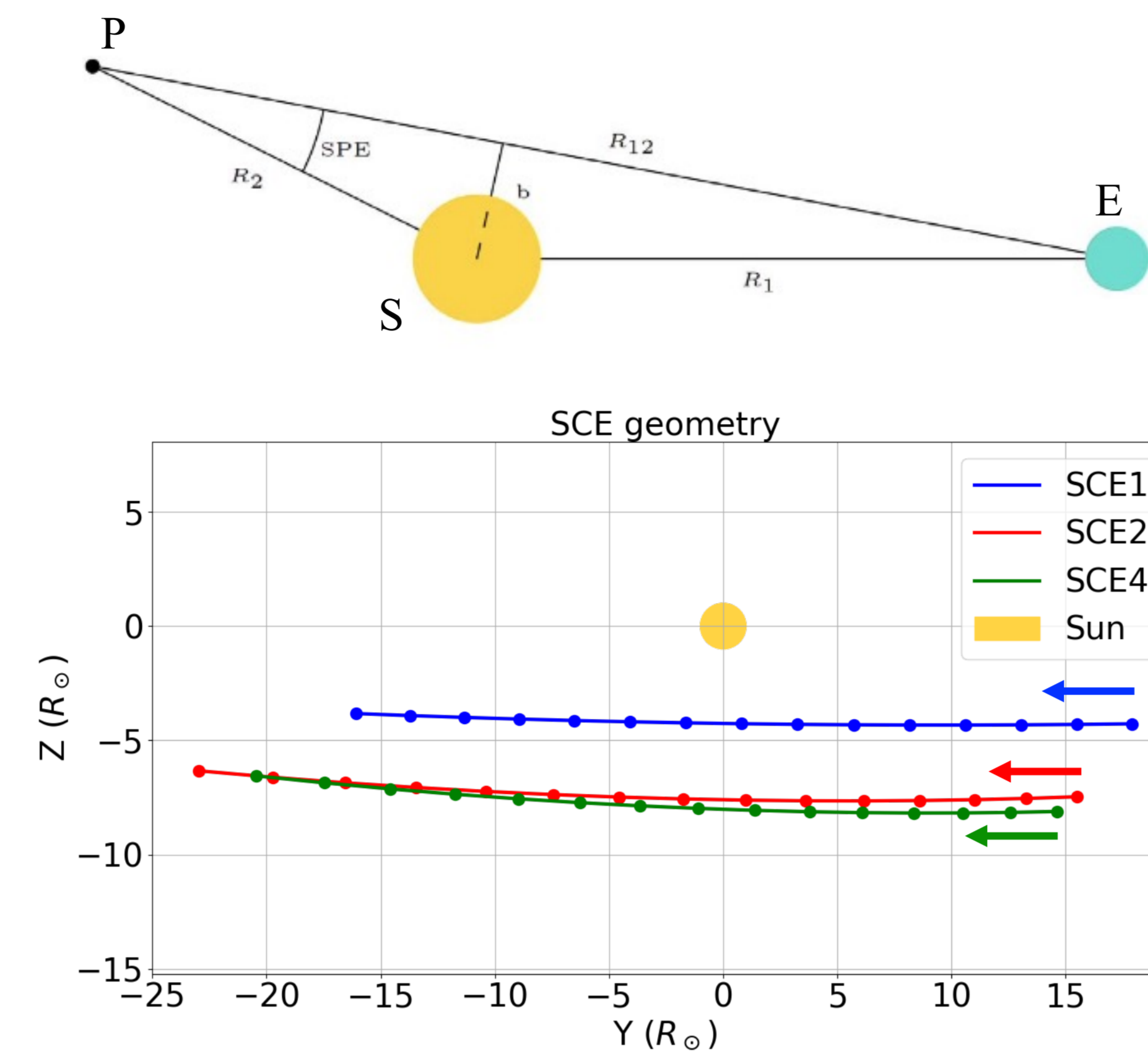




BepiColombo and MORE

- BepiColombo is a joint mission ESA/JAXA, launched on 20th October 2018. It is currently in cruise towards Mercury and it has 16 experiments on-board to characterize it.
- MORE (Mercury Orbiter Radioscience Experiment):
 - Static gravity field and gravitational tidal response of Mercury
 - Mercury's rotational state
 - Fundamental physics tests → determination of several post-newtonian parameters



