Exploiting the full potential of ocean tide models for space geodetic techniques

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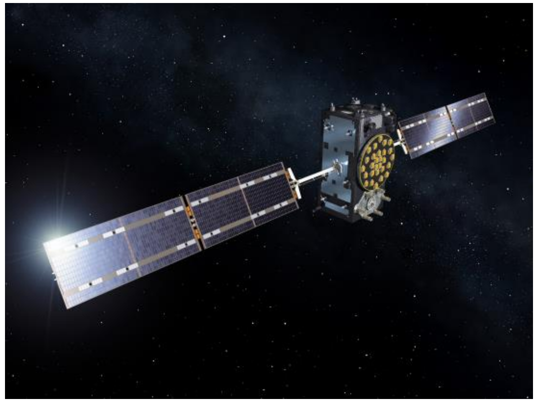
2023-04-24

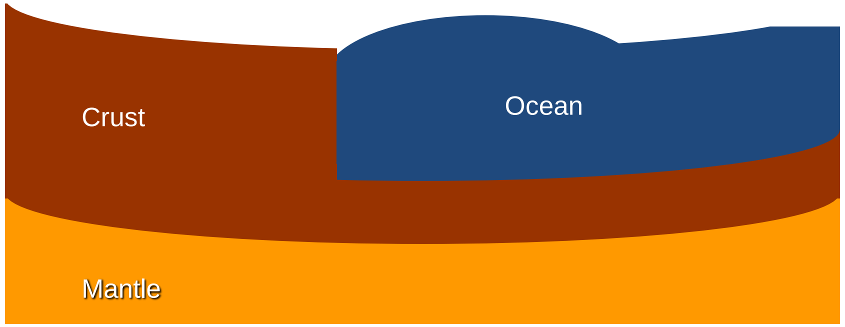
Introduction

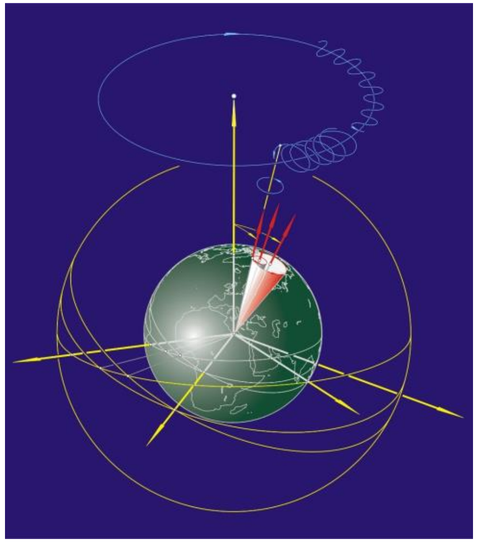
- Ocean tides effects
  - Orbit computation
  - Station tidal displacements
  - Earth rotation

