Collision efficiency of poly-dispersed charged spheres settling in a quiescent environment

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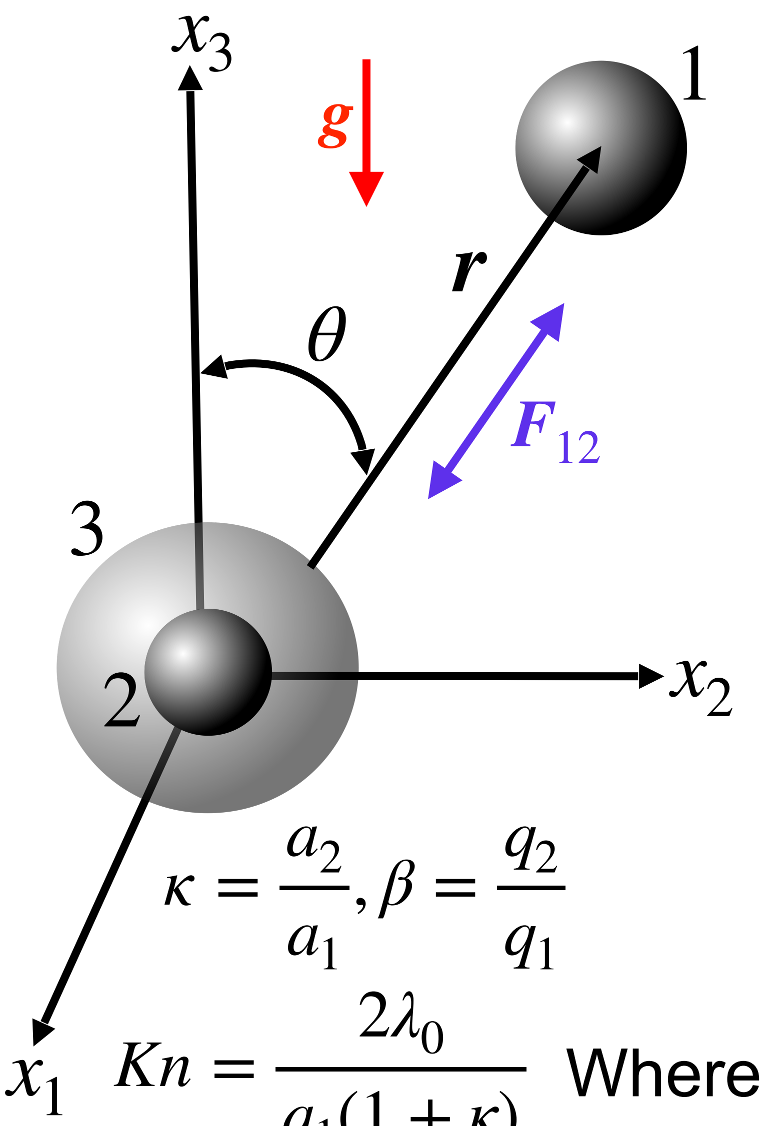
Relative velocity between two charged drops settling in quiescent air at $St = 0$ and $Re_p = 0$



$$\boldsymbol{V}_{12} = \frac{2\rho_p\left(a_1^2 - a_2^2\right)\boldsymbol{g}}{9\mu_f}\left[L\frac{r_i r_j}{r^2} + M\left(\delta_{ij} - \frac{r_i r_j}{r^2}\right)\right]$$

$$+\frac{1}{6\pi\mu_f}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)\left[G\frac{r_i r_j}{r^2} + H\left(\delta_{ij} - \frac{r_i r_j}{r^2}\right)\right]\cdot\boldsymbol{F}_{12}$$

Batchelor, *J. Fluid Mech.*, (1976,1982a)

Hydrodynamic mobilities L, M, G, H are functions of the non-dimensional centre-to-centre distance, r, the size ratio, κ, and the Knudsen number, Kn.

$\boldsymbol{F}_{12}$ = Electrostatic force on sphere 1 due to sphere 2. It depends on r, κ, β

$$\kappa = \frac{a_2}{a_1}, \beta = \frac{q_2}{q_1}$$

$$Kn = \frac{2\lambda_0}{a_1(1+\kappa)}$$ Where λ_0 = Mean free path of the suspending medium

Dimensionless velocity and trajectory equations in spherical coordinates

$$\frac{dr}{dt} = v_r = -L\cos\theta + N_e G f_e, \text{ and } r\frac{d\theta}{dt} = v_\theta = M\sin\theta$$

Here, $N_e = \dfrac{3q_1^2}{16\pi^2 a_1^5(\rho_p - \rho_f)\epsilon_0\kappa(1-\kappa)g}, f_e = \dfrac{|\boldsymbol{F}_{12}|}{q_1^2/(4\pi\epsilon_0 a_1^2)}$

We calculate f_e using bispherical coordinates solutions

Bichoutskaia, *J. Chem. Phys.*, (2014)

For a colliding trajectory, $v_r < 0$ for all r $\Longrightarrow$ $N_e < \dfrac{L\cos\theta}{Gf_e}$ for $r \in [2,\infty]$

Condition for getting at least one collision trajectory is $N_e < \dfrac{(L/G)}{Gf_e}$ for $r \in [2,\infty]$

