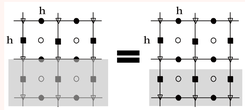
A discrete representation and the implementation for the finite-difference seismic waveform simulation with coarse grid

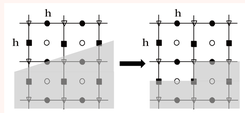
Luqian Jiang[1] (jianglq@mail.ustc.edu.cn) Wei Zhang[2] (zhangwei@sustech.edu.cn)

[1]University of Science and Technology of China, Hefei, China
[2]Southern University of Science and Technology, Shenzhen, China

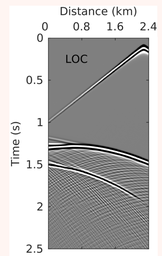
The problem: the interface error



subcell resolution?



stair-step representation



artefact diffraction caused by the staircase approximation

Many equivalent medium parametrization methods [5, 7] have been developed in recent years. Most of these methods are developed for the fourth-order staggered-grid scheme and may not be accurate enough for coarse grids when applying higher-order and optimized schemes.

In this work, we develop a tilted transversely isotropic (TTI) equivalent medium parametrization method [3, 4] to suppress interface errors and the artefact diffraction caused by the staircase approximation under the application of coarse grids. We also present an algorithm for equivalent medium parameterization implementation of complex layered model.

TTI Equivalent Medium Parameterization Method

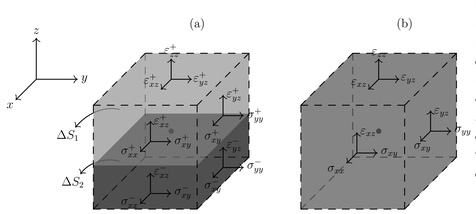


Figure 1: Illustration of a grid cell intersected by a horizontal interface between two media. (a) Discontinuous stress and strain components. (b) The overall stress and strain state of the grid cell. We need to find an equivalent medium filling the cell that can produce total values for the discontinuous stress and strain components along the faces same as those of the cell containing the interface (Fig. b); thus, the **total forces and deformations** of this cell acting on adjacent cells are the same.

the discontinuous stress and strain components at a horizontal interface

$$\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \varepsilon_{xz}, \varepsilon_{yz}, \varepsilon_{zz},$$

The equivalent medium parameters for the heterogeneous anisotropic medium

$$\begin{pmatrix} \Sigma_{NN} \\ \Sigma_{TT} \end{pmatrix} = \begin{pmatrix} \overline{C_{NN}} & \overline{C_{TN}} \\ \overline{C_{TN}}^T & \overline{C_{TT}} \end{pmatrix} \begin{pmatrix} E_{NN} \\ E_{TT} \end{pmatrix}, \tag{1}$$

where

$$\Sigma_{NN} = \begin{pmatrix} \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \end{pmatrix}, \quad \Sigma_{TT} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix}, \quad E_{NN} = \begin{pmatrix} \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \end{pmatrix}, \quad E_{TT} = \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{pmatrix}, \tag{2}$$

$$\begin{aligned} \overline{C_{NN}} &= \langle C_{NN}^{-1} \rangle^{-1}, \\ \overline{C_{TN}} &= \langle C_{NN}^{-1} \rangle^{-1} \langle C_{NN}^{-1} C_{TN} \rangle, \\ \overline{C_{TT}} &= \langle C_{TT} - C_{TN}^T C_{NN}^{-1} C_{TN} \rangle + \langle C_{TN}^T C_{NN}^{-1} \rangle \overline{C_{TN}}, \end{aligned} \tag{3}$$

$$C_{NN} = \begin{pmatrix} c_{33} & c_{34} & c_{35} \\ c_{34} & c_{44} & c_{45} \\ c_{35} & c_{45} & c_{55} \end{pmatrix}, \quad C_{TT} = \begin{pmatrix} c_{11} & c_{12} & c_{16} \\ c_{12} & c_{22} & c_{26} \\ c_{16} & c_{26} & c_{66} \end{pmatrix}, \quad C_{TN} = \begin{pmatrix} c_{13} & c_{23} & c_{36} \\ c_{14} & c_{24} & c_{46} \\ c_{15} & c_{25} & c_{56} \end{pmatrix}.$$

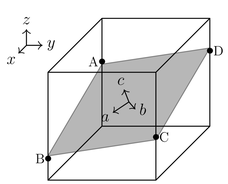


Figure 2: an interface arbitrarily passing through a half-grid cell.

$$C^{TTI} = R\overline{C}R^T, \tag{4}$$

Bond transformation matrix R [1],

$$R = \begin{pmatrix} \alpha_{xa}^2 & \alpha_{xb}^2 & \alpha_{xc}^2 & 2\alpha_{xb}\alpha_{xc} & 2\alpha_{xc}\alpha_{xa} & 2\alpha_{xa}\alpha_{xb} \\ \alpha_{ya}^2 & \alpha_{yb}^2 & \alpha_{yc}^2 & 2\alpha_{yb}\alpha_{yc} & 2\alpha_{yc}\alpha_{ya} & 2\alpha_{ya}\alpha_{yb} \\ \alpha_{za}^2 & \alpha_{zb}^2 & \alpha_{zc}^2 & 2\alpha_{zb}\alpha_{zc} & 2\alpha_{zc}\alpha_{za} & 2\alpha_{za}\alpha_{zb} \\ \alpha_{ya}\alpha_{za} & \alpha_{yb}\alpha_{zb} & \alpha_{yc}\alpha_{zc} & \alpha_{yb}\alpha_{zc}+\alpha_{zb}\alpha_{yc} & \alpha_{yc}\alpha_{za}+\alpha_{zc}\alpha_{ya} & \alpha_{ya}\alpha_{zb}+\alpha_{za}\alpha_{yb} \\ \alpha_{za}\alpha_{xa} & \alpha_{zb}\alpha_{xb} & \alpha_{zc}\alpha_{xc} & \alpha_{zb}\alpha_{xc}+\alpha_{xb}\alpha_{zc} & \alpha_{zc}\alpha_{xa}+\alpha_{xc}\alpha_{za} & \alpha_{za}\alpha_{xb}+\alpha_{xa}\alpha_{zb} \\ \alpha_{xa}\alpha_{ya} & \alpha_{xb}\alpha_{yb} & \alpha_{xc}\alpha_{yc} & \alpha_{xb}\alpha_{yc}+\alpha_{yb}\alpha_{xc} & \alpha_{xc}\alpha_{ya}+\alpha_{yc}\alpha_{xa} & \alpha_{xa}\alpha_{yb}+\alpha_{ya}\alpha_{xb} \end{pmatrix}, \tag{5}$$

α_{ij} is the directional cosine between the tilted symmetry axis $\mathbf{i} = (\vec{a}, \vec{b}, \vec{c})$ and a geographical axis $\mathbf{j} = (\vec{x}, \vec{y}, \vec{z})$.

Efficient Numerical Implementation of Equivalent Medium Parameterization

For which point(s) is it necessary to calculate the equivalent medium parameters?

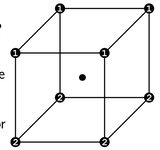


Figure 3: Illustration of the labels of medium layers. The black dot on the middle of the cell is the grid point, black lines are edges of the auxiliary half-grid cell. The numbers on the vertices of the cell are the medium layer labels. It can represent the cases of Fig. 1(a) and Fig. 2.

If the number of different labels of all vertices is greater than 1, then there must be an interface or interfaces crossing the cell.

How to compute the angle of the interface passing through grid cell?

- calculate the intersection points (e.g. A, B, C, D in Fig. 2) by a pre-built lookup table.

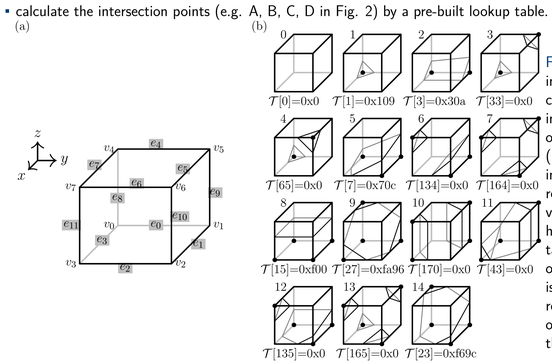
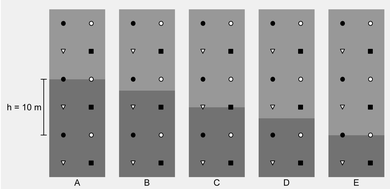
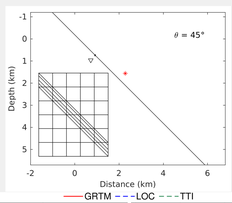


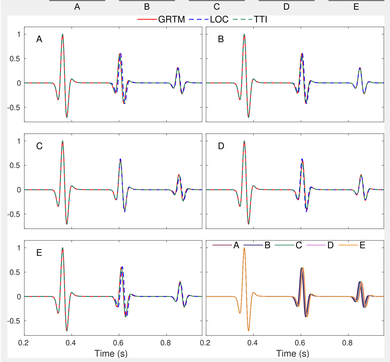
Figure 4: 15 basic cases of interfaces intersecting a half-grid cell and the corresponding values in the lookup table. (a) the index of vertices and edges of the cell. (b) 15 basic cases of interfaces intersecting the cell. Black dot represents the negative point. The value of $\mathcal{T}[i]$ below the cell is the hexadecimal value of the look-up table in this situation. The value on the upper left corner of the cell is the label of this case. The remaining 241 cases can be obtained by rotating and reflecting these 15 cases.

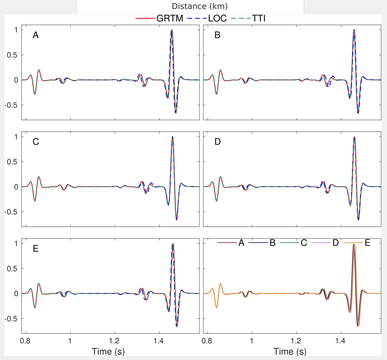
- Compute the least-square plane by the intersection points.
- Establish a new coordinate system based on the least squares plane (e.g. a-b-c axes in Fig. 2).
- Calculate the Bond transformation matrix of Eq. (5), then calculate Eq. (4).

subcell resolution?





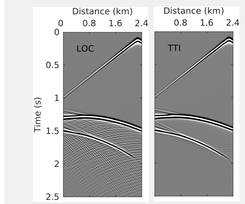




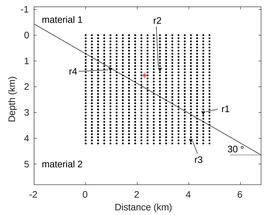
LOC: the local values of material parameters; **TTI**: TTI equivalent medium parameterization.
GRTM: the generalized reflection/transmission matrices method [2]

Reference: Luqian Jiang and Wei Zhang, 2021. TTI equivalent medium parametrization method for the seismic waveform modelling of heterogeneous media with coarse grids. Geophys. J. Int, 227(3), pp.2016-2043. doi: 10.1093/gji/ggab310

staircase approximation

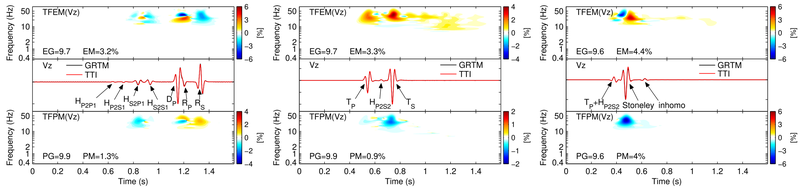


Accuracy of Seismic Phases

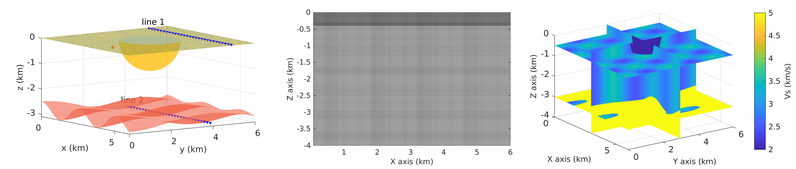


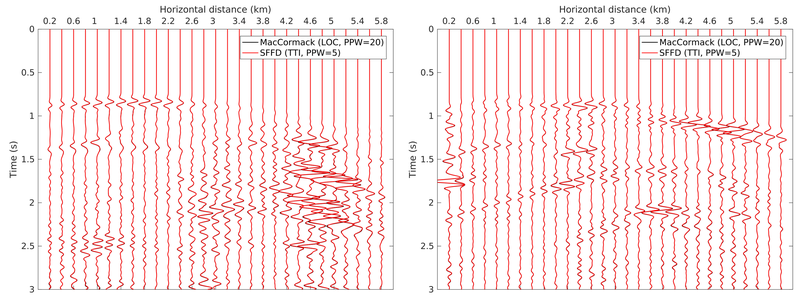
	$\rho(kg/m^3)$	$v_p(km/s)$	$v_s(km/s)$
material 1	1400.0	2550.0	1500.0
material 2	5600.0	10200.0	6000.0

source: $f_c = 20Hz$, Ricker, $t_0 = 0.1s$;
boundaries: ADE-CFS PML [9] on four sides;
FD operator: 14th-order optimal schemes [6, Table 6];
dx = dz = 10m, PPW=3.



Test - 3D Complex Layered Model





The reference results are calculated by the DRP/opt MacCormack scheme [8] with dense grids.

References

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