Inverse problems for climate policy mixes including geoengineering

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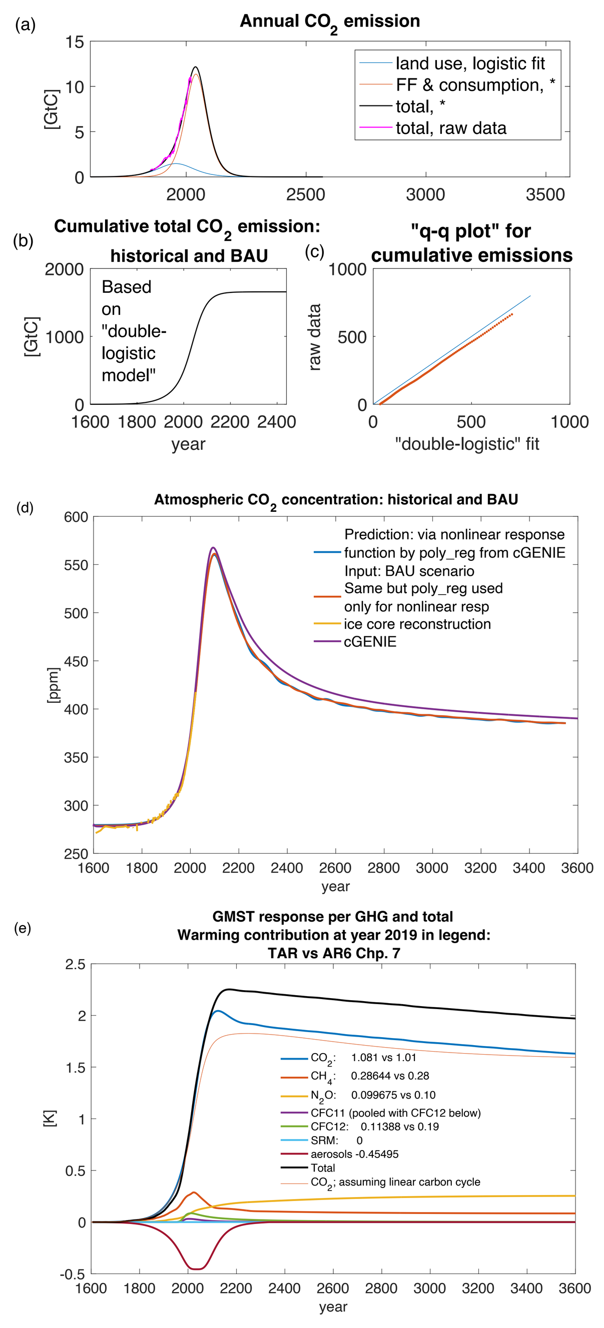


Business as usual (BAU) cumulative emission **scenario** is assumed to follow the logistic eq. of natural growth within limits (K):

$$\frac{dP}{dt} = rP\left(1-\frac{P}{K}\right)$$

whose solution is the logistic curve:

$$P(t) = \frac{KP_0e^{rt}}{K+P_0(e^{rt}-1)} = \frac{K}{1+\left(\frac{K-P_0}{P_0}\right)e^{-rt}}$$

$$\lim_{t\to\infty} P(t) = K$$

The momentary emission $E = dP/dt$ is a bell shape curve, also called the Hubbert curve after the pioneer of peak oil theory. The spark of the indsutrial revolution ($P(t=0)=P_0$) initiates an exponential initial growth

$$E(t) \sim rP_0e^{rt} \ (t\to 0),$$

which later slows down and decays.

Attainment of the Paris15 targets

We include both abatement and geoengineering in the policy mix.

