Effects of topography on the evolution of the coastal upwelling and downwelling jets in the Baltic Sea

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Set of diagnostics, I

In the active phase, assuming small displacements of the interfaces and linearized shallow water equations [Csanady [1977]]:

$$U = \frac{u_*^2}{f}\left[1 - \left(Ae^{-x/R_1} + Be^{x/R_1}\right)\frac{h}{h+h'} - \left(Ae^{-x/R_2} + Be^{x/R_2}\right)\frac{h'}{h+h'}\right] \tag{1}$$

$$U' = \frac{u_*^2}{f}\frac{h'}{h+h'}\left[Ae^{-x/R_2} + Be^{x/R_2} - \left(Ae^{-x/R_1} + Be^{x/R_1}\right)\right] \tag{2}$$

giving $U + U' = 0$

Two layer model, (Csanady [1977]):

Decomposition in two modes:

$$\frac{\partial}{\partial t}\left[\left(\frac{\partial^2}{\partial t^2}+f^2\right)\eta_{(bt,bc)}-\nabla\cdot(c_i^2\nabla\eta_{(bt,bc)})+\nabla\cdot\vec{F_{xi}}\right]=-fJ(c_i^2,\eta_{(bt,bc)})-f\nabla\times\vec{F_{xi}}$$

The free surface η and the interface η' displacement is given by the combination of the baroclinic and barotropic mode (neglecting inertial oscillations):

Free-surface displacement.

$$\eta=\eta_{bt}-\epsilon\frac{h'}{h+h'}\eta_{bc}=\frac{u_*^2 t}{c_{bt}}\left[Ae^{-x/R_{bt}}+Be^{x/R_{bt}}+\epsilon\frac{c_1}{c_2}\left(\frac{h'}{h+h'}\right)^2\left(Ce^{-x/R_{bc}}+De^{x/R_2}\right)\right]$$

Interface displacement.

$$\eta'=\eta'_{bc}+\frac{h'}{h+h'}\eta'_{bt}=\frac{u_*^2 t}{c_2}\frac{h'}{h+h'}\left[Ce^{-x/R_{bc}}+De^{x/R_{bc}}-\frac{c_2}{c_1}\left(Ae^{-x/R_{bt}}+Be^{x/R_{bt}}\right)\right]$$

$$A=\frac{1-e^{+L/R_{bt}}}{2\sinh\frac{L}{R_{bt}}},\ B=\frac{1-e^{-L/R_{bt}}}{2\sinh\frac{L}{R_{bt}}},\ c_2=\sqrt{g\epsilon\frac{hh'}{h+h'}},\ c_1=\sqrt{gH}/f,\ \epsilon=(\rho_2-\rho_1)/\rho_1$$

Set of diagnostics

Defining, mean quantity as follows:

$$\{\phi(x,z,t)\} = \frac{1}{L}\int_{y_0}^{y_0+L} \phi(x,y,z,t)dy \tag{3}$$

together with its fluctuation:

$$\phi'(x,y,z,t) = \phi(x,y,z,t) - \{\phi(x,z,t)\} \tag{4}$$

and applying the averaging procedure to the equations of motion, the time change of eddy kinetic energy can be expressed as:

$$\int_O^L \int_O^H K_{et}\, dzdx = C_{pke} + C_{mke} + Diss. \tag{5}$$

Energy conversion rates (Orlanski and Cox [1973]):

$$C_{pke}(t) = -\frac{g}{\rho_0 A}\int_0^W \int_{-h}^0 \overbrace{\{\rho' w'\}}^{\text{baroclinic instability}} dzdx \tag{6}$$

$$C_{mke}(t) = \frac{1}{A}\int_0^W \int_{-h}^0 \overbrace{(\{v_x\}\{u'v'\} + \{u_x\}\{u'u'\}}^{\text{barotropic instability}} + + \overbrace{\{v_z\}\{w'v'\} + \{u_z\}\{w'u'\}}^{\text{KH instability}} dzdx \tag{7}$$

