Imperial College London

Firedrake

ICE PACK

Point Data Assimilation in Firedrake and Icepack

Reuben W. Nixon-Hill[1,2], Daniel Shapero[3], Colin J. Cotter[2], David A. Ham[2]

[1] Science and Solutions for a Changing Planet DTP, Grantham Institute for Climate Change and the Environment, Imperial College London
[2] Department of Mathematics, Imperial College London
[3] Polar Science Center, Applied Physics Laboratory, University of Washington

Grantham Institute
Climate Change and the Environment
An institute of Imperial College London

Science and Solutions for a Changing Planet DTP

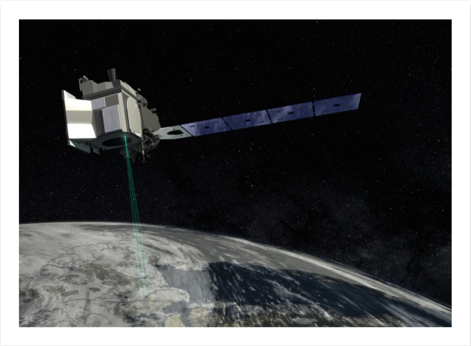
UKRI Natural Environment Research Council

Point data are everywhere. Are we using them well?

Can measure

Sparse Data

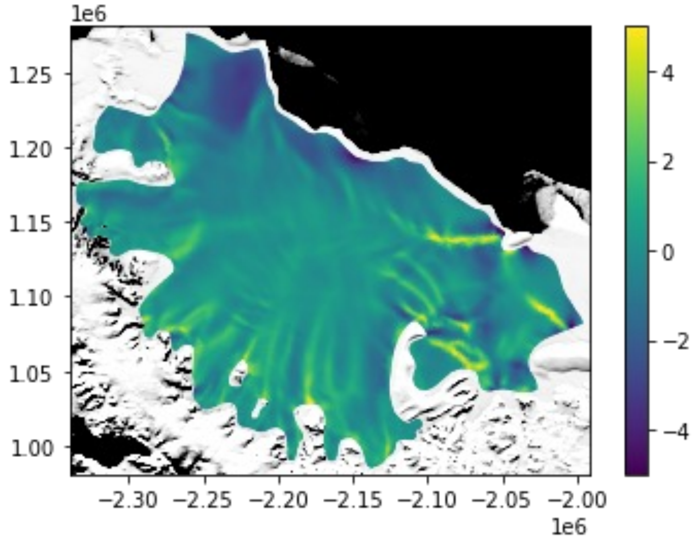
Dense Data



Data Assimilation

Can't measure

Log-Fluidity



Sparse and Dense Data

Accurate results

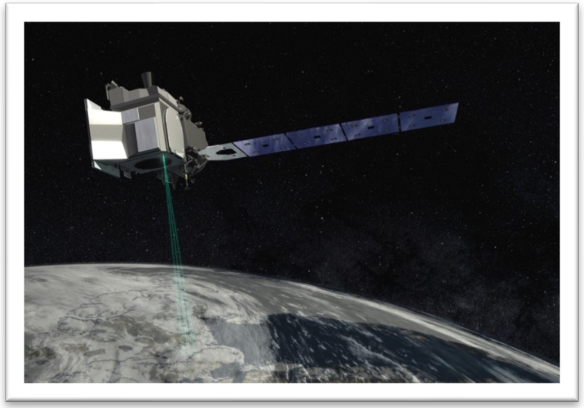
More Data

Better accuracy

ICESat-2 Animation Credit: NASA's Goddard Space Flight Center, Animator Chris Meaney (KBRwyle) [Lead], Scientist Thorsten Markus (NASA/GSFC), Producer Ryan Fitzgibbons (KBRwyle), Project support Aaron E. Lepsch (ADNET)

Data Assimilation is an Inverse Problem: *'Control Method'* or Constrained Optimisation