Superior solar conjunction geometry

- During a superior solar conjunction, the probe and the Earth are almost aligned with the Sun in-between
- The line of sight of the signal passes really close to the Sun, in the inner solar corona → the multifrequency link allows us to isolate the plasma contribution on the uplink and downlink leg
- Three solar conjunctions campaigns are considered: SCE1 (2021), SCE2 (2022) and SCE4 (2023)

Intensity spectrum analysis

- From the open-loop data of SCE1, intensity spectra are computed using the Welch method
- The following theoretical model for the intensity spectrum is assumed (Scott et al. 1983):

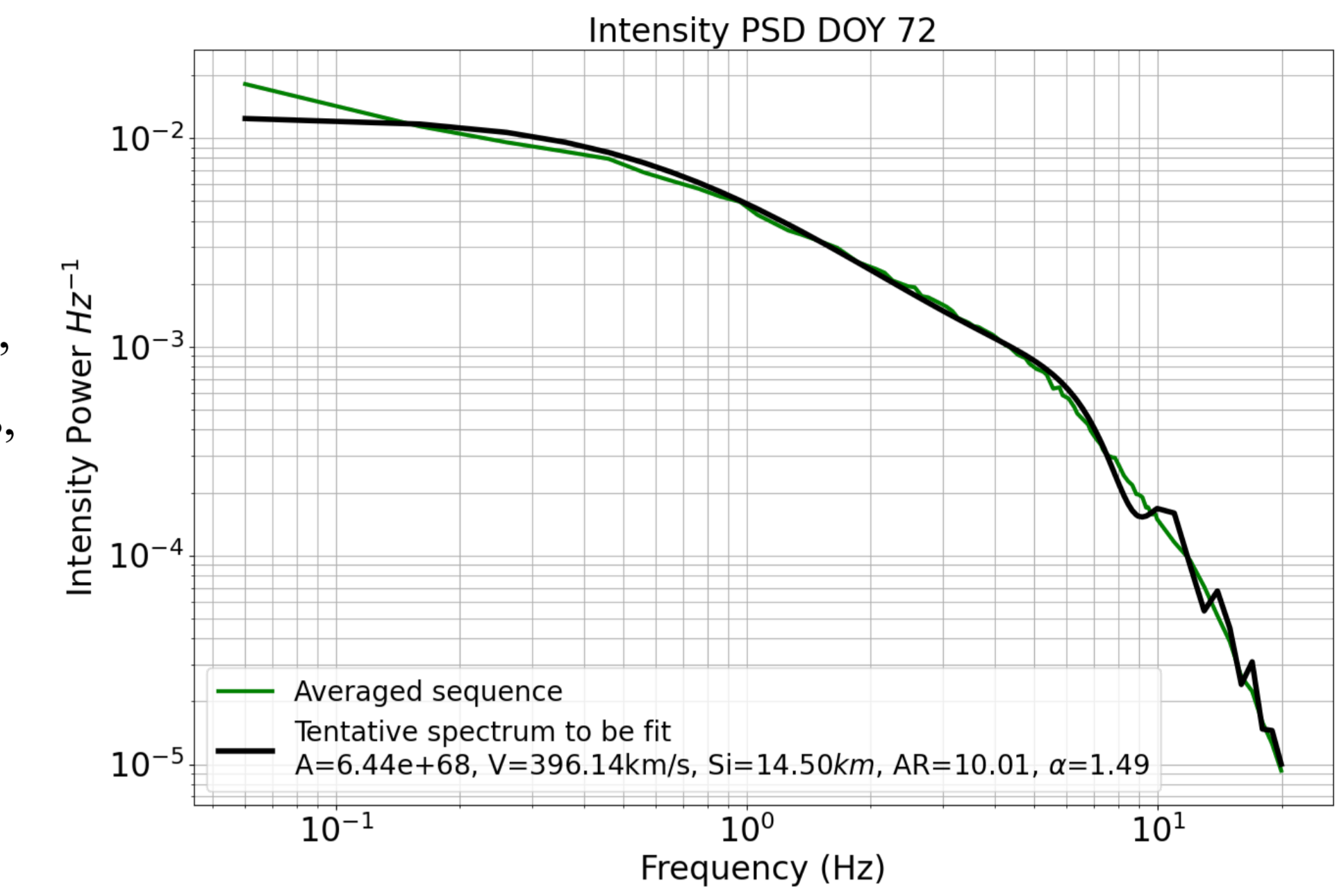
$$P(f) = \frac{2\pi(\lambda r_e)^2}{V} \int_{-\infty}^{+\infty} \Phi_n(k_x = \frac{2\pi}{V} f, k_y) F_{diff}(k) dk_y$$

