The anisotropy of internal tide generation: Global estimates for the M2 tide and implications for tidally driven mixing parameterizations

Friederike Pollmann, Jonas Nycander, Carsten Eden, Dirk Olbers

Supplementary material

EGU General Assembly
April 25, 2025

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

IDEMIX: compartment model

Treat high modes and low modes separately (Eden & Olbers, 2014):

$$E_{tot}(x,y,t) = \int_{-h}^{0} E_{high}(x,y,z,t)\mathrm{d}z + \int_{0}^{2\pi} \underbrace{E_{low}}_{E_{NIW}+E_{M_2}} (x,y,\Phi,t)\mathrm{d}\Phi$$

vertical column model for high modes:

$$\frac{\partial E_{high}}{\partial t} = \frac{\partial}{\partial z}\left(c_0 \tau_v \frac{\partial c_0 E_{high}}{\partial z} \right) - \epsilon_{IW} + \underbrace{W_c}_{=+W_{low}+T_{low}}$$

W_c: energy gain through interaction with low modes

2 compartments for low modes (NIW, M_2) resolve horizontal direction:

$$\frac{\partial E_{low}}{\partial t} = -\nabla \cdot c_g E_{low} + \frac{\partial}{\partial \Phi}\dot{\Phi} E_{low} \underbrace{-W_{low} - T_{low}}_{=-W_c} + F_{low}$$

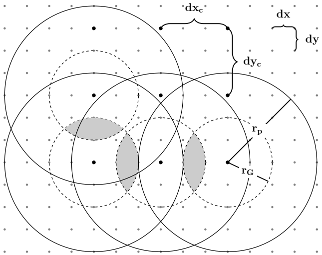
$\mathbf{k} = |\mathbf{k}|(\sin\Phi, \cos\Phi)$ $\mathbf{c_g} = \partial\omega / \partial\mathbf{k}$

W_{low}: wave-wave interactions T_{low}: topographic scattering

F_{low}: forcing by tides (M_2) and winds (NIW)

Method: Anisotropic internal tide generation

data: WOCE+Argo ($N^2 \rightarrow a_n$); TPXO9.1 ($\mathbf{U}$); SRTM30+ (h)
Gouretski & Koltermann, 2004; Egbert & Erofeeva, 2002; Becker et al., 2009



- consider high-resolution topography (grid spacing: dx,dy) in overlapping circular patches (centers on grid dx_c, dy_c)
- patch radius $r_p \approx 5\lambda_n$
- taper topography (2D Gaussian) toward patch mean at patch boundary
- assume constant parameters $\mathbf{U}$ and κ_n in each patch $\rightarrow$ weighted average (Gaussian)

Pollmann et al., 2019: *Resolving the horizontal direction of internal tide generation* (JFM)

Linearity assumptions: Supercritical slopes

Linear theory is valid for subcritical slopes $\gamma < 1$:

$$\gamma = \frac{|\nabla h|}{\alpha} = \left(\frac{N_B^2 - \omega^2}{\omega^2 - f^2} \right)^{\frac{1}{2}} \tag{1}$$

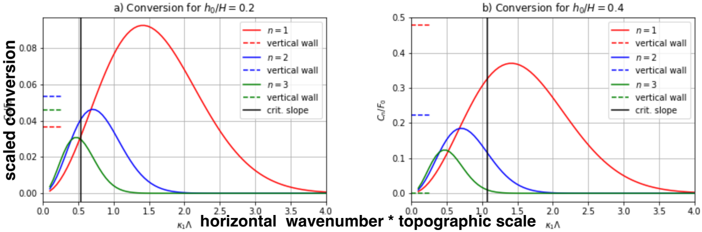
α: slope of tidal beam; f: Coriolis parameter; N_B: buoyancy frequency at the bottom; ω tidal frequency

Supercritical slope correction (Melet et al., 2013):
Where $\gamma > 1$, divide C by γ^2
(C $\sim \gamma^2$ for $\gamma < 1$ and saturation for $\gamma > 1 \rightarrow$ linear theory might overestimate conversion at steep slopes)

$\Rightarrow$ Revisit for (a) isolated ridges and (b) continental slopes

Supercritical slopes: Isolated seamounts and ridges

Conversion at 1D Gaussian ridge: $h(x) = h_0 exp(-x^2/\Lambda)$



supercritical slopes (linear theory is invalid) on left side of black vertical lines → for small n, this is only where conversion is small anyways as long as h_0/H is moderate