- Spurious tidal signals visible in almost every product

⇒ Hypothesis:
Incomplete conventions leads to incomplete/wrong implementations







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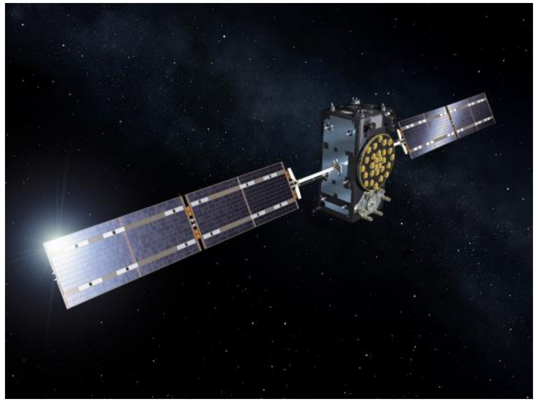
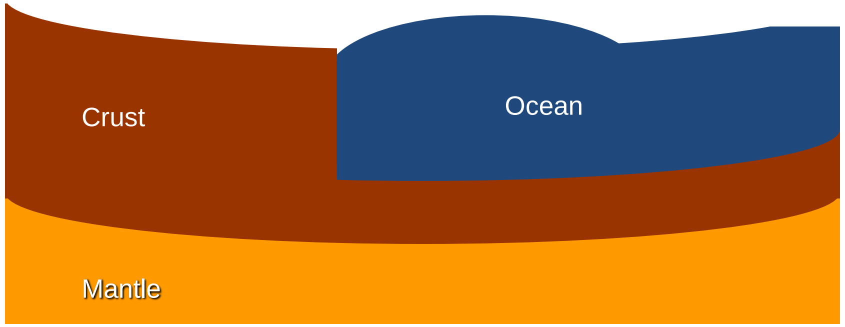
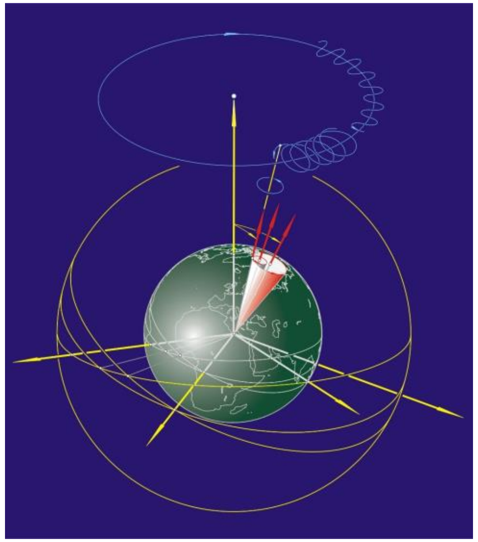




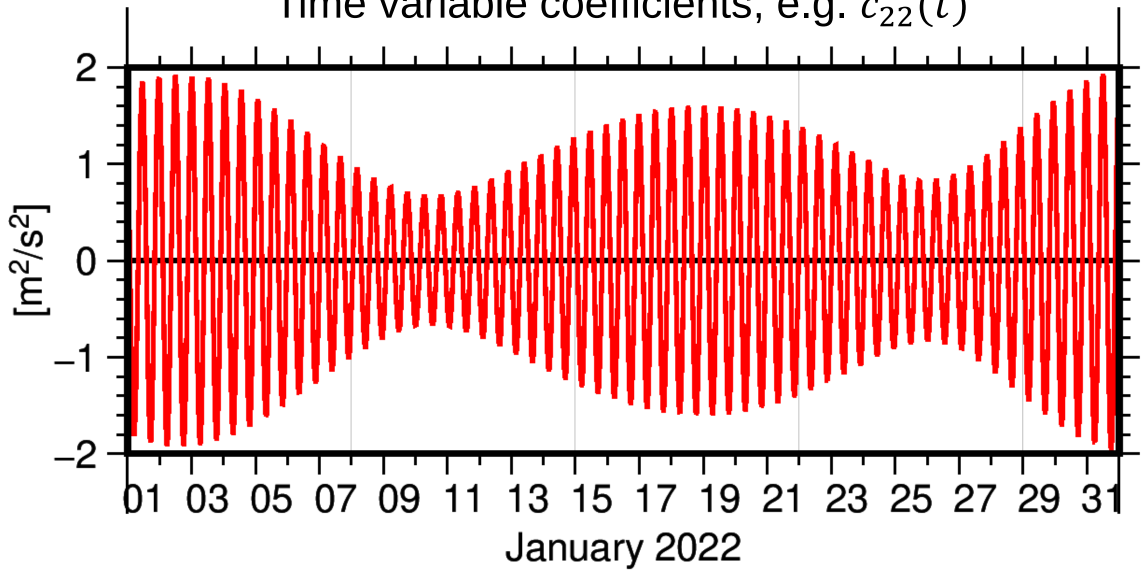


A crash course on ocean tides

- Ocean tides are forced by the Tide Generating Potential (TGP)

$$V(\lambda,\vartheta,r,t) = \sum_{n=2}^{\infty}\left(\frac{r}{R}\right)^n \sum_{m=0}^{n} c_{nm}(t)C_{nm}(\lambda,\vartheta) + s_{nm}(t)S_{nm}(\lambda,\vartheta)$$