$$F_{diff}(k) = 4 \sin^2\left(k^2 \frac{\lambda z}{4\pi}\right)$$

$$\Phi_n(k) = A \left(\sqrt{k_x^2 + \frac{k_y^2}{AR^2}} \right)^{-\alpha} \exp\left(-\frac{k^2 S_i^2}{9}\right) R^{-4}$$

- where λ is the wavelength of the carrier signal, V the solar wind velocity, AR the axial ratio, A a constant, S_i the inner scale, α constant of the power law, R the impact parameter, $z = \frac{L_1 L_2}{L_1 + L_2}$ (L_1 and L_2 distances of Earth and spacecraft from closest approach), k_x wavenumber in radial direction
- A least square fitting would be very sensitive to the initial conditions. We use a particle swarm optimization (PSO) algorithm to find a global minimum and then run a least square fitting starting from the output of the PSO.

- Uncertainties:
 - $\sigma_A = 1.97 \cdot 10^{69}$,
 - $\sigma_V = 30.05$ km/s,
 - $\sigma_{S_i} = 7.39$ km,
 - $\sigma_{AR} = 19.24$ and
 - $\sigma_\alpha = 0.995$



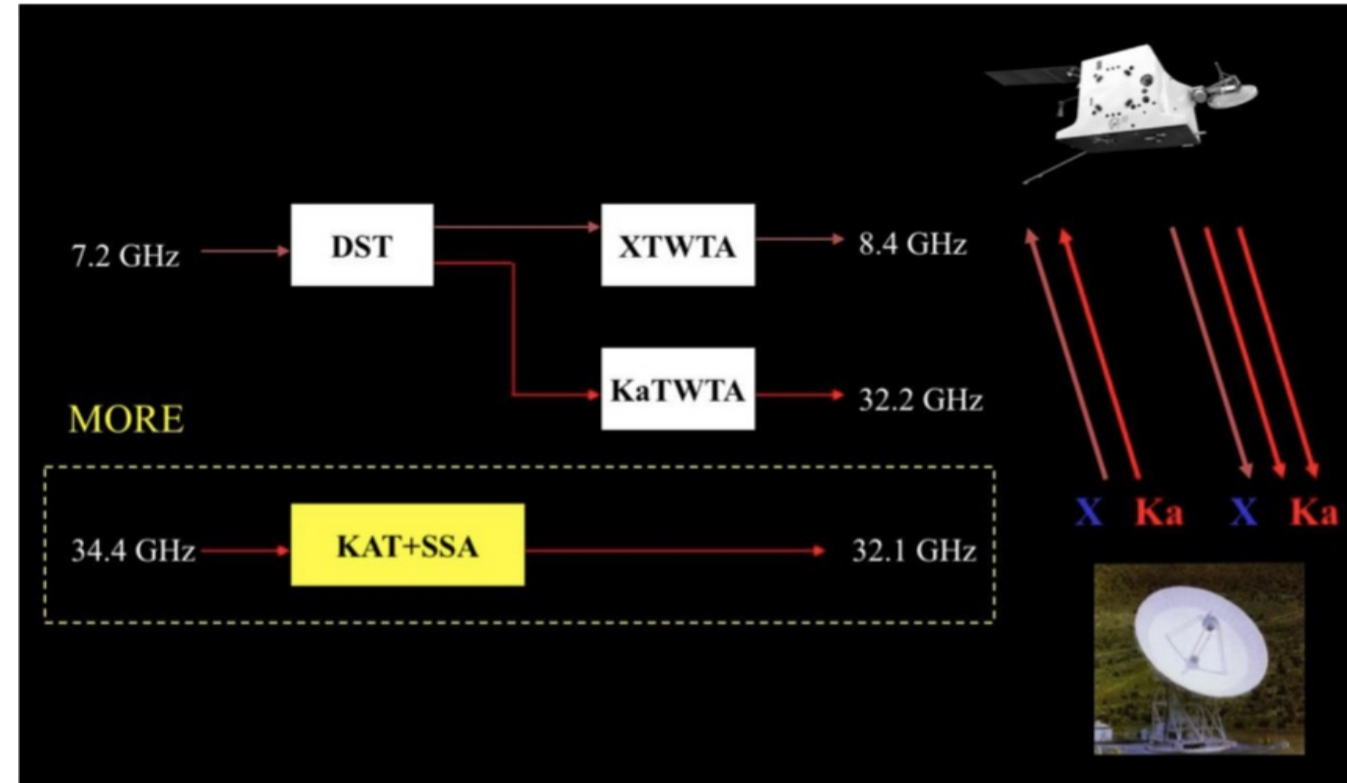
- The only parameter that we can constrain is the velocity. The uncertainties on the other parameters are too large