Data at specific points
u^i_{obs} at X_i

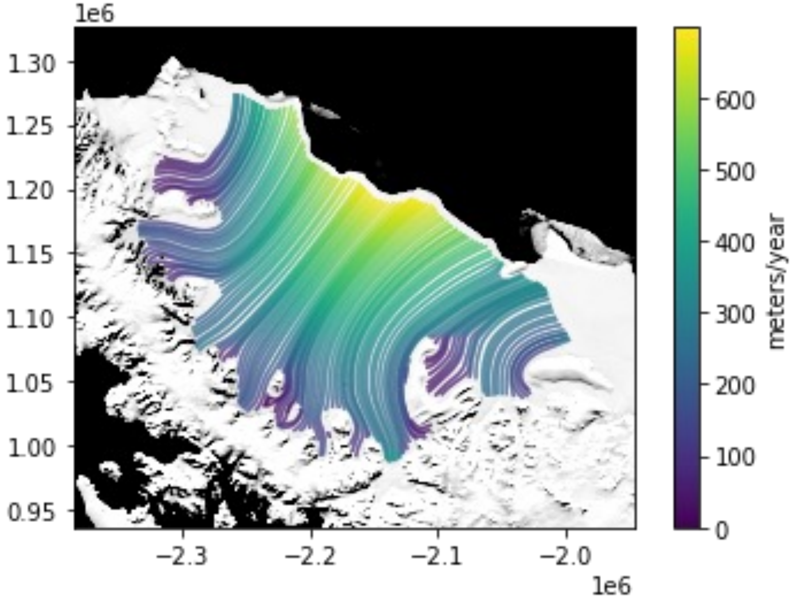


Change q until $u(q)$ matches u^i_{obs}
e.g. via gradient descent

$$\min_q J = J_{\text{model-data misfit}} + J_{\text{regularisation}}$$

$u(q)$ from q
e.g. velocity from fluidity

q is the 'control'



Key Question: Which model-data misfit?

Reconstruct u^i_{obs} to $u_{\text{interpolated}}$

$$J_{\text{model-data misfit}} = \|u_{\text{interpolated}} - u\|_N$$

or

Point Evaluate $u(X_i)$

$$J_{\text{model-data misfit}} = \|u^i_{\text{obs}} - u(X_i)\|_N \ \forall\ i$$

e.g. $J[\beta] = \frac{1}{2}\int_\Gamma n \cdot (\sigma^N - \sigma^D) \cdot (u^N - u^D)dA$ (Shapero et al., 2016)

Firedrake

Mathematics…

$$-\nabla \cdot k \nabla u = f \qquad k = k_0 e^{q(x)}$$

$$\int_\Omega k_0 e^q \nabla u(x) \cdot \nabla v - f v \, \mathrm{d}x = 0 \qquad \forall v \in \mathrm{P2CG}(\Omega)$$