Time variable coefficients, e.g. $c_{22}(t)$



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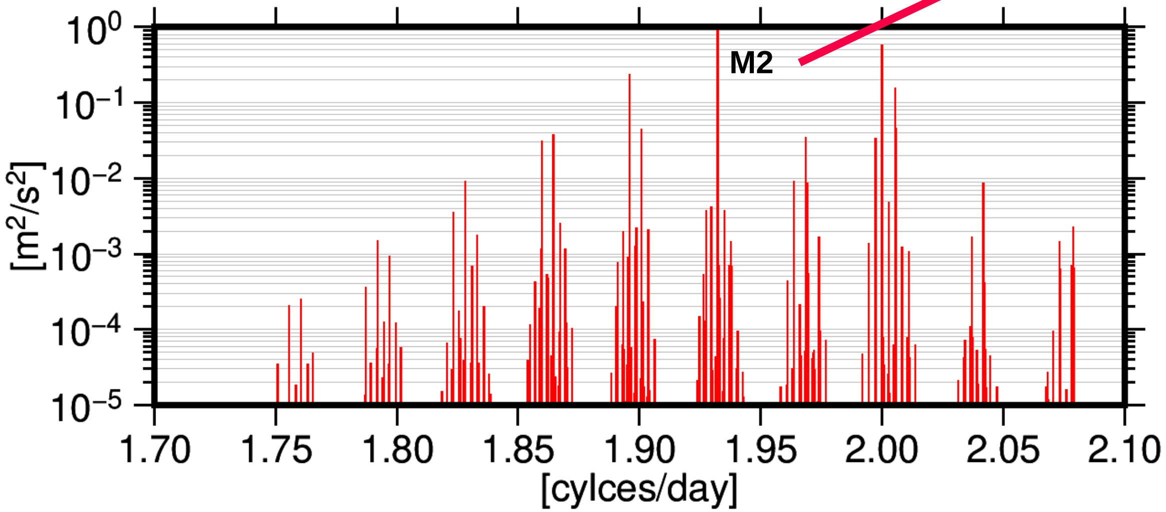
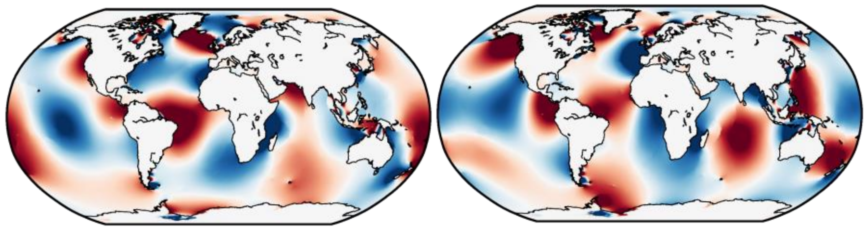
Amplitude spectrum, e.g. $c_{22}(t)$