1. Abatement is modelled by a nonautonomous logistic eq.

$$K(t) = K_{BAU} + (P - K_{BAU})\min(r_at, 1)$$

$K(0) = K_{BAU},\ t=0$ at year 2000

2. CDR is modelled as a negative emission goeverned by a sigmoid function:

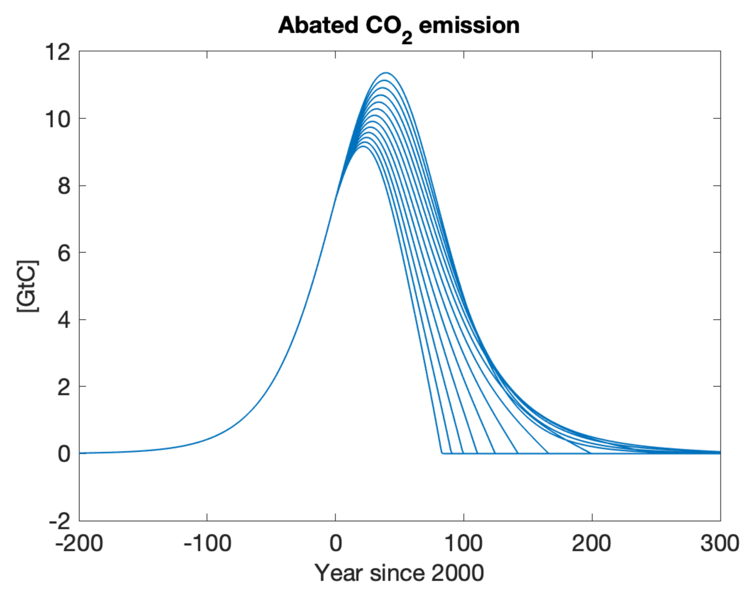
$$CDR = \frac{A_{CDR}}{1+\exp[(t-T_{CDR})/\tau_{CDR}]}$$

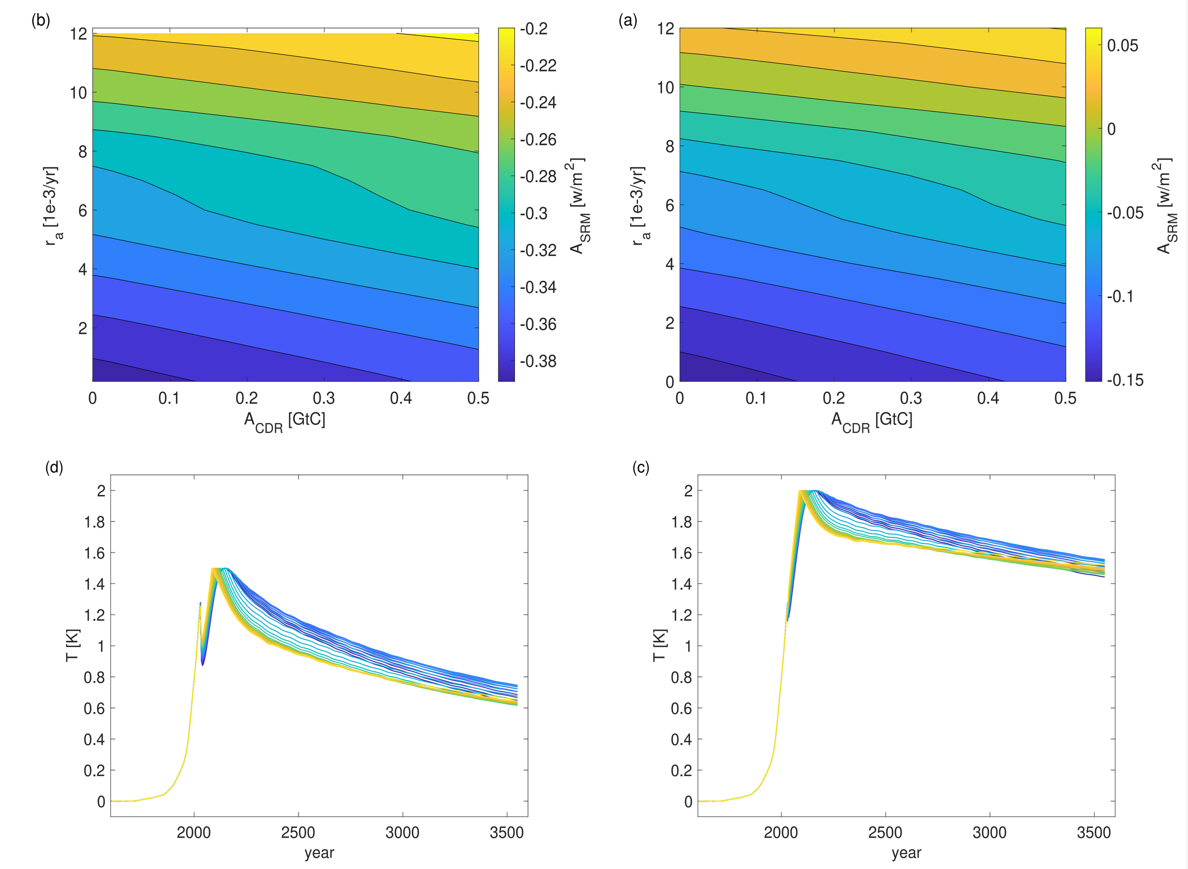
$$\tau_{CDR} = 10 \text{ yr},\ T_{CDR} = 2\tau_{CDR},\ t=0 \text{ at } 2000$$