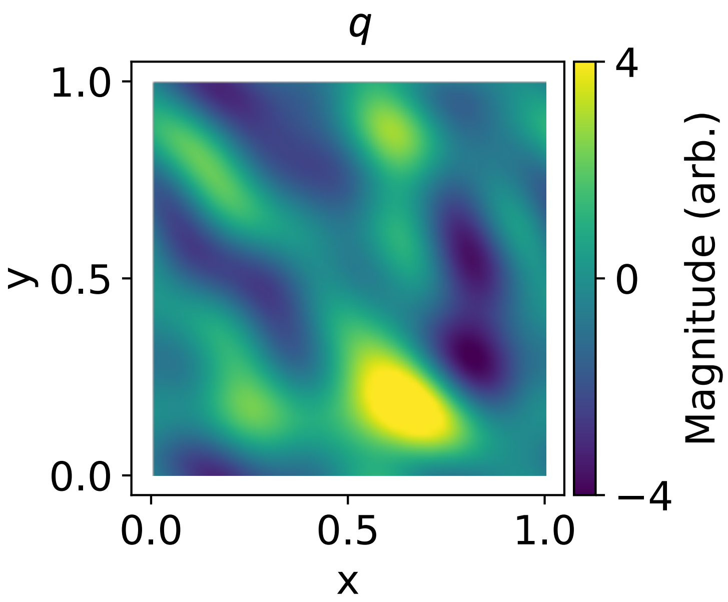
…as (differentiable) code

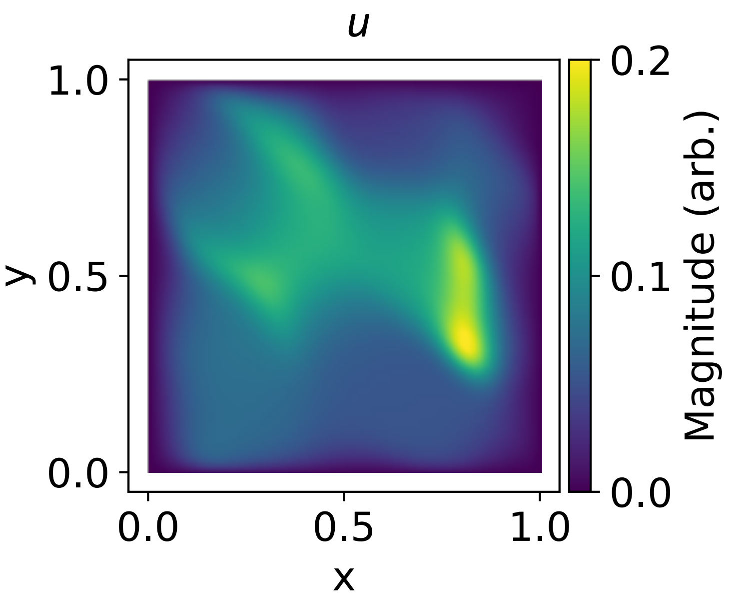
```
from firedrake import *

omega = UnitSquareMesh(20, 20)
P2CG = FunctionSpace(omega, family="CG", degree=2)
u = Function(P2CG)
v = TestFunction(P2CG)

f = Constant(1.0)
k0 = Constant(0.5)
q = Function(P2CG).assign(...)
bc = DirichletBC(P2CG, 0, "on_boundary")

F = (k0 * exp(q) * inner(grad(u), grad(v)) - f * v) * dx
solve(F == 0, u, bc)
```





Which Misfit To Use?