Plasma calibrations

- The MORE radiotracking system relies on a multi-frequency radio link: three downlinks (X, Ka1, Ka2) and two uplinks (X, Ka).
- The signal due to the solar corona plasma can be isolated through precise calibrations exploiting the dispersive nature of the plasma itself:

$$\begin{cases} y_{X/X}^{obs} = y_{nd} + y_{pl,up} + \frac{1}{\alpha_{X/X}^2} y_{pl,dn} \\ y_{X/Ka}^{obs} = y_{nd} + y_{pl,up} + \frac{1}{\alpha_{X/Ka}^2} y_{pl,dn} \\ y_{Ka/Ka}^{obs} = y_{nd} + \frac{1}{\beta^2} y_{pl,up} + \frac{1}{\beta^2 \alpha_{Ka/Ka}^2} y_{pl,dn} \end{cases}$$

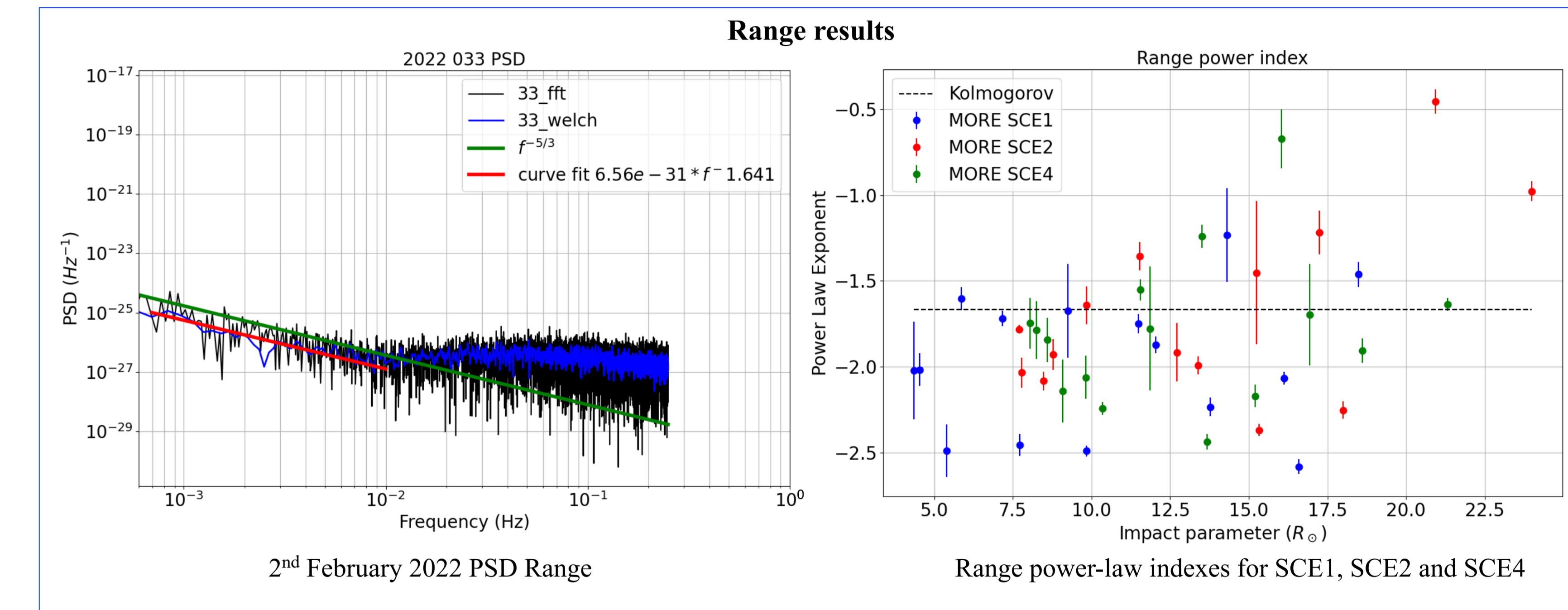
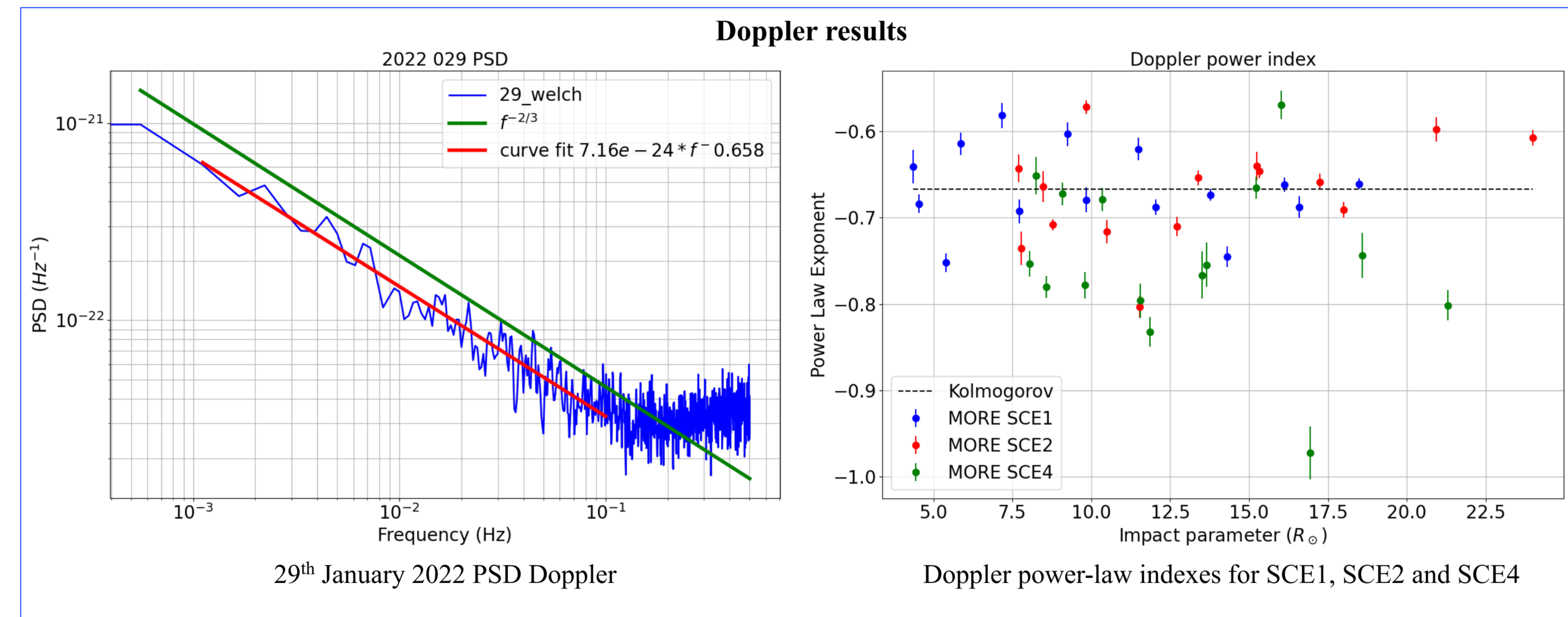
$y_{X/X}^{obs}$, $y_{X/Ka}^{obs}$ and $y_{Ka/Ka}^{obs}$ are the Doppler or range observables and $y_{pl,up}$ and $y_{pl,dn}$ are the plasma contributions.