3. SRM is modelled also by a sigmoid function, such that it's available from about 2030.

Inverse problem: for A_{SRM}, T_{SRM} and τ_{SRM} such that the minimum amount of sulfate is used to attain a maximal GMST level.

Solution: SRM needs to start ASAP and most vigorously.



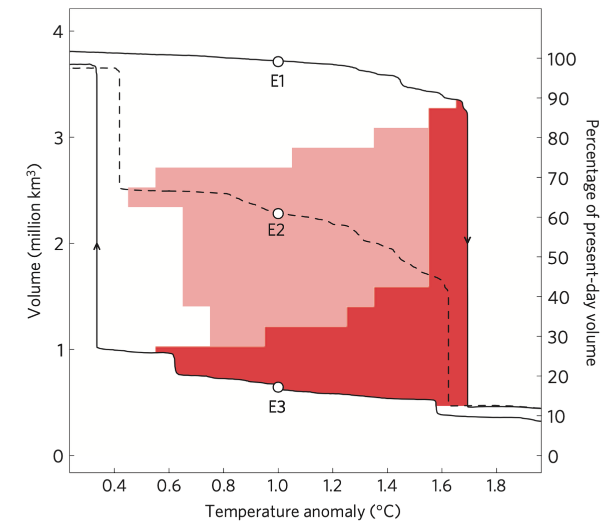


Conservation of the Greenland ice sheet

Approximate bifurcation diagram of the tetra-stable system by the normal form:

$$f^*(x) + \mu = 0,\ f^*(x) = \sum_{i=0}^{8} p_i x^i$$

With this, the "coordinates" of the saddle-node bifurcation points imply the parameters via the linear system of eqs.



Robinson et al. NCC 2012

$$\mu_i = -f^*(x_i),$$

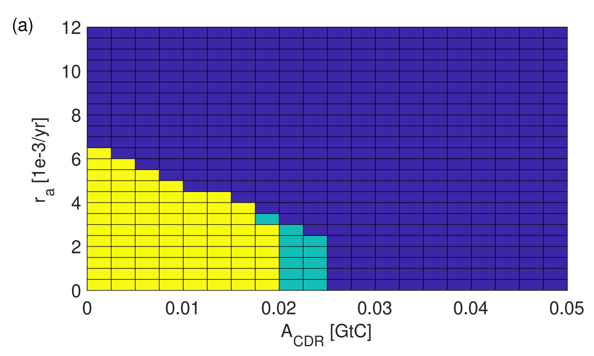
$$\left.\frac{\mathrm{d}\mu}{\mathrm{d}x}\right|_{x_i} = -\left.\frac{\mathrm{d}f^*}{\mathrm{d}x}\right|_{x_i} = 0, i=1,\dots,4$$

$$\mu_1 = 1.7,\ x_1 = 3.4,\ \mu_2 = 0.41,\ x_2 = 2.4,$$
$$\mu_3 = 1.6,\ x_3 = 1.6,\ \mu_4 = 0.38\ [^\circ\mathrm{C}],\ x_4 = 1\ [\text{Mega km}^3 = \text{Peta m}^3]$$