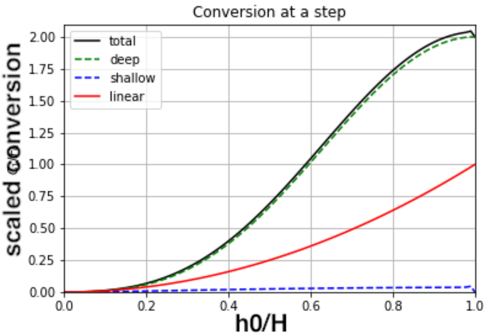
estimate of nonlinear solution (dashed): conversion at a wall (Llewellyn Smith and Young, 2003) → overestimation by linear theory only seen for steeper slopes ($h_0/H = 0.4$, right) and higher modes

⇒ for n=1, supercriticality is not a serious issue

Pollmann et al., 2023: *Resolving the horizontal direction of internal tide generation: Global application for the M_2-tide's first mode* (in press)

Supercritical slopes: Continental slopes

Conversion at a sharp step: $h(x) = \frac{h_0}{\pi} \tan^{-1}\left(\frac{x}{\Lambda}\right)$ (red)
estimate of nonlinear solution: St. Laurent et al, 2003 (black and dashed)

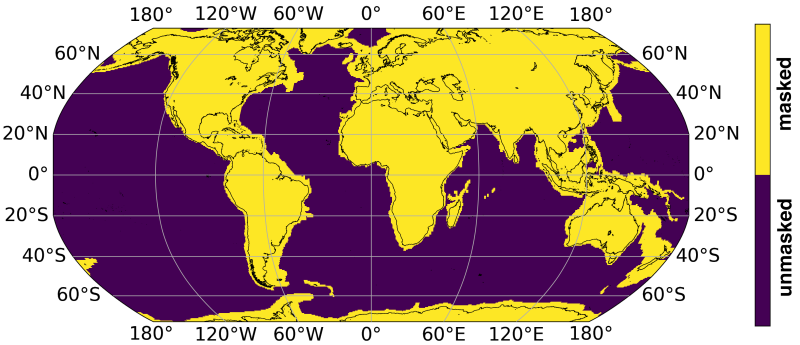


→ the linear solution underestimates the total nonlinear conversion
→ linear theory does not differentiate between shallow and deep side

⇒ Continental slopes need to be treated differently, but not by capping ⇒ exclude them entirely

Pollmann et al., 2023: *Resolving the horizontal direction of internal tide generation: Global application for the M_2-tide's first mode* (in press)

Masking continental shelves and slopes

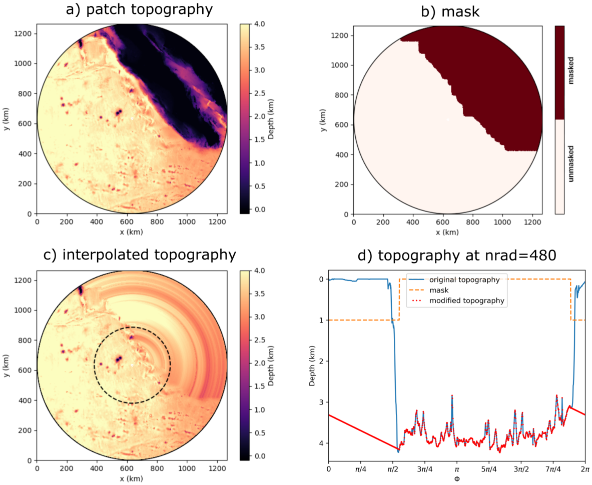


Mask land and continental shelves and slopes and deep ocean trenches based on geomorphology data of Harris et al., 2014. Plus manual "correction" (e.g. Mediterranean, Zealandia)

Pollmann et al., 2023: *Resolving the horizontal direction of internal tide generation: Global application for the M_2-tide's first mode* (in press)

Masking continental shelves and slopes

Partially masked patches: Replace masked topography by smooth artificial topography



Pollmann et al., 2023: *Resolving the horizontal direction of internal tide generation: Global application for the M_2-tide's first mode* (in press)