Ocean tide response cos sin

M2



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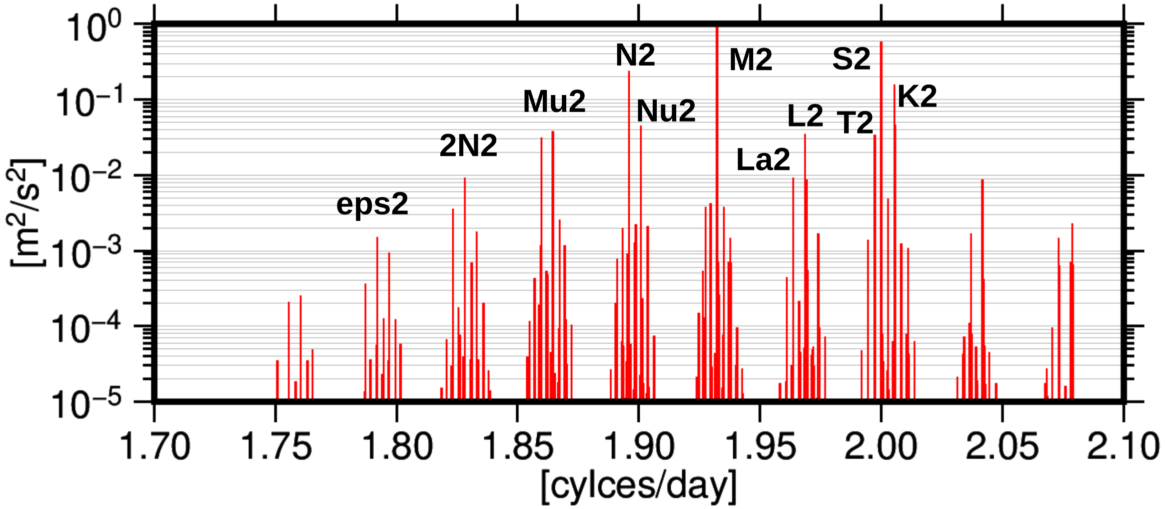
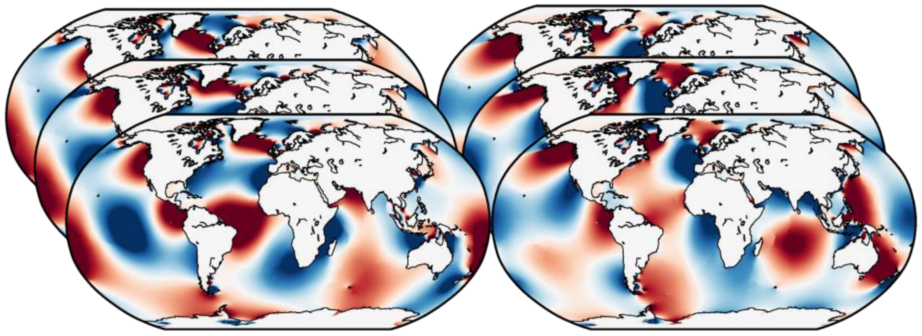


- missing lines must be interpolated (admittance)

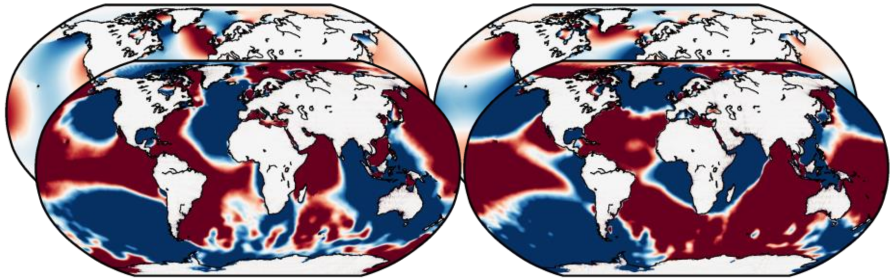
Ocean tide atlas — cos — sin

M2, 2N2, eps2, J1, K1, K2, L2, La2, Mu2, N2, Nu2, O1, P1, Q1, R2, T2, Mf, Mm, MSf, …



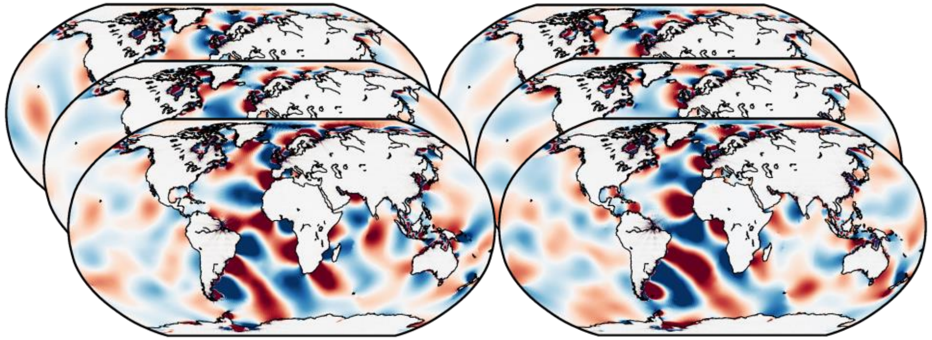
With additional atmospheric forcing (radiation tides)

SA, S1, S2, …



Non-linear tides (over tides)