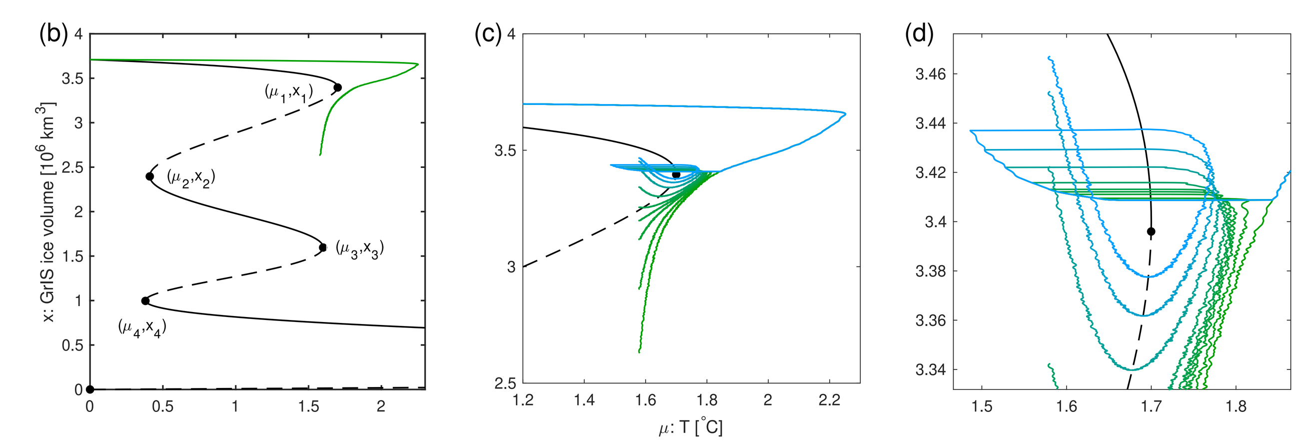
Thus the 1D ODE model is:

$$C\frac{\mathrm{d}x}{\mathrm{d}t} = f(x) = -(f^*(x)+\mu)$$

"Regime diagram": which atttractor we end up on depending on the forcing scenario.



Inverse problem: Start geoengineering in year 5000 at A_{SRM} = -0.1 [W/m²]. How long do we need to **maintain** it in order to avoid the collapse of the GrIS?



Response theory

Consider the nonautonomous system of ODEs

$$\dot{x} = g(x,t) = g_0(x) + \epsilon g_1(x) f(t)$$

In the weak forcing limit (small ϵ), the response can be obtained in a perturbative framework as

$$\Psi(t) = h_0 + \lim_{N\to\infty}\sum_{n=1}^{N}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} \underbrace{h_n(\tau_1,\dots,\tau_n)}_{\text{Volterra kernel}} \prod_{j=1}^{n} f(t-\tau_j)\,d\tau_j$$

called **Volterra series**, where $\Psi = \Psi(x)$.