N = 256 measurements

$$J_{\text{regularisation}} = \alpha^2 \int_\Omega |\nabla q|^2 dx$$

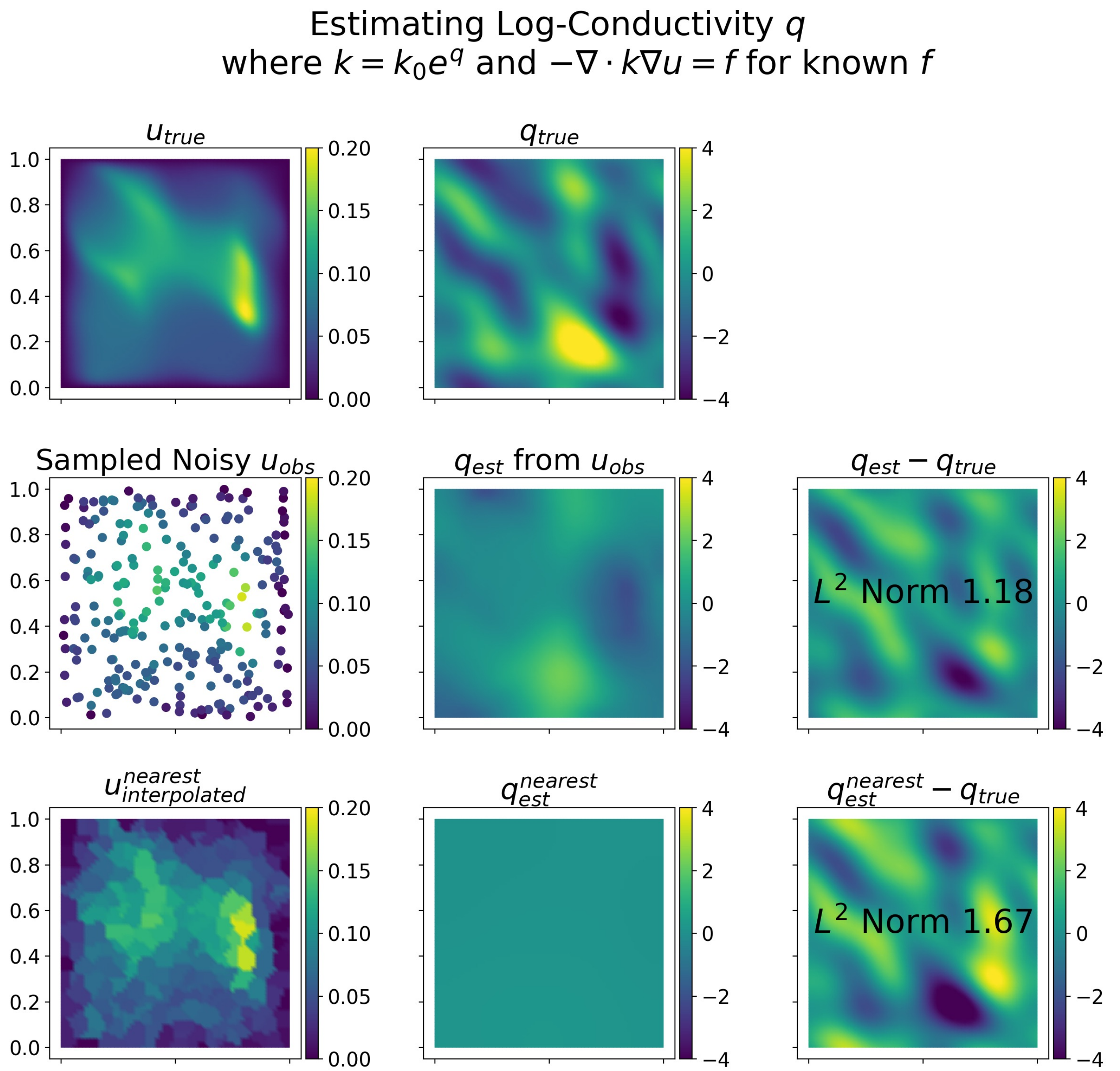
Evaluate $u(X_i)$

$$\min_q J = \sum_{i=0}^{N-1} (u_{\text{obs}}^i - u(X_i))^2 + J_{\text{regularisation}}$$

Reconstruct u_{obs}^i to $u_{\text{interpolated}}$

$$\min_q J' = \int_\Omega (u_{\text{interpolated}} - u)^2 dx + J_{\text{regularisation}}$$