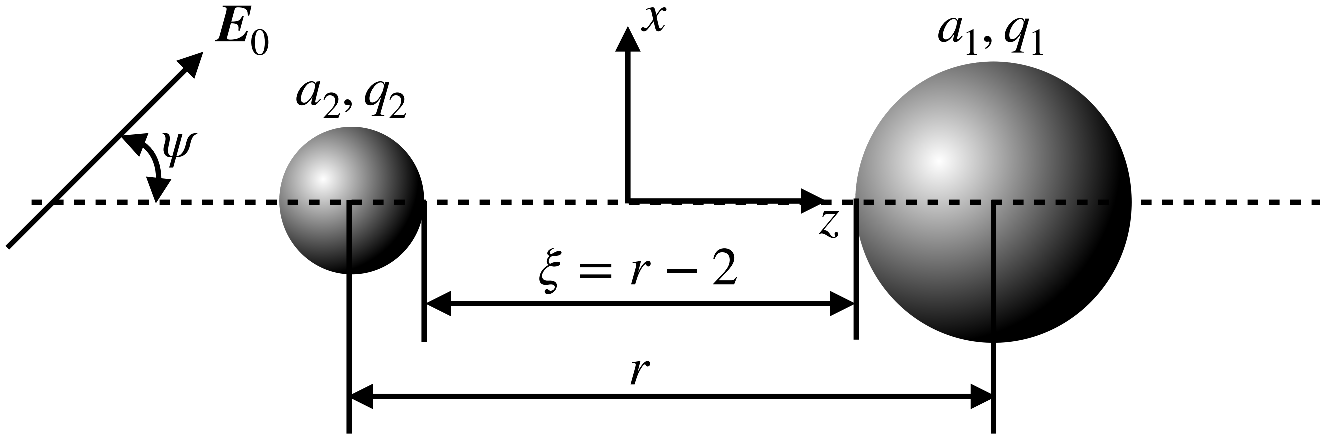
Therefore, we calculate the critical value of N_e above which there will be no collision trajectories

Electrostatic force between two charged conducting spheres in a uniform electric field

$$F_z^{(2)} = \left\{ \epsilon_0 a_2^2 E_0^2 \left(F_1 \cos^2 \psi + F_2 \sin^2 \psi \right) + E_0 \cos \psi \left(F_3 q_1 + F_4 q_2 \right) + \frac{1}{\epsilon_0 a_2^2} \left(F_5 q_1^2 + F_6 q_1 q_2 + F_7 q_2^2 \right) \right\} + E_0 q_2 \cos \psi,$$

$$F_x^{(2)} = \left\{ \epsilon_0 a_2^2 E_0^2 F_8 \sin 2\psi + E_0 \sin \psi \left(F_9 q_1 + F_{10} q_2 \right) \right\} + E_0 q_2 \sin \psi$$

$$\boldsymbol{F}^{(1)} = \boldsymbol{E}_0 \left(q_1 + q_2 \right) - \boldsymbol{F}^{(2)}$$



Davis, *Q. J. Mech. Appl. Math.*, (1964)

Electrostatic potential energy and force between two charged conducting spheres

$$\Phi_{\text{el}} = \frac{1}{2}\sum_{i=1}^{N} q_i V_i \qquad \Phi_{\text{el}} = \frac{q_2^2 C_{11} - 2q_1 q_2 C_{12} + q_1^2 C_{22}}{2(C_{11}C_{22} - C_{12}^2)}, \quad F_{\text{el}} = -\frac{d\Phi_{\text{el}}}{dr}$$

$$q_i = \sum_{j=1}^{N} C_{ij} V_j \quad (i = 1,2,3,\cdots,N)$$

$$C_{11} = 4\pi\epsilon_0 a_1 a_2 \sinh\eta \sum_{n=0}^{\infty} [a_1 \sinh n\eta + a_2 \sinh(n+1)\eta]^{-1}$$

$$C_{22} = 4\pi\epsilon_0 a_1 a_2 \sinh\eta \sum_{n=0}^{\infty} [a_2 \sinh n\eta + a_1 \sinh(n+1)\eta]^{-1},$$

$$C_{12} = -\frac{a_1 a_2}{r} \sinh\eta \sum_{n=1}^{\infty} [\sinh n\eta]^{-1}$$

$$\cosh\eta = \frac{r^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

Lekner, *Proc. R. Soc. A*, (2012)

Electrostatic potential energy and force between two charged conducting spheres (lubrication form)

Let say, $a_1 = a_2 = a$

$$\Phi_{\mathrm{el}} = -\frac{1}{2a}\frac{(q_1+q_2)^2\ln(a/\xi)+4q_1q_2\gamma-2q_2^2\Psi(1/2)-2q_1^2\Psi(1/2)}{(\gamma+\Psi(1/2))\ln(a/\xi)+\gamma^2-(\Psi(1/2))^2}+O(\xi)$$

$$F_{\mathrm{el}} = -\frac{(q_1-q_2)^2}{2a\xi\left[\ln(4a/\xi)+2\gamma\right]^2}+O(1)$$