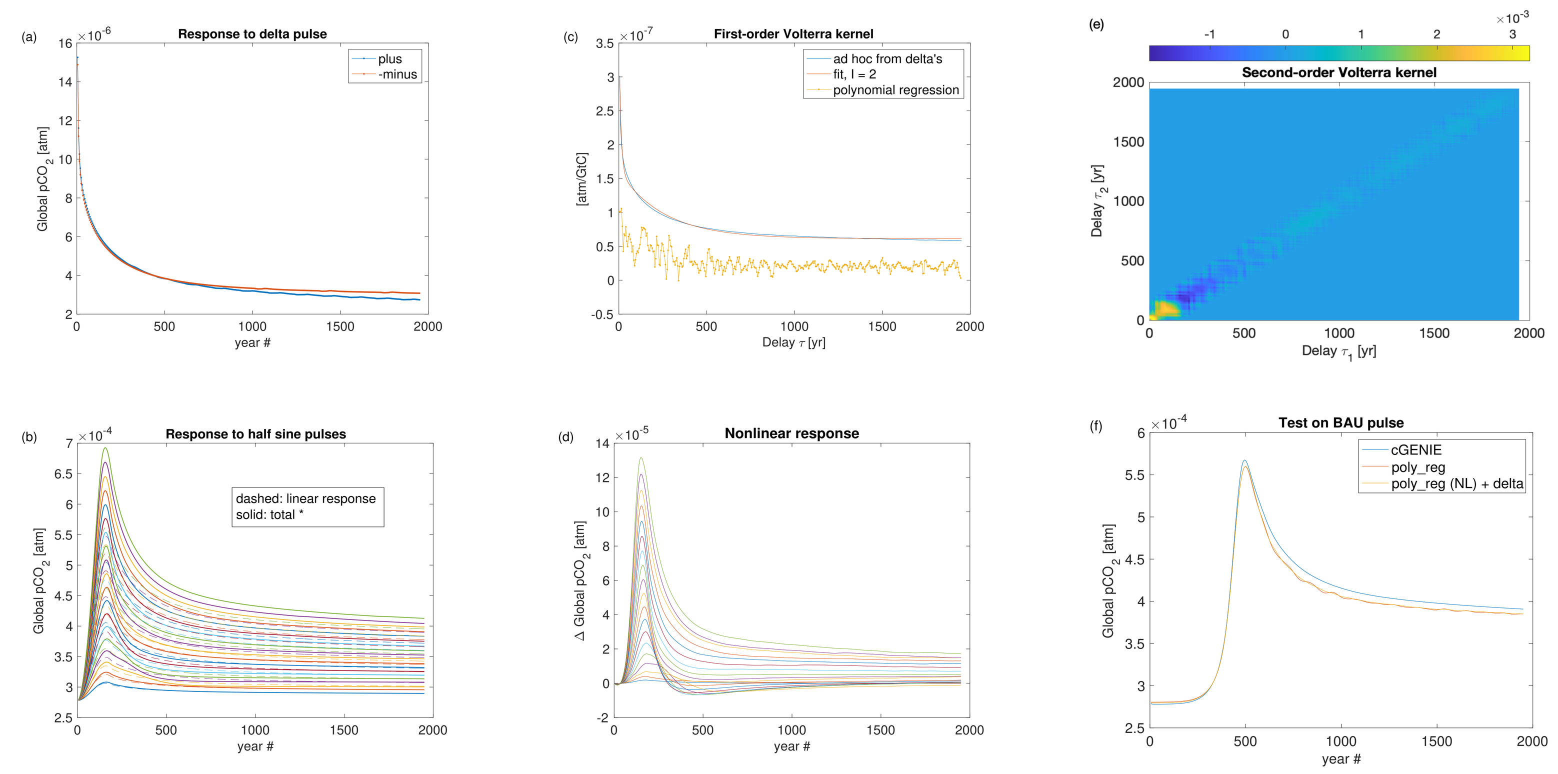
Originally developed for nonchaotic systems. However, the same formalism applies to stochastic (Risken) and chaotic (Ruelle) systems, too, but considering only the *ensemble mean* of the observable, $\langle\psi\rangle$.

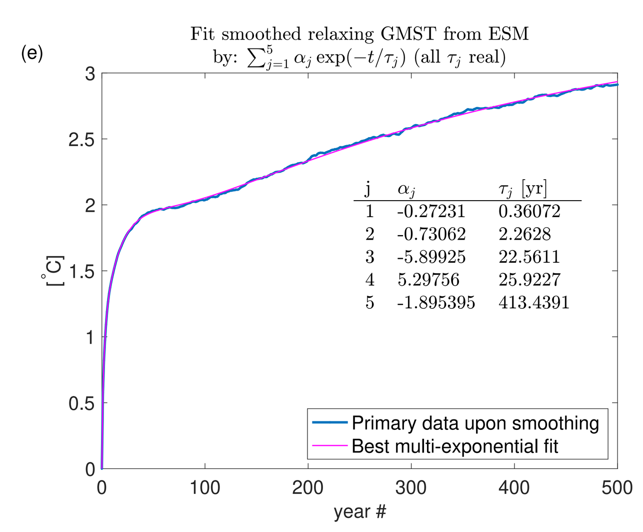
We "assume" that

- The climate system is chaotic and linear (model: MPI-ESM), and
- The carbon cycle is nonchaotic and nonlinear (model: cGENIE).

Volterra kernels for the carbon cycle are inferred by "polynomial kernel regression" (Matthias O. Franz and Bernhard Schölkopf, Neural Computations, 2006) using the *poly_reg* software package (github).

Carbon cycle





Climate system. The 1st-order Volterra kernel, i.e., the linear impulse response function, is inferred from the output of the simulation when the CO_2 concentration is abruptly doubled. It is also convenient to fit it with an analytic form.