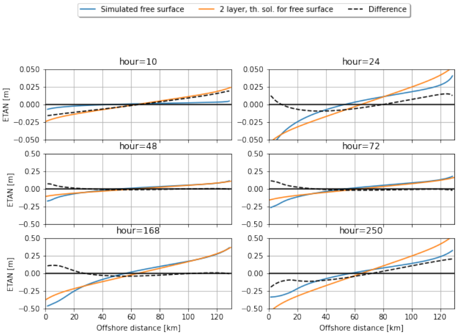
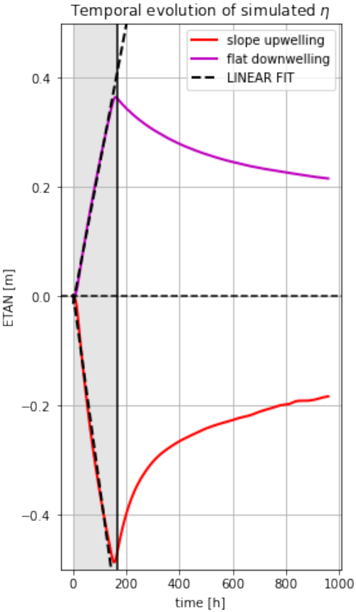
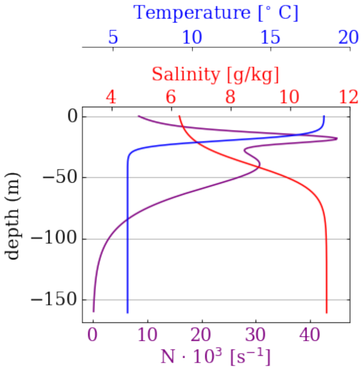


Figure: Averaged simulated sea level anomaly compared with the analytical solution from two-layer shallow water equations in channel



Initialization is representative of the summer conditions with a developed thermocline at 20 m superimposed on the permanent halocline at 60 m giving a two-pycnocline structure, typical for the proper Baltic during summer months. The stratification profile is taken by CMEMS reanalysis.



Eigenvalue problem :

$$R_{bc}\frac{\partial^2 w(z)}{\partial^2 z^2} - \frac{N(z)^2}{f^2} w = 0 \qquad (8)$$

giving R_{bc} =8.13 km

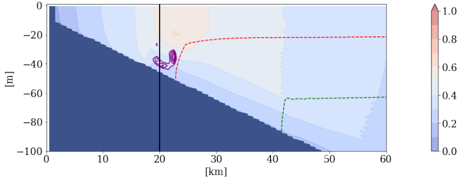
Discrete 2 layer model :

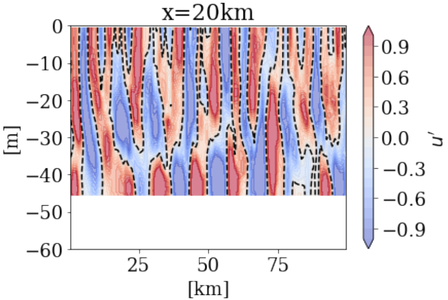
h' =20 m $\rightarrow R_{bc}$ =5.15 km
h' =60 m $\rightarrow R_{bc}$ =12.2 km

Temperature/salinity profile:

$$S = \frac{1}{2}(S_0 + S_1 - (S_0 - S_1)\tanh\left(\frac{(z-z_0)}{l}\right))$$

Onset of instabilities: downwelling, $t' = 18$

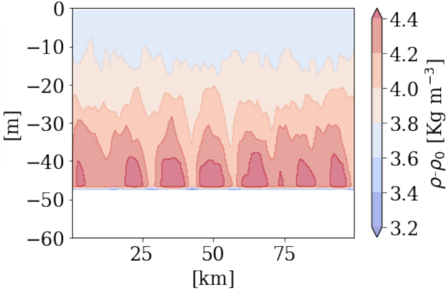




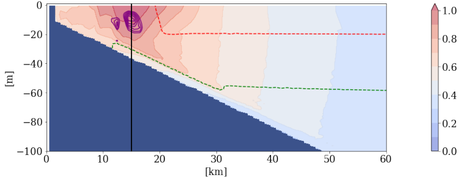
Energy conversion rates:

$$\int_O^L \int_O^H K_{et}\, dzdx = C_{pke} + C_{mke} + Diss. \tag{9}$$

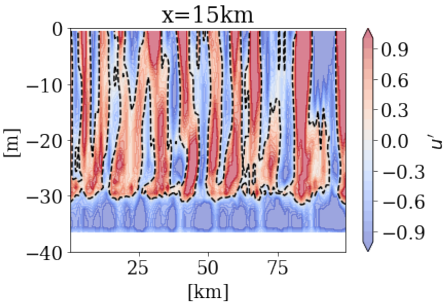
$$C_{pke}(t) = -\frac{g}{\rho_0 A} \int_0^W \int_{-h}^0 \overbrace{\{\rho' w'\}}^{\text{baroclinic instability}} dzdx \tag{10}$$

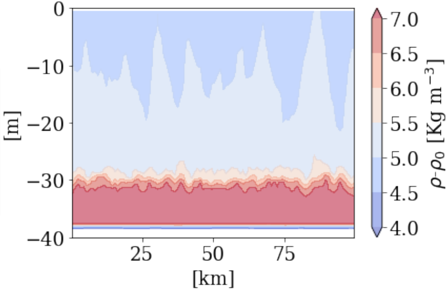


Onset of instabilities: upwelling, $t' = 12$



Figure



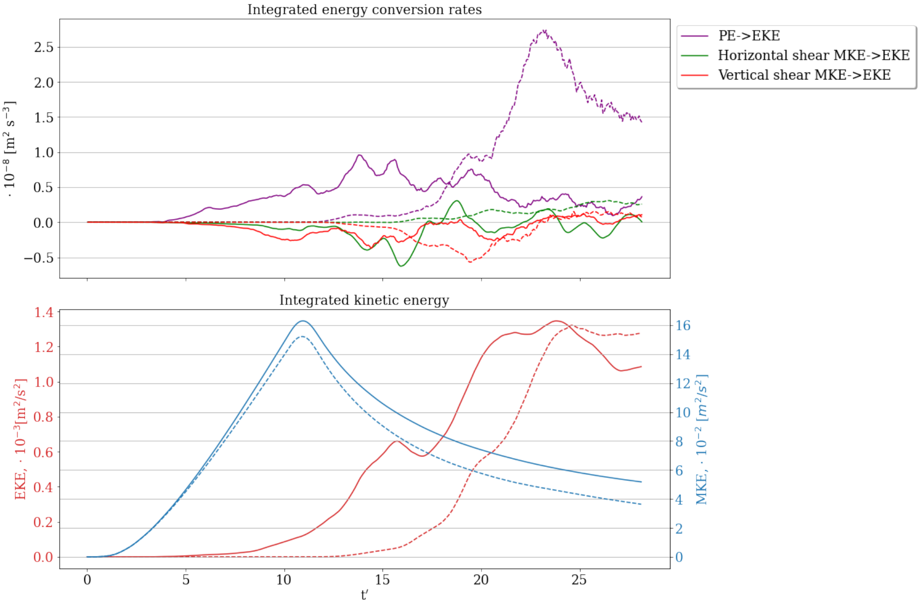


Energy conversion rates:

$$\int_O^L \int_O^H K_{et}\, dzdx = C_{pke} + C_{mke} + Diss. \qquad (11)$$

$$C_{pke}(t) = -\frac{g}{\rho_0 A} \int_0^W \int_{-h}^0 \overbrace{\{\rho' w'\}}^{\text{baroclinic instability}} \, dzdx \qquad (12)$$

Averaged conversion rates



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Isidoro Orlanski and Michael D. Cox. Baroclinic instability in ocean currents. *Geophysical Fluid Dynamics*, 4(4):297–332, 1973. doi: 10.1080/03091927208236102. URL https://doi.org/10.1080/03091927208236102.