Ψ : digamma function

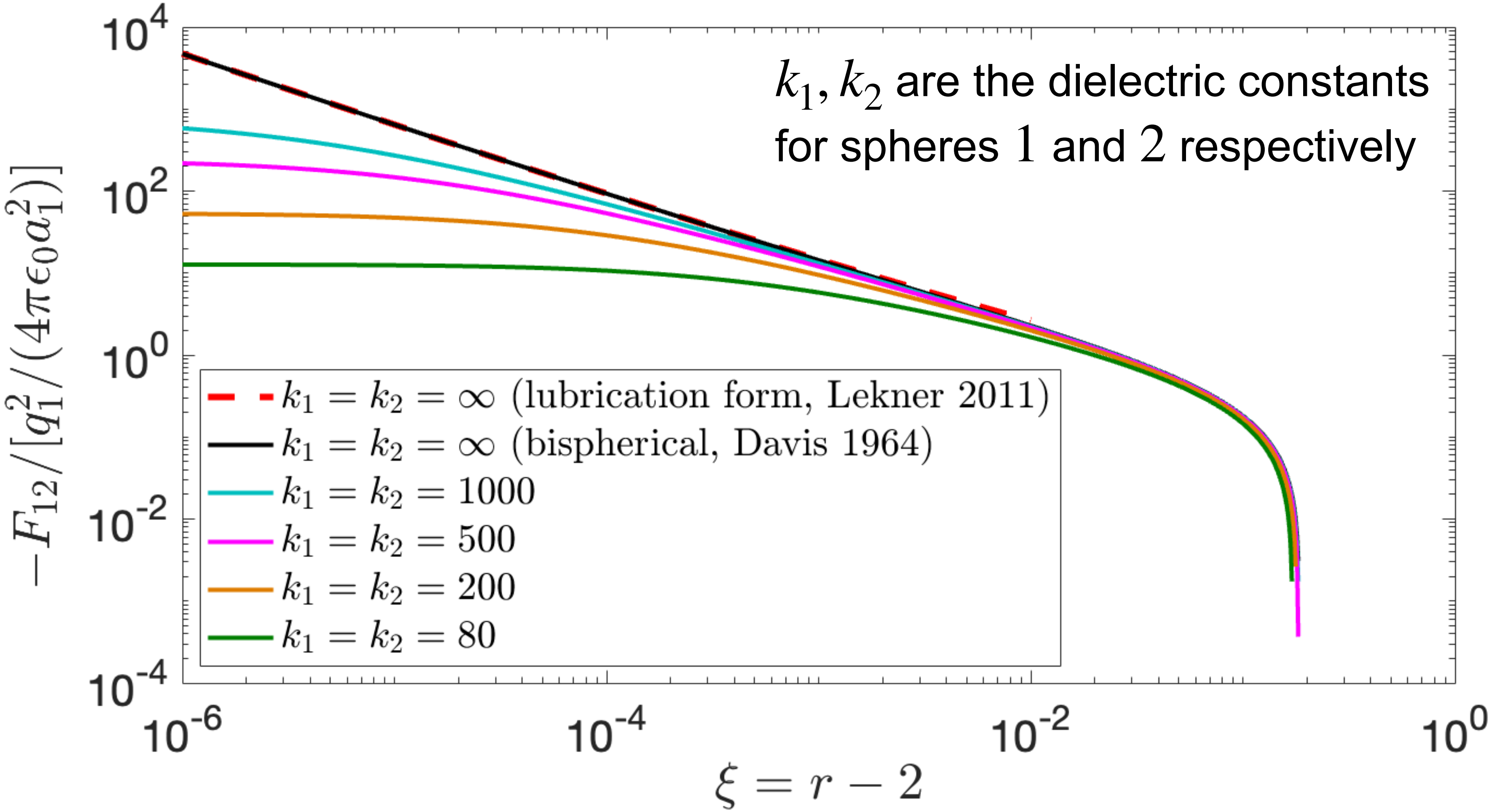
γ : Euler constant

Relation between charge ratio and size ratio for which spheres will repel each other at all separation is:

$$\beta = \frac{q_2}{q_1} = \frac{\gamma+\Psi\left(\frac{a_1}{a_1+a_2}\right)}{\gamma+\Psi\left(\frac{a_2}{a_1+a_2}\right)} = \frac{\gamma+\Psi\left(\frac{1}{1+\kappa}\right)}{\gamma+\Psi\left(\frac{\kappa}{1+\kappa}\right)}$$

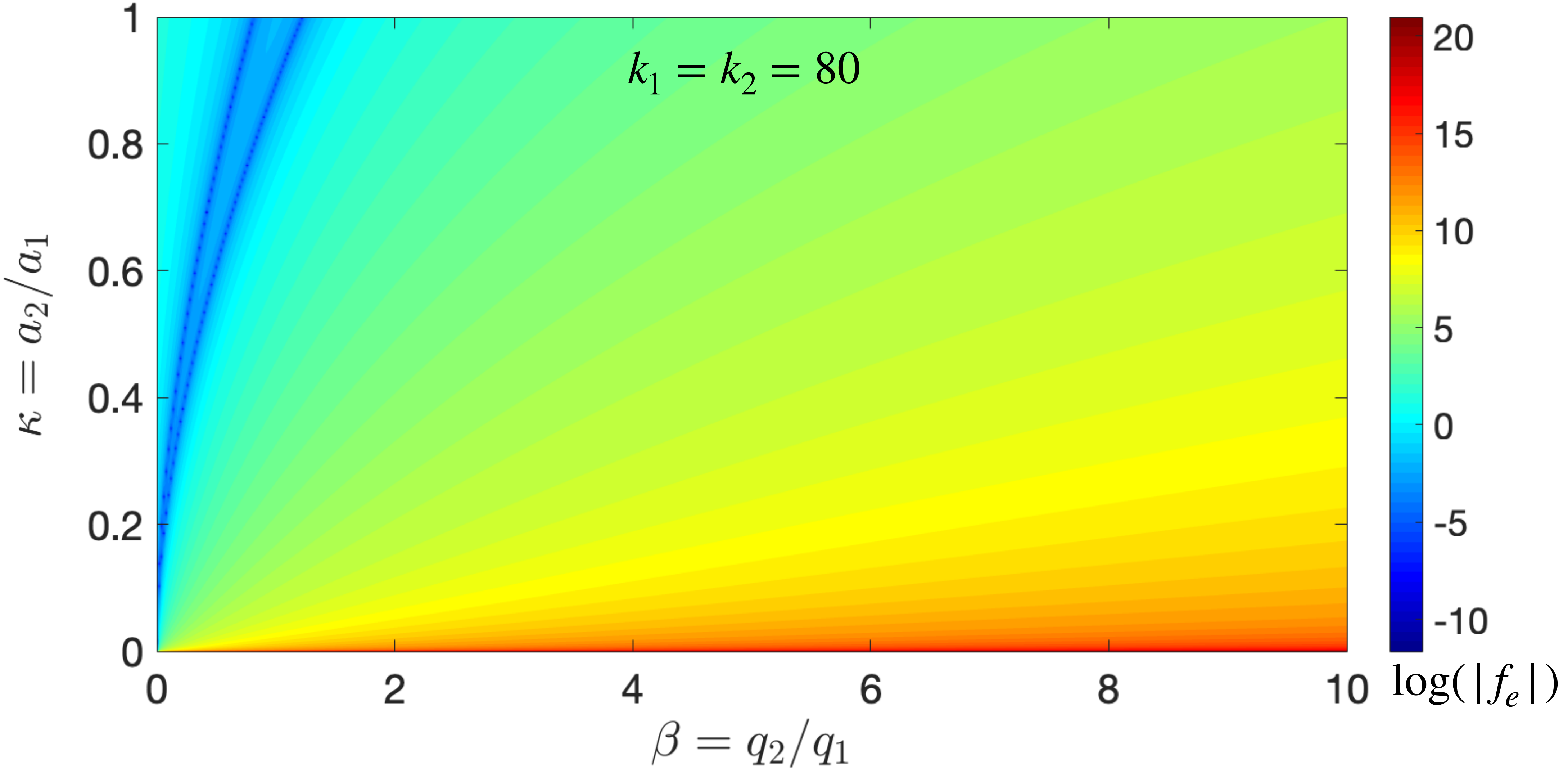
Lekner, *Proc. R. Soc. A*, (2012)