MN4, M4, MS4, S4, N4, M6, …



Torsten Mayer-Gürr

A crash course on ocean tides

Ocean tide atlas cos sin

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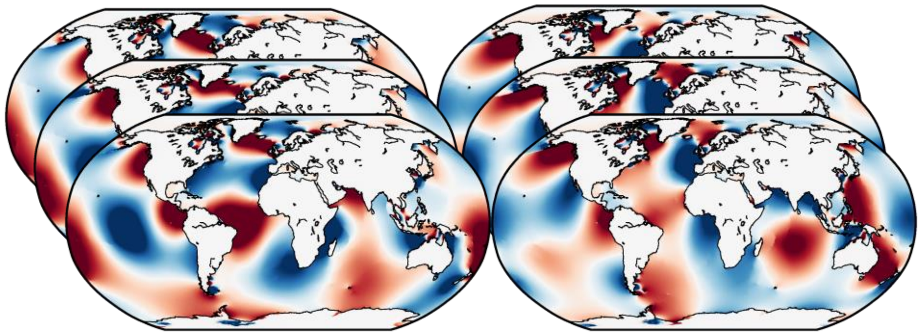

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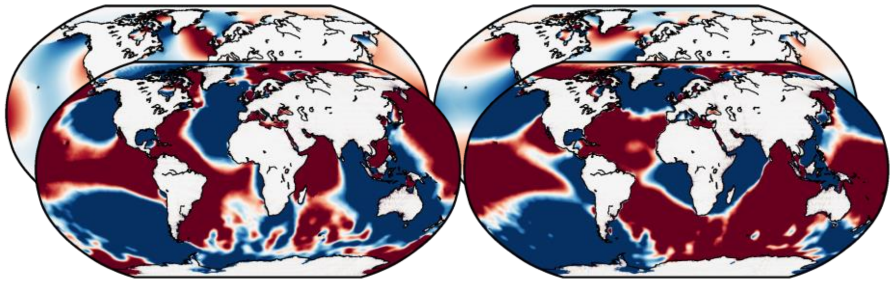

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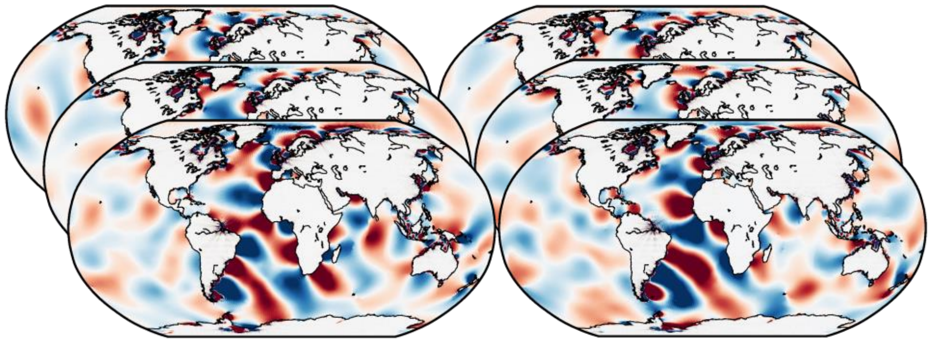


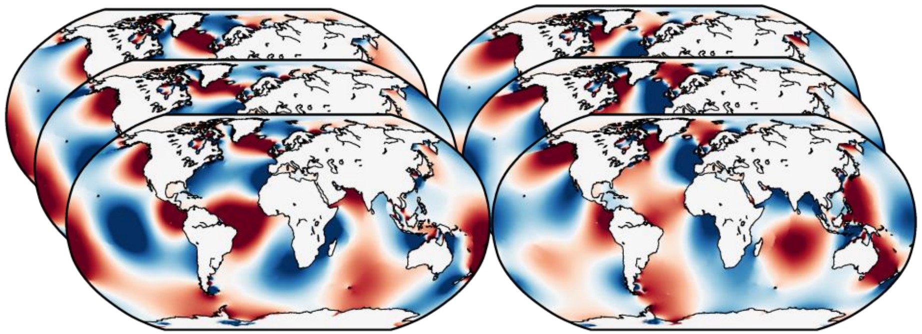
Current status

- Gravity field / Orbit computation
  - Only tidal atlas is provided
  - Admittance implementation is on user side
    - BERNESE, NAPEOS, GROOPS
    - EPOS, GIPSY, GINS, …
  - Unclear formular, not straight forward
    - IERS conventions only for old FES2004
  - New ocean tide model needs adjustment of source code

  - Complicated phase definition
    - Doodson-Warburg, needs additional tables
    - ambiguous phase definitions for S1, minor tides
  - Ambiguous tide definition
    - e.g. S1: 164.556 or 164.555?