`scipy.interpolate.NearestNDInterpolator`

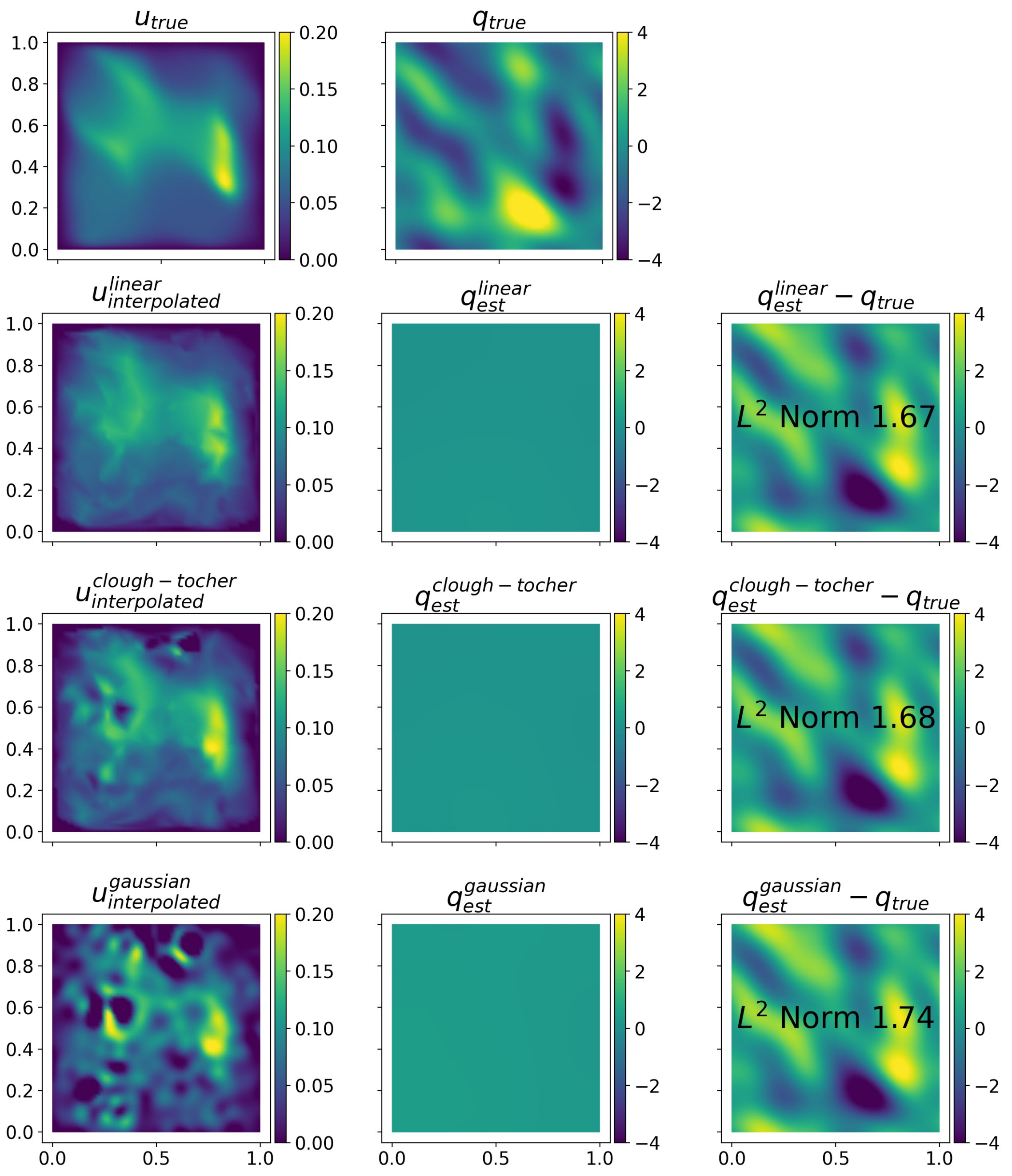


Which Misfit To Use?