Non-dimensional Electrostatic force between two charged spheres



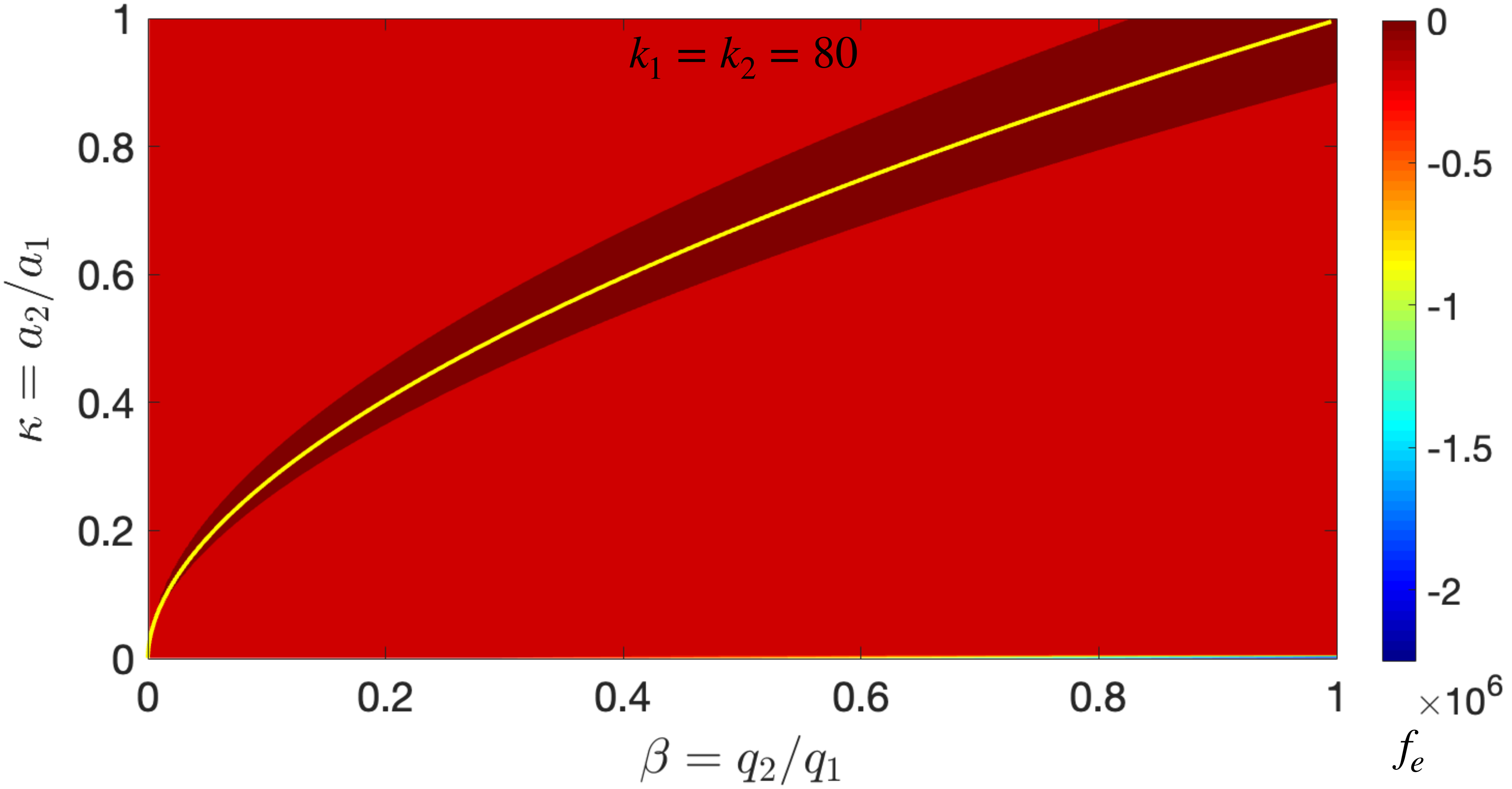
Davis, *Q. J. Mech. Appl. Math.*, (1964)
Lekner, *Proc. R. Soc. A*, (2012)
Bichoutskaia, *J. Chem. Phys.*, (2014)

Non-dimensional Electrostatic force between two charged spheres at contact



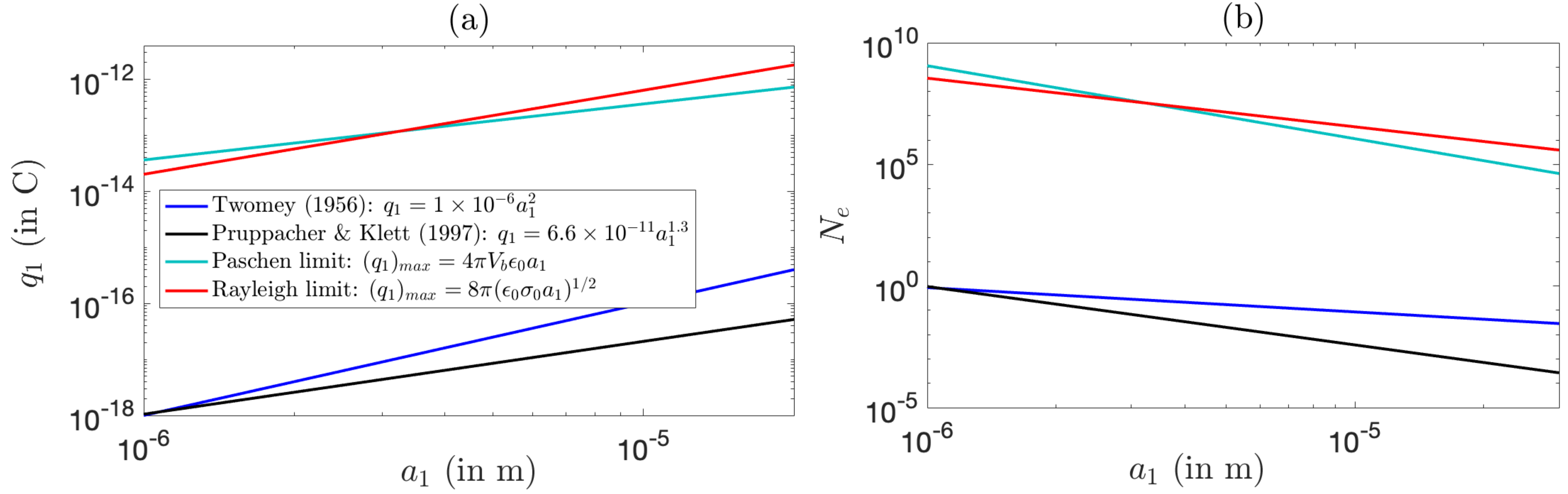
Bichoutskaia, *J. Chem. Phys.*, (2014)

Non-dimensional Electrostatic force between two charged spheres at contact

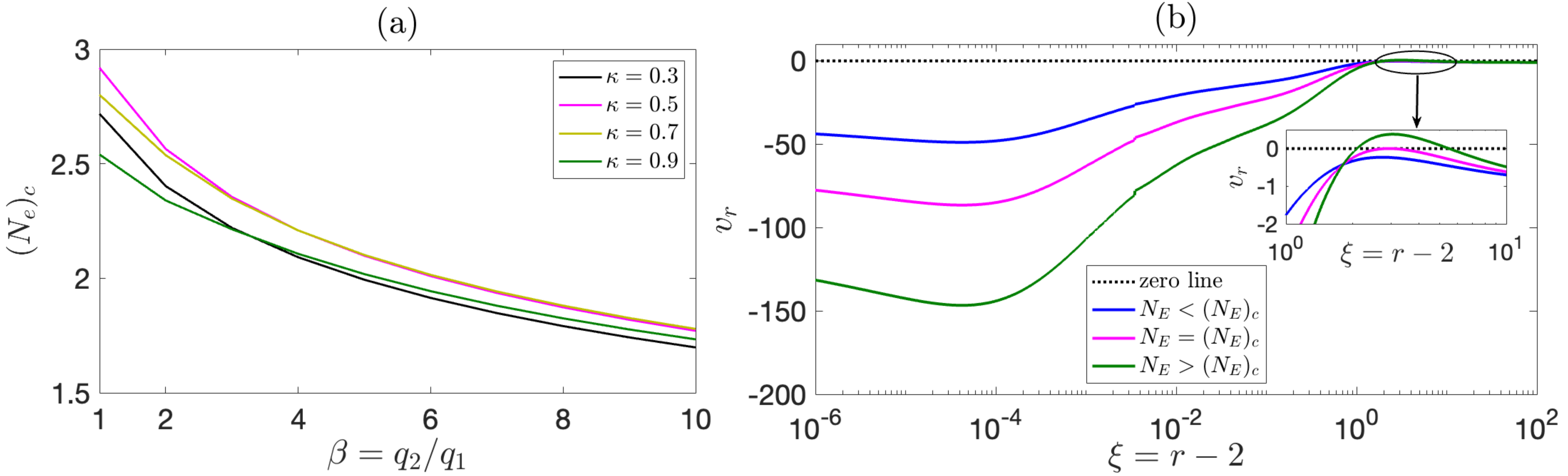


Bichoutskaia, *J. Chem. Phys.*, (2014)