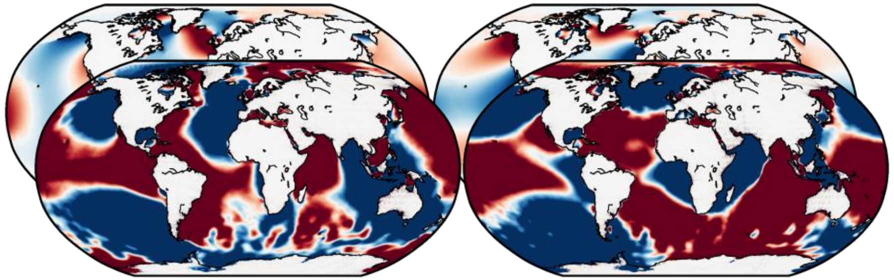
Ocean tide atlas cos sin

M2, 2N2, eps2, J1, K1, K2, L2, La2, Mu2, N2, Nu2, O1, P1, Q1, R2, T2, Mf, Mm, MSf, …



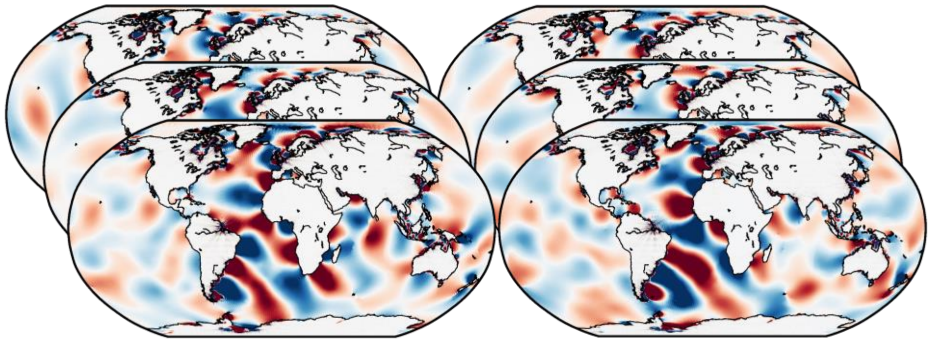
With additional atmospheric forcing (radiation tides)

SA, S1, S2, …



Non-linear tides (over tides)

MN4, M4, MS4, S4, N4, M6, …



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Goal: User friendly
- conventions
- file formats
  - Standard ICGEM format (*.gfc)
  - 2 matrices in simple free format

Ocean tide atlas cos sin

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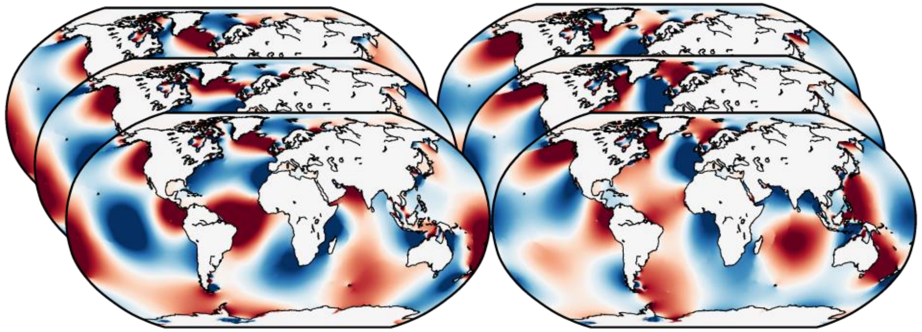
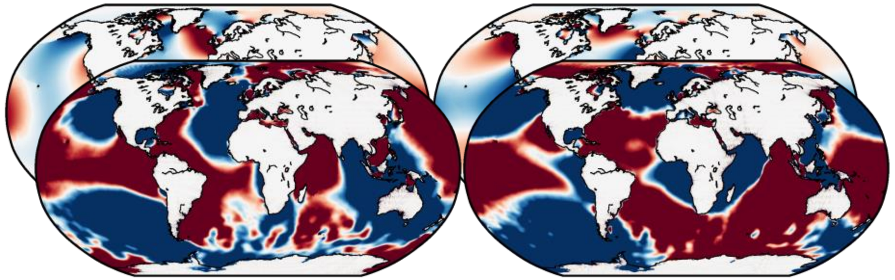


atmospheric forcing (radiation tides)