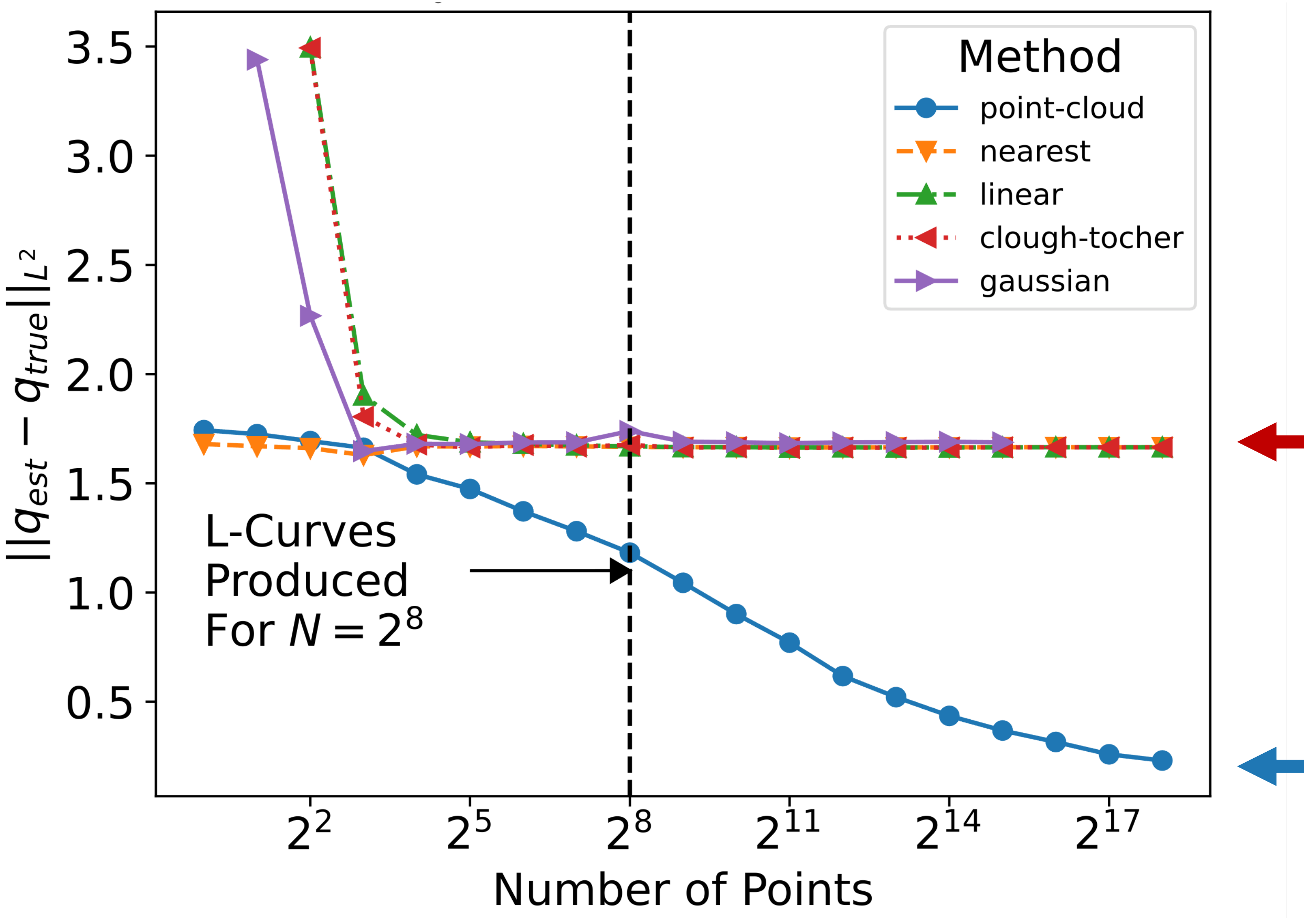
scipy.interpolate.LinearNDInterpolator

scipy.interpolate.CloughTocher2DInterpolator
(fill_value=0.0)

scipy.interpolate.Rbf
(Gaussian Radial Basis Function)



Posterior Consistency:
Do more points give me more accurate results?



