Superstatistical analysis of sea surface currents in the Gulf of Trieste, measured by HF Radar, and its relation to wind regimes

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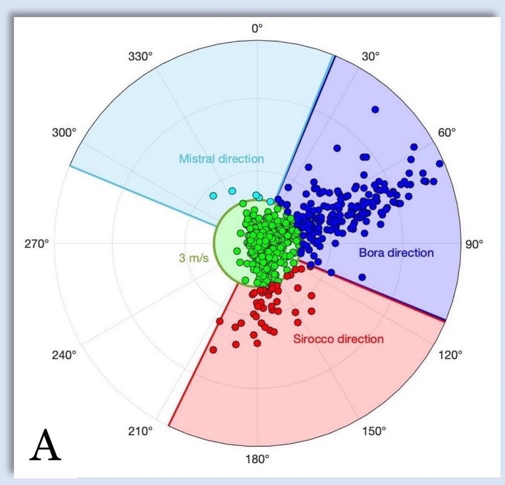
Research Aims

Earth observing systems provide us with an **ever increasing amount of data**. This allows us today to **move** from **considering** low order moments of fluctuating observations to their **Probability Density Functions** (PDFs).



A

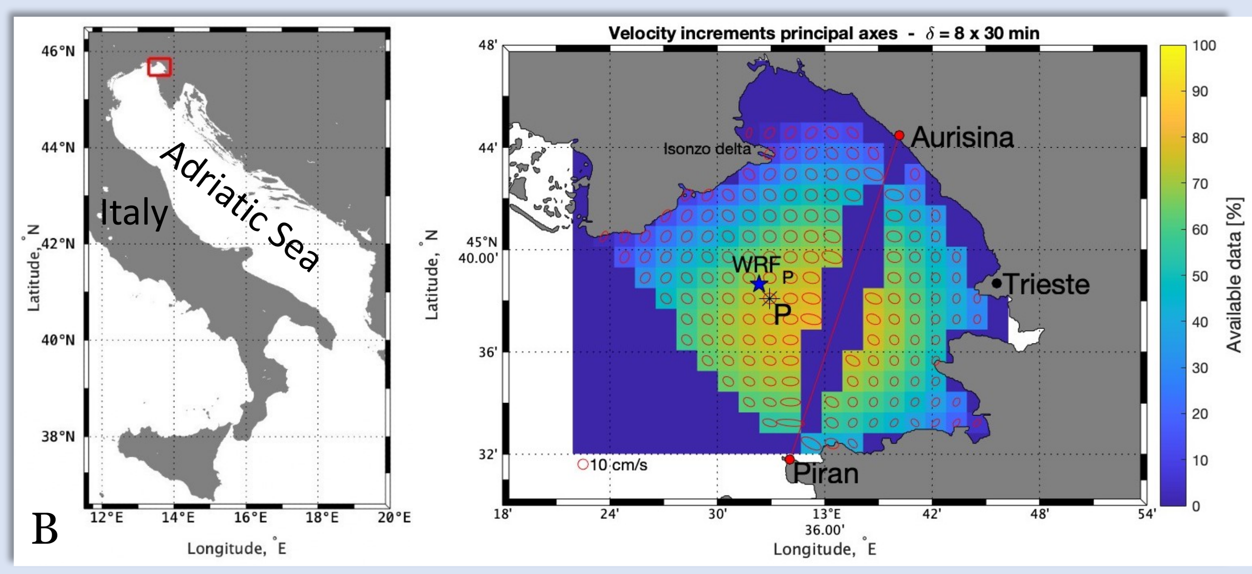
We used **ocean currents data** from the Gulf of Trieste (Adriatic Sea, Italy, Fig. B), where **three distinct wind forcing regimes** blow [PR01]: Bora, Sirocco and low wind (Fig. A), asking for a **superstatistical analysis** [BCS05] of the sea surface current data.

We further developed an idealized deterministic air-sea interaction model. The next step consists in the integration of a stochastic model governing the evolution of the PDF.

The Data

Sea surface currents consist of **High Frequency Radar current data** in the Gulf of Trieste. One station is located in Aurisina (Italy - OGS) and the other in Piran (Slovenia – first NIB, later ARSO) (Fig. B). The P grid point was selected as the best candidate for the analysis (Fig. B). The data cover a time range of almost two years (2021-2022) with a time resolution of 30 minutes.

The horizontal wind data are provided by the **WRF model forecastings** (ARPA FVG, [G18]) with a time resolution of 1 hour. The WRF_P grid point was selected for the analysis.



B

https://thredds.hfrnode.eu:8443/thredds/NRTcurrent/HFR-NAdr/HFR-NAdr_catalog.html

Superstatistics

Driven nonequilibrium systems of sufficient complexity are often effectively described by a **superposition of different dynamics on different time scales**. They result in a **superposition of different statistics**, or in a short, a superstatistics [BCS05].