Typical charges on cloud droplets and corresponding values of N_e



Numerically calculated values of $(N_e)_c$ for different κ and β

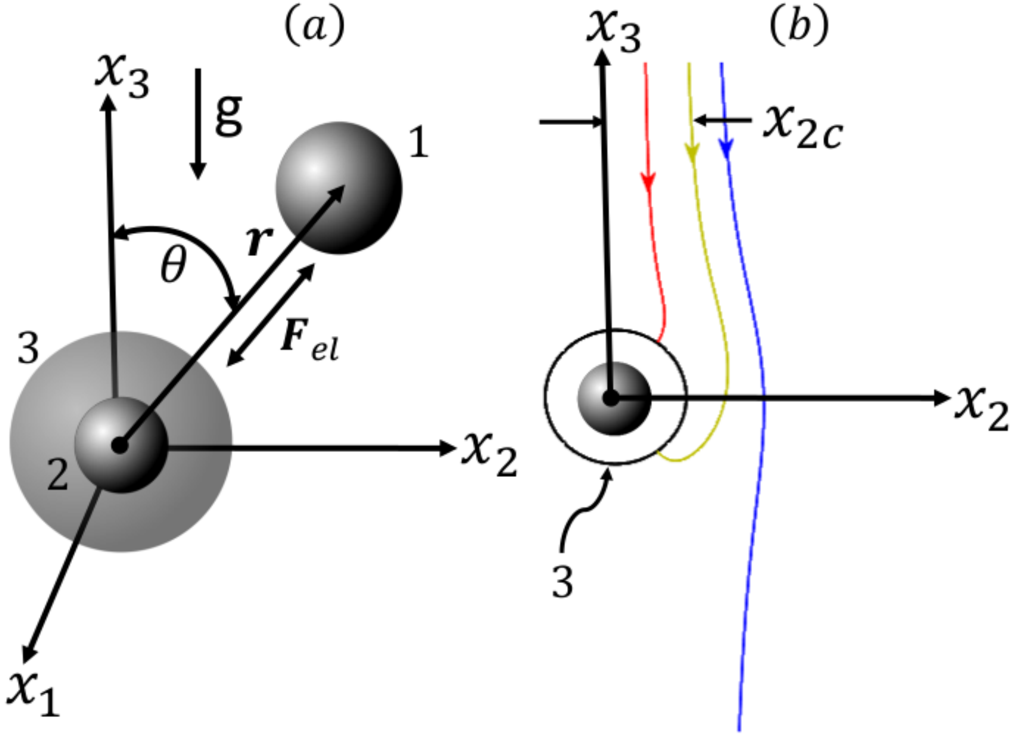


Relative trajectories and collision efficiency

Relative trajectory equation: $$\frac{d\theta}{dr} = \frac{M\sin\theta}{r\left(-L\cos\theta + N_e G f_e\right)}$$

Collision efficiency: $$E_{12} = \frac{\text{Collision rate with interaction } (K_{12})}{\text{Collision rate without interaction } (K_{12}^0)} = \frac{1}{4}\bar{x}_{2c}^2$$

where $$\bar{x}_{2c} = \frac{2x_{2c}}{(a_1 + a_2)}$$

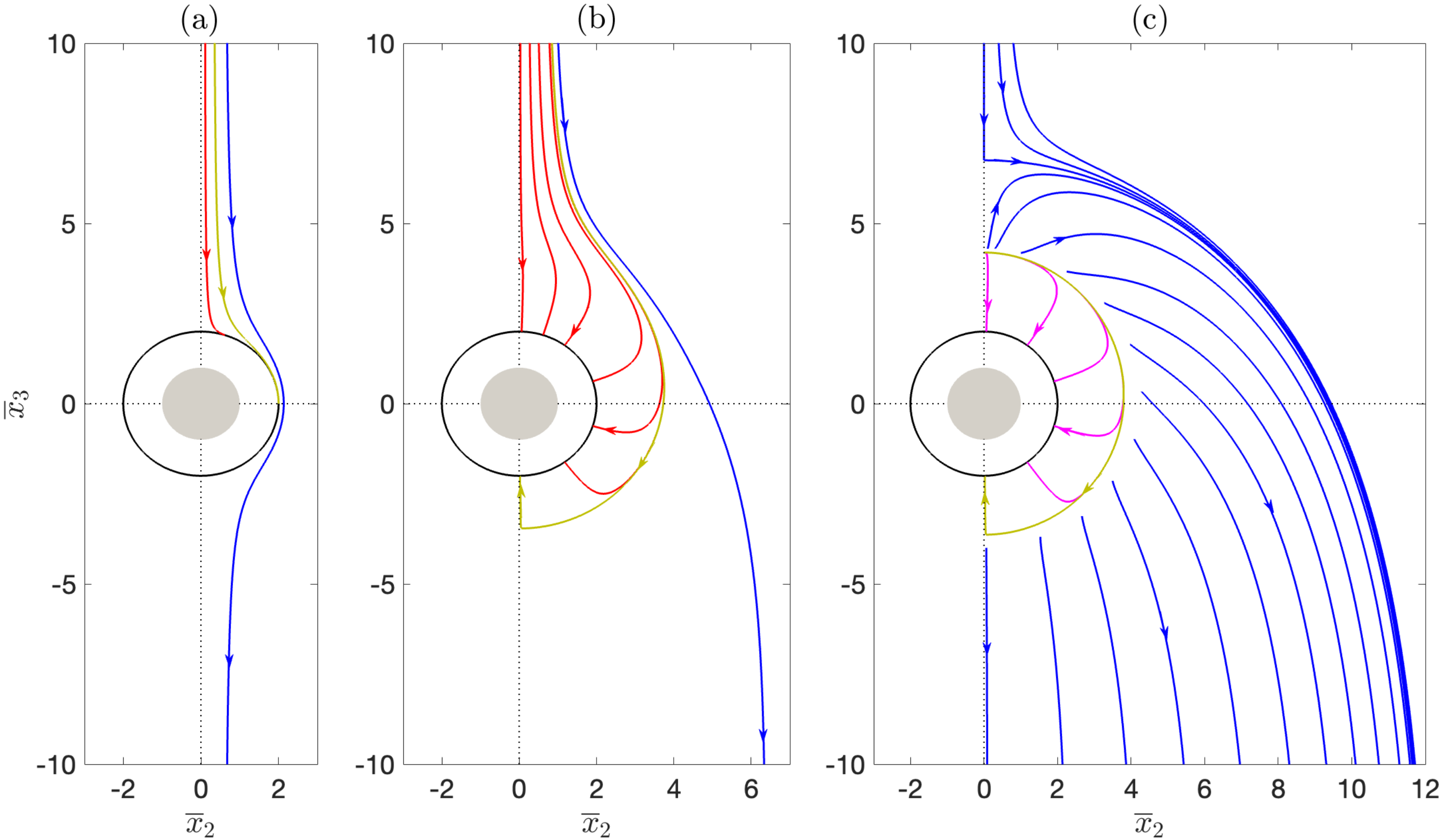


Analytical expression for the collision efficiency in the $N_e = 0$ limit:

$$E_{12} = \exp\left(-2\int_2^\infty \frac{M-L}{rL}dr\right)$$

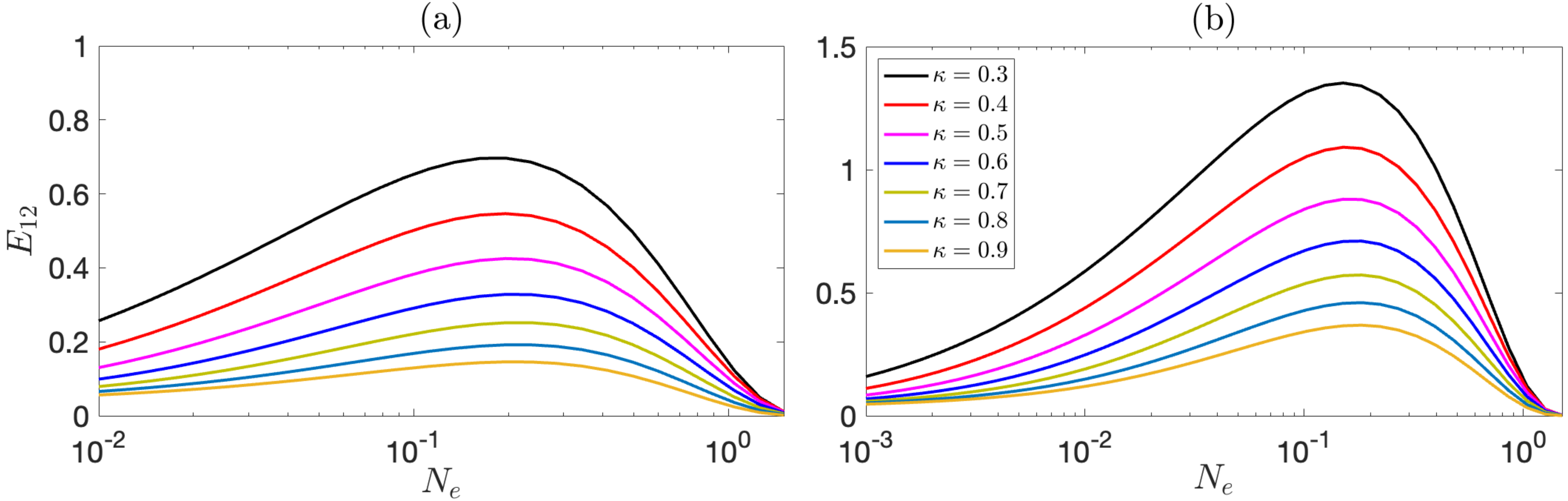
Davis, *J. Fluid Mech.*, (1984)

Relative trajectories for different values of N_e when $\kappa = 0.5$ and $\beta = 10$



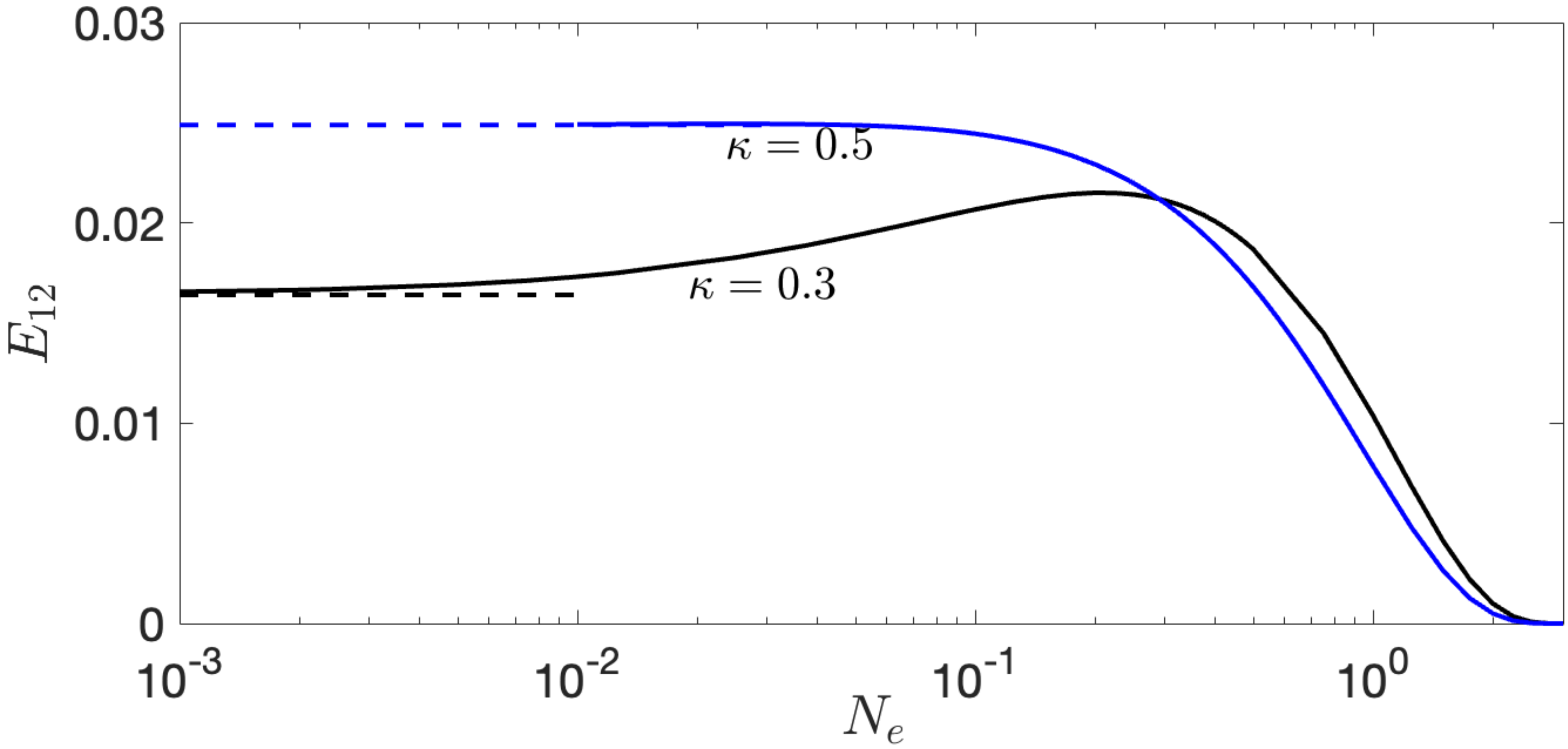
$N_e = 0$ $\quad$ $N_e \leq (N_e)_c$ $\quad$ $N_e > (N_e)_c$

