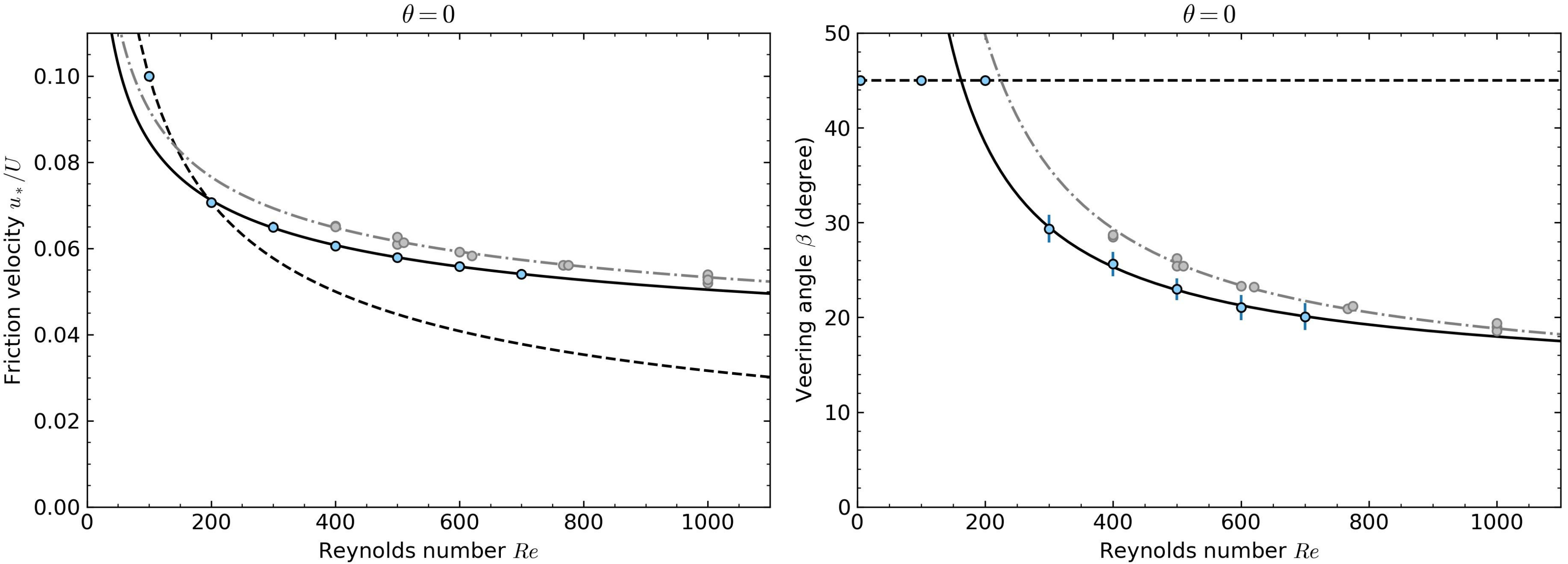
$$\frac{U}{u_*}\cos\vartheta + \frac{2}{0.41}\ln\frac{U}{u_*} = \frac{2}{0.41}\ln Re - \frac{1}{0.41}\ln 2 + B \qquad \sin\vartheta = \frac{A}{U/u_*} \qquad \vartheta = \beta + \frac{2C_5}{Re^2}\left(\frac{U}{u_*}\right)^2$$

Best fit:

A = 5.74

B = 1.55

C_5 = -25



Local Dissipation