Following the procedure described by [BCS05], we obtained:

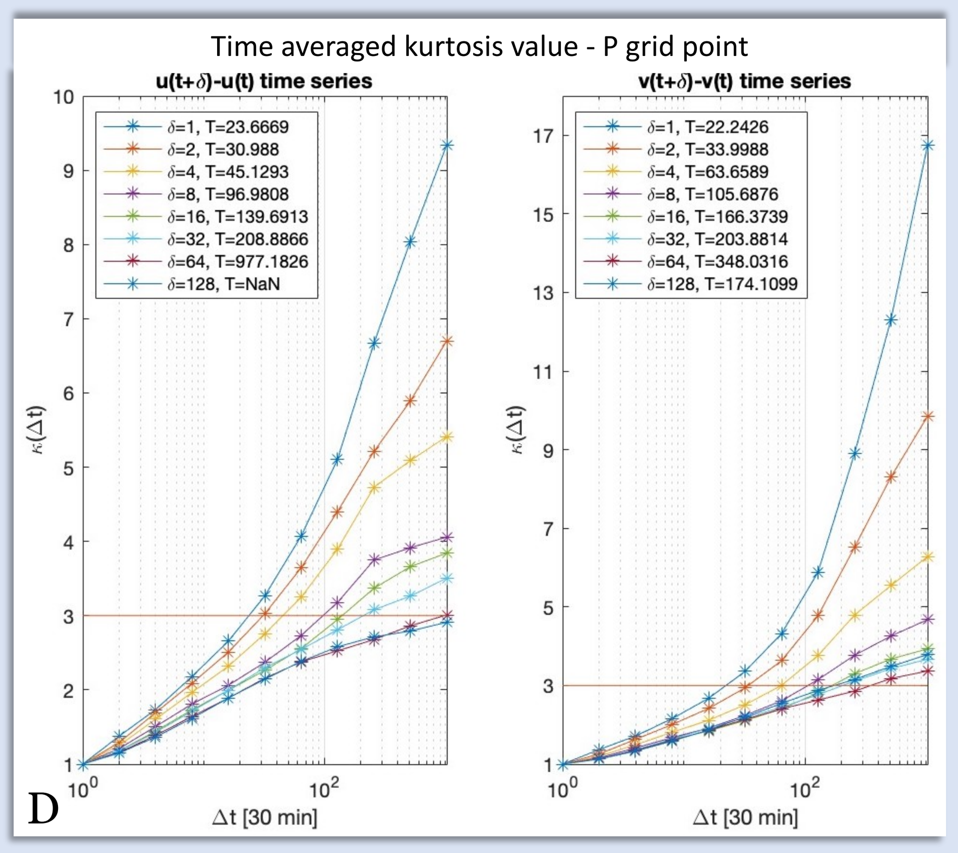
- **fat-tailed PDF $p(\delta u)$**, where $\delta u(t) = u(t+\delta) - u(t)$ is the velocity increment of the sea surface current $u(t)$:

$$p(\delta u) = \int_0^\infty f(\sigma^2) p(\delta u|\sigma^2) d\sigma^2;$$

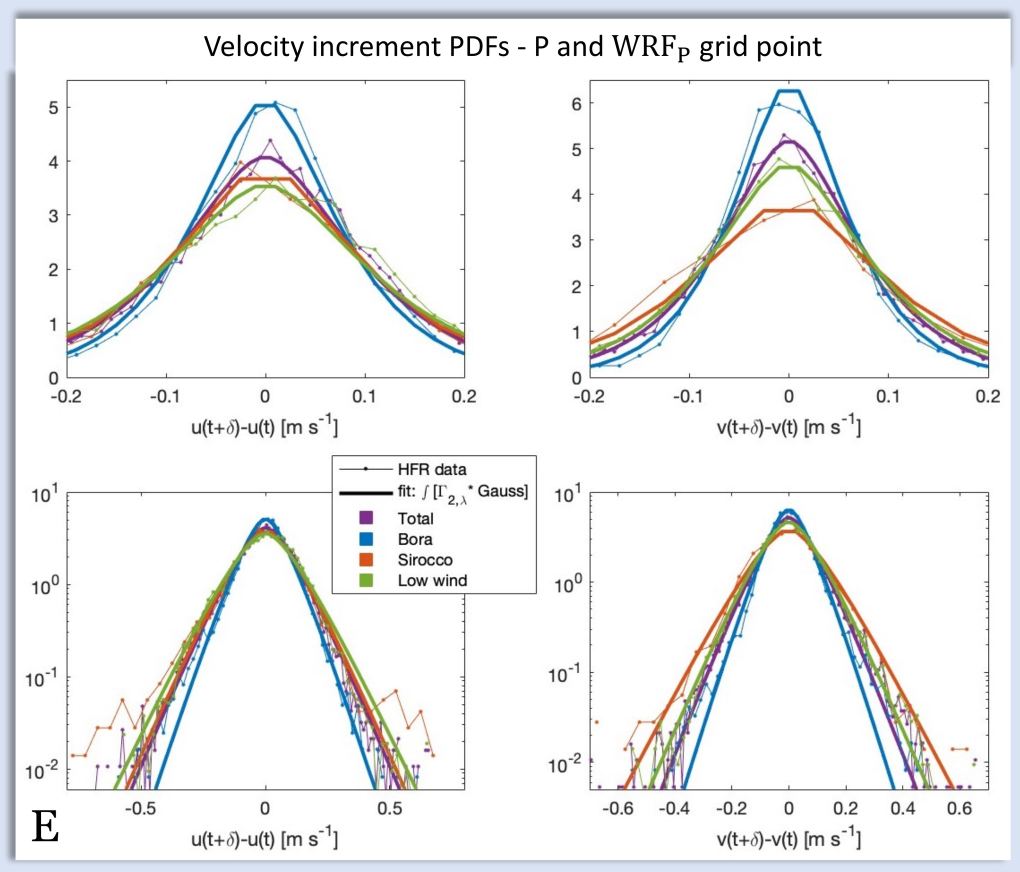
  where the slowly varying (Fig. C) stochastic process $\sigma^2(t)$ represents the **local gaussian variance**.

- **Time scale separation** $\tau \ll T$: a short time scale τ represented by the relaxation time and a **longer time scale T for which $p(\delta u|\sigma^2)$ is gaussian** (kurtosis value equal to 3, Fig. D) (**local entropy maximization**).

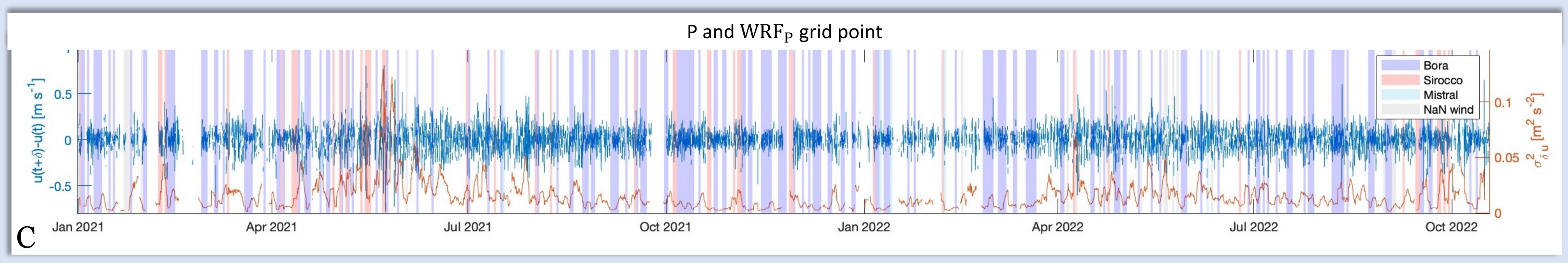
Choosing the velocity time increment $\delta = 8 \times 30\ min = 4\ h$ leads to $T_\star \simeq 2\ days$, which is in agreement with the ocean surface-layer turbulent and the synoptic atmospheric time scales, respectively. For this δ value, $\tau_\star \simeq 1\ h\ 50\ min$.



D



E



C

Which is the shape of $f(\sigma^2)$?

We **maximized entropy on the two degrees of freedom** (a convolution of two exponentials), obtaining a gamma distribution with fixed shape parameter equal to 2:

$$f(\sigma_\star^2) = \Gamma_{2,\lambda_\star}(\sigma_\star^2) = \lambda_\star^2 \sigma_\star^2 e^{-\lambda_\star \sigma_\star^2}$$

So we found a **new analytical class** which fits well the observed data (Fig. E):

$$p(\delta u) = \frac{\sqrt{2\lambda_u} e^{-\sqrt{2\lambda_u}|\delta u|}(\sqrt{2\lambda_u}|\delta u| + 1)}{4}$$

Note that the analytical fat-tailed $p(\delta u)$ is given maximizing entropy twice: first locally (local gaussianity of the velocity increments) and secondly on the two degrees of freedom (convolution of two exponentials of the variance)

Does $p(\delta u)$ depend on the wind forcings?

The **same analytical law of the PDF** is obtained with different second order moment (the only free parameter in the fit) **for the different wind regimes** (Fig. E) with the **strongest wind forcing (Bora)** leading to the **lowest fluctuations**. This fact points towards a **universal behaviour**.