Field Reconstruction Misfit

$$\int_\Omega (u_{\text{interpolated}} - u)^2 dx$$

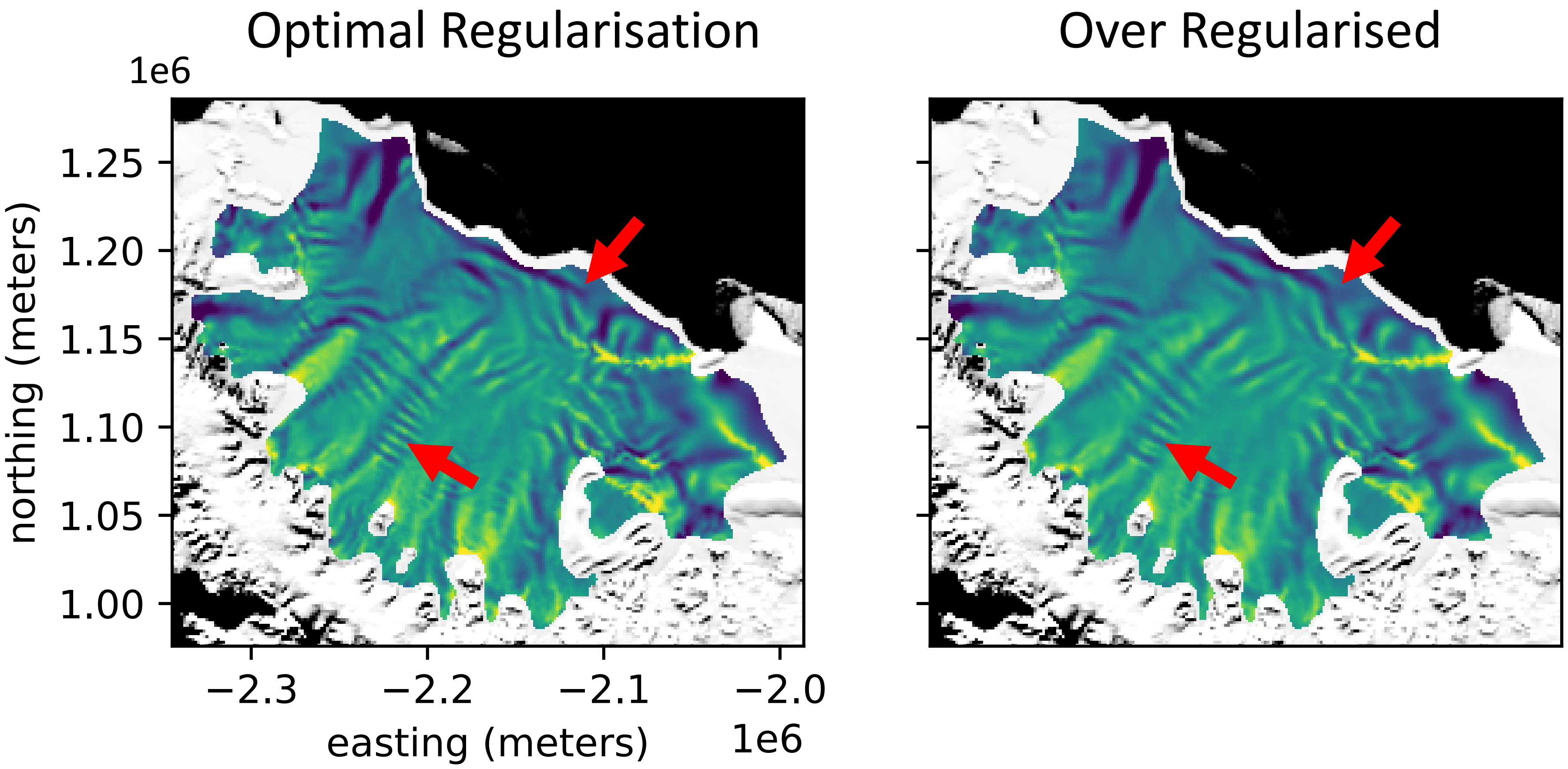
Point Evaluation Misfit

$$\sum_{i=0}^{N-1} (u^i_{\text{obs}} - u(X_i))^2$$

Cross Validation Data Assimilation – Larsen C

ICE PACK

Log-fluidities at different regularisation



Stated error on velocity data from remote sensing seems too low…

$$\sigma_k^{\text{true}} \approx 3.4 \times \sigma_k$$

Bad Data?

Bad Physics?

Advantages of Using a Point Evaluation Approach with Firedrake and ICE PACK

Data

- Sparse and Dense Data
- More Data
- Accurate Data?
- Where/how often should I measure?

$$\sum_{i=0}^{N-1}(u_{\text{obs}}^{i} - u(X_i))^2$$

$$u(X_i)$$

Models

- Accurate results
- Better accuracy ✓
- Accurate Physics?
- Easy
- Extensible

For much more see the paper!

DOI: https://doi.org/10.48550/arXiv.2304.06058
arXiv: https://arxiv.org/abs/2304.06058

Get in touch!

reuben.nixon-hill10@imperial.ac.uk

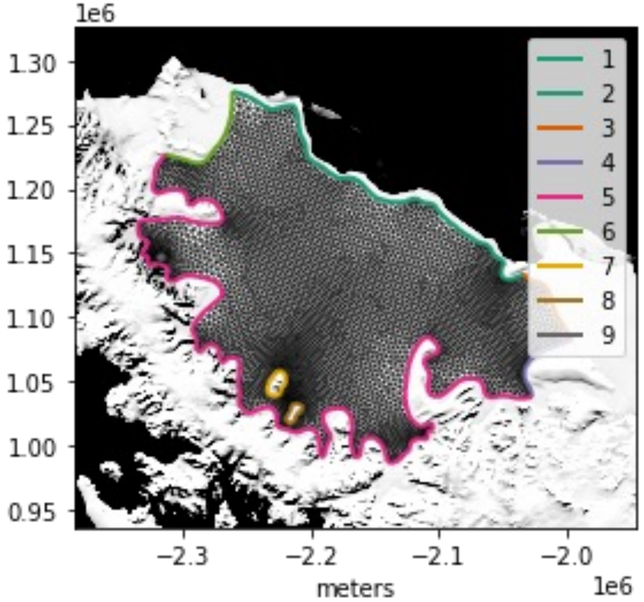
References

Shapero, D. R., Joughin, I. R., Poinar, K., Morlighem, M., and Gillet-Chaulet, F. Basal resistance for three of the largest Greenland outlet glaciers, Journal of Geophysical Research: Earth Surface, 121, 168–180, 2016.