Configuration scheme of the BepiColombo radio system

PSD of plasma content on Doppler and range signals

- The PSD of the plasma contribution is computed using the Welch method in order to filter the thermal noise: Hanning window of length 1800 seconds for Doppler signals and 3600 for range signals.
- The resulting PSD is expected to follow the exponential law of the Kolmogorov spectrum: $f^{-2/3}$ for the Doppler observables and $f^{-5/3}$ for the range ones.
- In order to compute the power-law index, it's important to correctly choose the interval of frequencies, where the PSD follows the Kolmogorov spectrum.



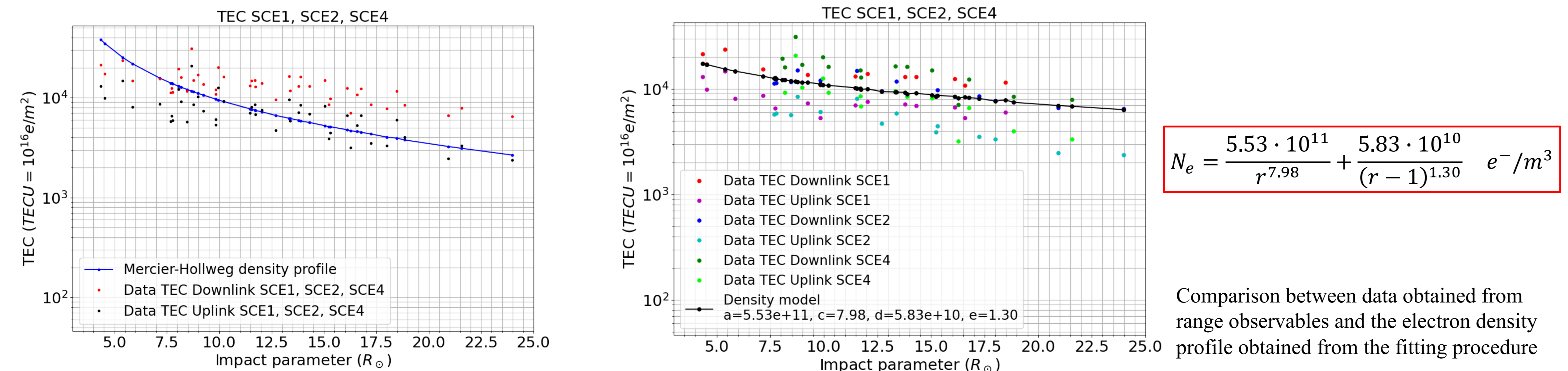
Electron density model

- The Mercier-Hollweg electron density model has been considered (David B. Wexler et al., 2019):

$$N_e = \left(\frac{65}{r^{5.94}} + \frac{0.768}{(r-1)^{2.25}} \right) \times 10^{12} \text{ e}^-/\text{m}^3$$
- Relation between electron density and Total Electron Content (TEC), which is the total number of free electrons contained in a column (1 m² cross section) joining satellite and receiver:

$$TEC = \int_{sat}^{rec} N_e(s) ds$$
- Moreover, the TEC can be computed from range observables with the following expression:

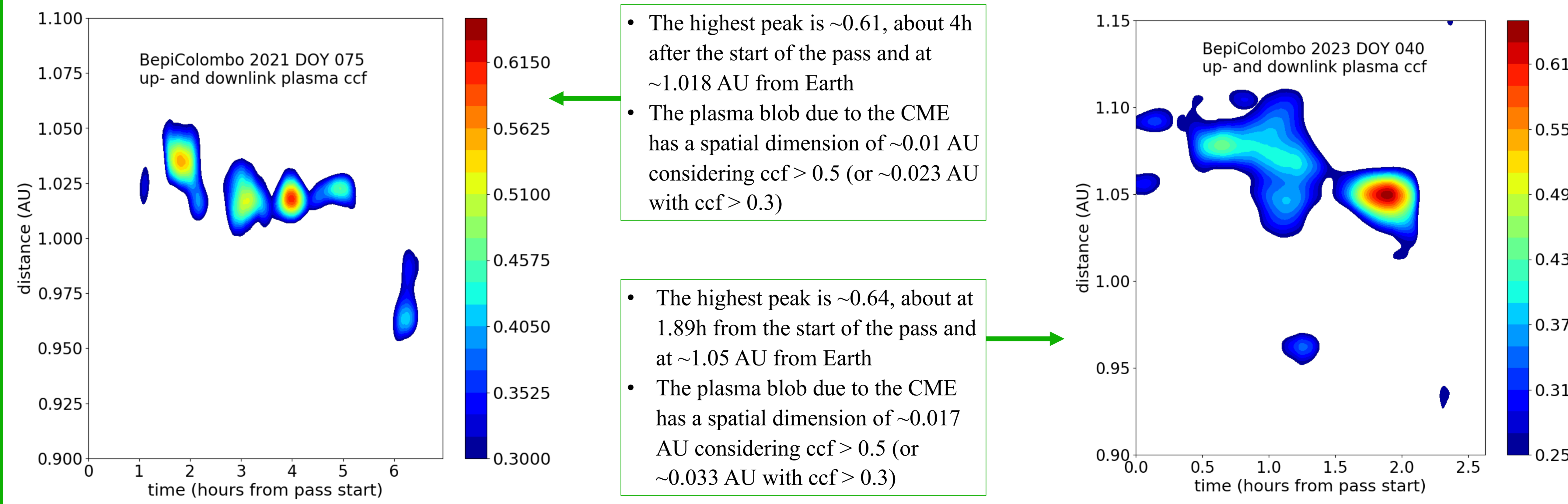
$$TEC = \frac{\Delta s \cdot f^2}{k}, \quad k = \frac{e^2}{8\pi m_e \epsilon_0}$$
- where f is the carrier frequency and Δs is the delay induced by the solar plasma (retrieved from the calibrations on range observables).
- The TEC data obtained with this last formula are then fitted with the theoretical model for electron density $N_e = \frac{a}{rc} + \frac{d}{(r-1)^e} \text{ e}^-/\text{m}^3$ (the parameters to be estimated are a , c , d and e). The initial conditions for the least squares fitting are retrieved from the PSO algorithm.



Plasma features space-time localization

- Cross-correlation between uplink and downlink Doppler time series allows to localize in space and time plasma structures, like Coronal Mass Ejections (CME); in particular, the method proposed by Richie-Halford has been considered (A. C. Richie-Halford et al., 2009). The steps are the following:
 - Bandpass filter: order 5, lowcut=0.001 Hz, highcut=0.05 Hz
 - Triangular time window:

$$\begin{cases} \Lambda(t) = 0 & \text{if } |t| > 1 \\ \Lambda(t) = 1 - |t| & \text{if } |t| < 1 \end{cases} \Rightarrow \begin{cases} \Lambda_{down}(t) = \Lambda\left(\frac{t-t'}{1800 \text{ s}}\right) \\ \Lambda_{up}(t) = \Lambda\left(\frac{t-t'-T_2+1000 \text{ s}}{1800 \text{ s}}\right) \end{cases}$$
 where T_2 is the round-trip light-time (1500 s) and the offset of 1000 s in the uplink is due to the apriori expectation that the main plasma contribution is at ~1AU
 - Cross-correlation between the two time series
- The time window is then advanced by 10 s and the process is repeated until the end of the time series is reached.



References
 • David B. Wexler et al., Spacecraft Radio Frequency Fluctuations in the Solar Corona: A MESSENGER-HELIOS Composite Study, *ApJ* **871** 202, 2019
 • A. C. Richie-Halford et al., Space-time localization of inner heliospheric plasma turbulence using multiple spacecraft radio links, *Space Weather*, **7**, S12003, 2009
 • S. L. Scott, W. A. Coles and G. Bourgois, "Solar wind observations near the sun using interplanetary scintillation", *Astronomy and Astrophysics* **123**, 207-215, 1983