We found that the PDF of the fast velocity fluctuations is gaussian and the PDF of the velocity variance at time scales larger than T is $\Gamma_{2,\lambda_\star}$.
Superstatistics is more than a tool for fitting fat-tailed PDFs: it gives the characteristic time scales and when combined with the maximum entropy principle it indicates the number of the degrees of freedom.
The analytic form of the PDF of the velocity increments can be used to develop parameterizations of the dynamics in the ocean surface layer at time scales faster than a few days and can discriminate between existing parameterizations.

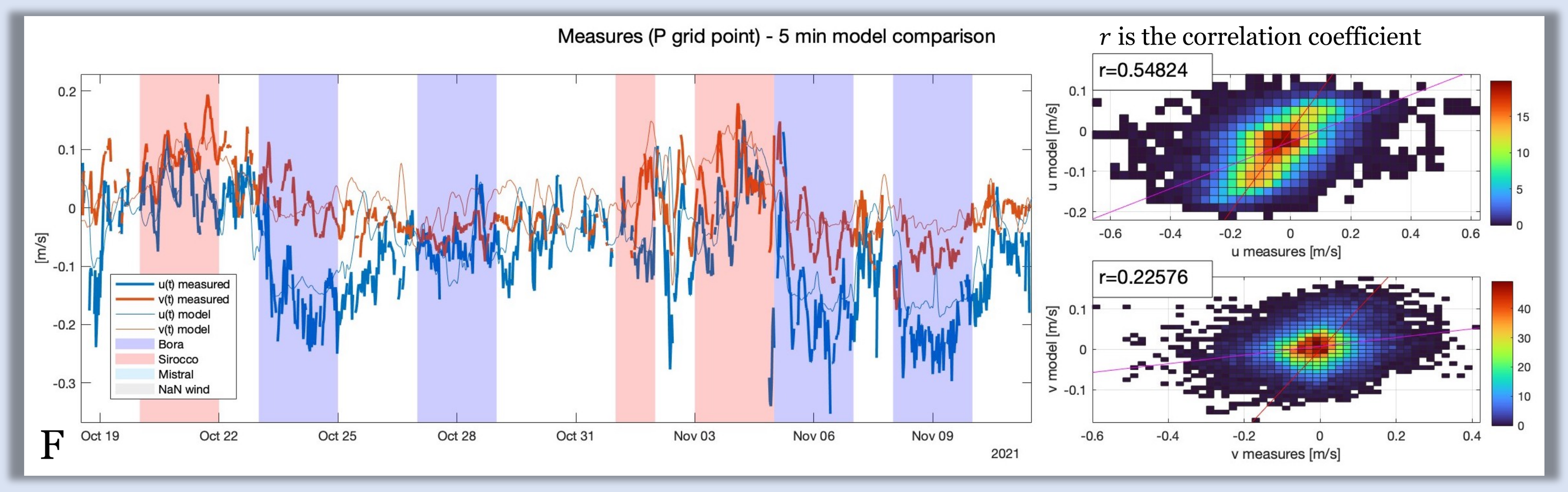
Deterministic Model

We developed an **idealized** wind driven, time-dependent **model** including a quadratic drag and the Coriolis force, **for the deterministic sea surface current** $\vec{u_D} = (u_D, v_D)$:

$$\tilde{h}\begin{pmatrix}\partial_t u_D \\ \partial_t v_D\end{pmatrix} = \begin{pmatrix}-C_B|\vec{u_D}| & \tilde{h}f \\ -\tilde{h}f & -C_B|\vec{u_D}|\end{pmatrix}\begin{pmatrix}u_D \\ v_D\end{pmatrix} + \begin{pmatrix}F_u \\ F_v\end{pmatrix}$$

explaining about the **50% of the observed variability** (Fig. F),

where: $\tilde{h} = \rho_o h$ (ρ_o is the ocean density, h is the considered ocean surface layer depth), $C_B = \rho_o c_b$ (c_b is the bottom/underlying layer drag coefficient), f is the Coriolis parameter and $\vec{F} = \rho_a c_a |\vec{u_a}| \vec{u_a}$ is the wind forcing (ρ_a is the atmospheric density, c_a is the atmospheric drag coefficient, $\vec{u_a}$ is the wind speed close to the ocean surface). The C_B coefficient and the $\tilde{h}$ density-depth have been computed combining the data and the model.



F

Future steps

Include uncertinities and unresolved processes in the model by adding an equation for a new stochastic current $\vec{u_S}$ (where $\vec{u} = \vec{u_D} + \vec{u_S}$) **through Stochastic Differential Equations**, in order to reproduce the observed superstatistics.

Bibliography

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