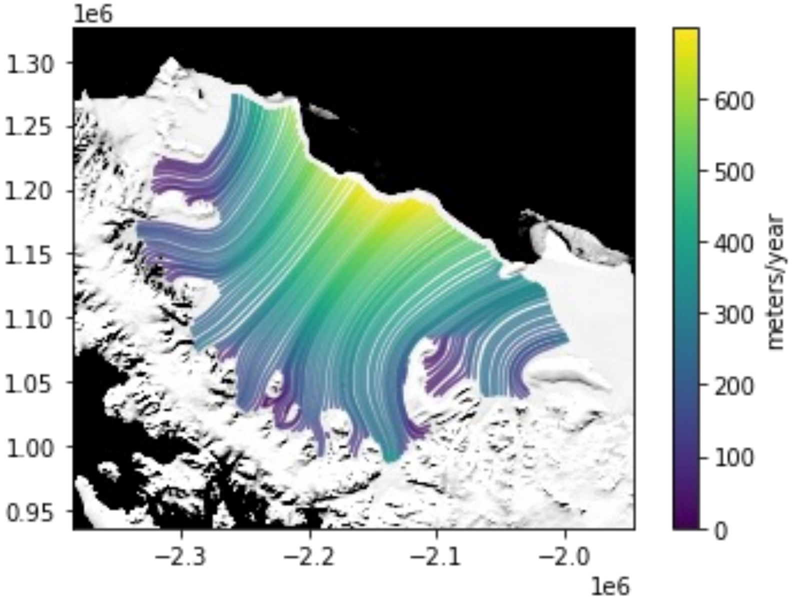
ICE PACK

Solve simple and complex ice flow models	Customizable stress balance models	Inverse solvers for data assimilation	icepack.github.io

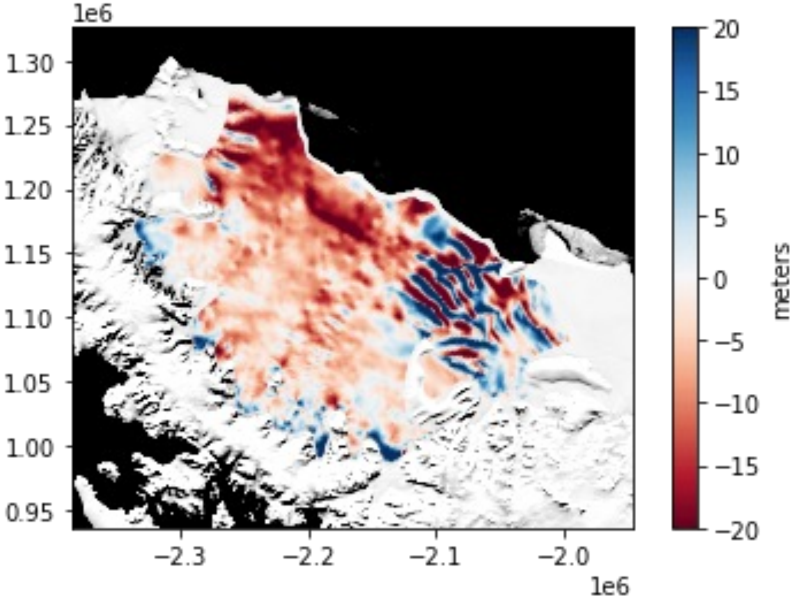
Meshed Domain



Final Computed Velocity



Computed Thickness Change



From https://icepack.github.io/notebooks/tutorials/03-larsen-ice-shelf

$$J_{\text{regularisation}}(\theta) = \frac{\alpha^2}{2}\int_\Omega |\nabla\theta|^2 dx$$