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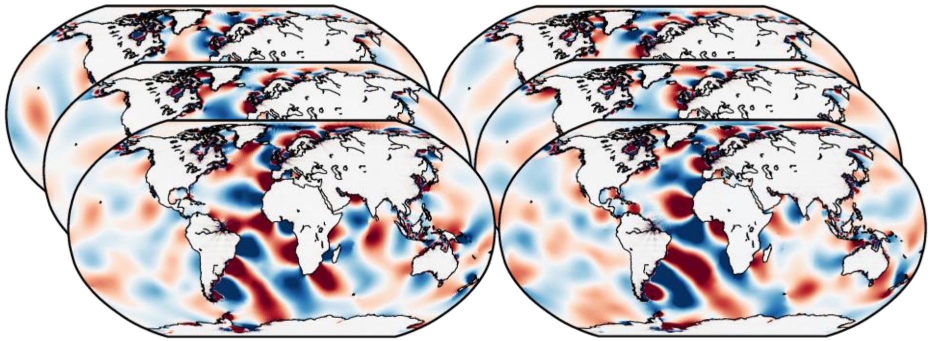
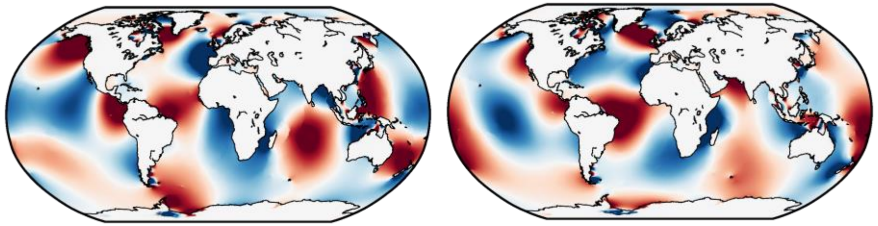


Gravity: new approach

- Ocean tide synthesis at time t

$$\begin{bmatrix} c_{nm} \\ s_{nm} \end{bmatrix}(t) = \sum_k f_k^{cos}(t) \begin{bmatrix} c_{nm} \\ s_{nm} \end{bmatrix}_k^{cos} + f_k^{sin}(t) \begin{bmatrix} c_{nm} \\ s_{nm} \end{bmatrix}_k^{sin}$$



Coefficients from the tidal atlas

- Standard ICGEM format (*.gfc)
  - `eot20_055.565_om1_cos.gfc`
  - `eot20_055.565_om1_sin.gfc`
  - `eot20_055.575_om2_cos.gfc`
  - `eot20_055.575_om2_sin.gfc`
  - `eot20_056.554_sa_cos.gfc`
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  - …

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Temporal changing factors

$$f_k^{cos}(t) = \sum_f A_{k,f} \cos \theta_f(t),$$

$$f_k^{sin}(t) = \sum_f A_{k,f} \sin \theta_f(t)$$

Phase arguments for all tidal lines

$$\theta_f(t) = \sum_{i=1}^{6} D_{f,i}\, \beta_i\,(t)$$ ← 6 Doodson arguments

Admittance matrix

```
1.00000e+00  0.00000e+00  0.00000e+00  …
0.00000e+00  1.00000e+00 -1.12052e-01  …
0.00000e+00  0.00000e+00 -1.48522e-03  …
…
```

Matrix with Doodson multipliers

```
0  0  0  0  1  0
0  0  0  0  2  0
0  0  0  2  1  0
0  0  1  0 -1 -1
0  0  1  0  0 -1
…
```

- All tidal lines are treated in the same way
- Flexible: different interpolation schemes, adding non TGP tides, equillibrium tides, resoncances…
- Fast