Collision efficiency as a function of N_e for different κ when $Kn = 10^{-2}$

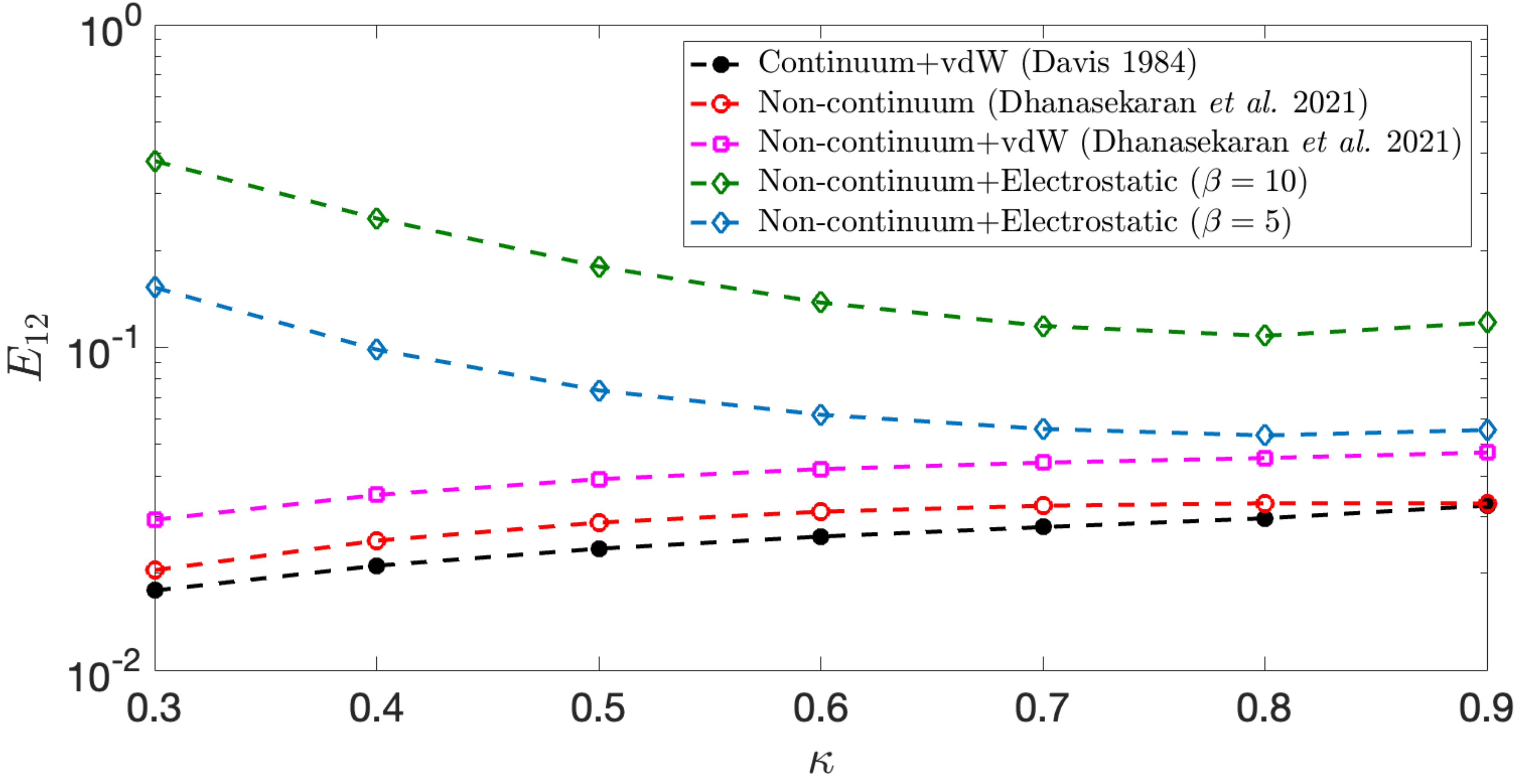


$\beta = 5$ $\beta = 10$

Collision efficiency as a function of N_e when $\kappa = \beta = 0.3, 0.5$



Comparisons of collision efficiencies with previous studies for $a_1 = 10\ \mu m$ and $q_1 = 2 \times 10^{-16}\ C$



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Thank You!