Do not care about
- Darwin names / Doodson codes
- Doodson-Warburg phase shifts

Example MATLAB Code

- Ocean tide synthesis at time t

```
thetaf = doodsonMatrix * doodsonArguments(t);
fCos   = admittanceMatrix * cos(thetaf);
fSin   = admittanceMatrix * sin(thetaf);

for i=1: size(fileList{1}, 1)
  cnm = cnm + fCos(i) * cnmCos{i} + fSin(i) * cnmSin{i};
  snm = snm + fCos(i) * snmCos{i} + fSin(i) * snmSin{i};
end
```

$$\theta_f(t) = \sum_{i=1}^{6} D_{f,i}\,\beta_i\,(t)$$
$$f_k^{cos}(t) = \sum_f A_{k,f} \cos\theta_f(t),$$
$$f_k^{sin}(t) = \sum_f A_{k,f} \sin\theta_f(t)$$

$$\begin{bmatrix} c_{nm} \\ s_{nm} \end{bmatrix}(t) = \sum_k f_k^{cos}(t) \begin{bmatrix} c_{nm} \\ s_{nm} \end{bmatrix}_k^{cos} + f_k^{sin}(t) \begin{bmatrix} c_{nm} \\ s_{nm} \end{bmatrix}_k^{sin}$$

Summary

- New approach to discuss
  - Standard file formats (ICGEM. *.gfc)
  - Few lines of code needed
  - Switch to newer ocean tide models without changing the source code
  - Fast (only matrix multiplications)
  - Flexible, can include future additional tidal lines
  - CON: lots of files
  - CON: breaks with old phase definition

- Next steps
  - Conversion of current ocean tide models to new file formats
  - Paper is in preparation
  - First draft for IERS conventions
  - Updates will be presented at IUGG

- Your feedback is needed

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1 Effect of the ocean and atmospheric tides

The dynamical effects of ocean and atmpospheric tides inclusive loading effects are most easily incorporated as periodic variations in the normalized Stokes' coefficients $\Delta\bar{C}_{nm}$ and $\Delta\bar{S}_{nm}$ of degree n and order m. These variations can be evaluated as

$$\begin{bmatrix}\Delta\bar{C}_{nm}(t)\\ \Delta\bar{S}_{nm}(t)\end{bmatrix} = \sum_k f_{k,cos}(t)\begin{bmatrix}\bar{C}_{nm}\\ \bar{S}_{nm}\end{bmatrix}_{k,cos} + f_{k,sin}(t)\begin{bmatrix}\bar{C}_{nm}\\ \bar{S}_{nm}\end{bmatrix}_{k,sin} . \qquad (1)$$

The $\left[\bar{C}_{nm}, \bar{S}_{nm}\right]_k$ are normalized Stokes' coefficients of the model for each tidal line provided separatedly for the cos and sin component. The modulated periodic factors for the cos/sin components of the model tides are computed using an admittance matrix $\mathbf{A} = (A_{k,f})$:

$$f_{k,cos}(t) = \sum_f A_{k,f}\cos\theta_f(t) \quad \text{and} \quad f_{k,sin}(t) = \sum_f A_{k,f}\sin\theta_f(t). \qquad (2)$$

The phase argument $\theta_f(t)$ of all considered major, minor and nonlinear tides f can be computed with

$$\theta_f(t) = \sum_{i=1}^{6} D_{f,i}\,\beta_i(t), \qquad (3)$$

where $\mathbf{D} = (D_{f,i})$ is a matrix with 6 columns containing the multipliers in a row for each tidal frequency f. The $\beta(t) = (\tau, s, h, p, N_0, p_s)$ is a six-vector of Doodson's fundamental arguments, see the explanatory text below Equation (**??**)(6.8e).
The tide model might be given in the Center of Earth (CE) frame or Center of Figure (CF) frame. For orbit computations in the Center of Mass (CM) frame the degree 1 coefficients $\Delta\bar{C}_{10}, \Delta\bar{C}_{11}, \Delta\bar{S}_{11}$ must be set to zero in Eq. (1).

1.1 Background information for conversion of models provided in gridded water heights

Ocean tide models are conventionally expressed for each tide k as water heights h_k on a geographical (ellipsoidal) grid either as amplitude A and phase ψ or as in-phase a and quadrature term b: