Modelling Magnetic Turbulence with log-normal Intermittency by continuous Cascades

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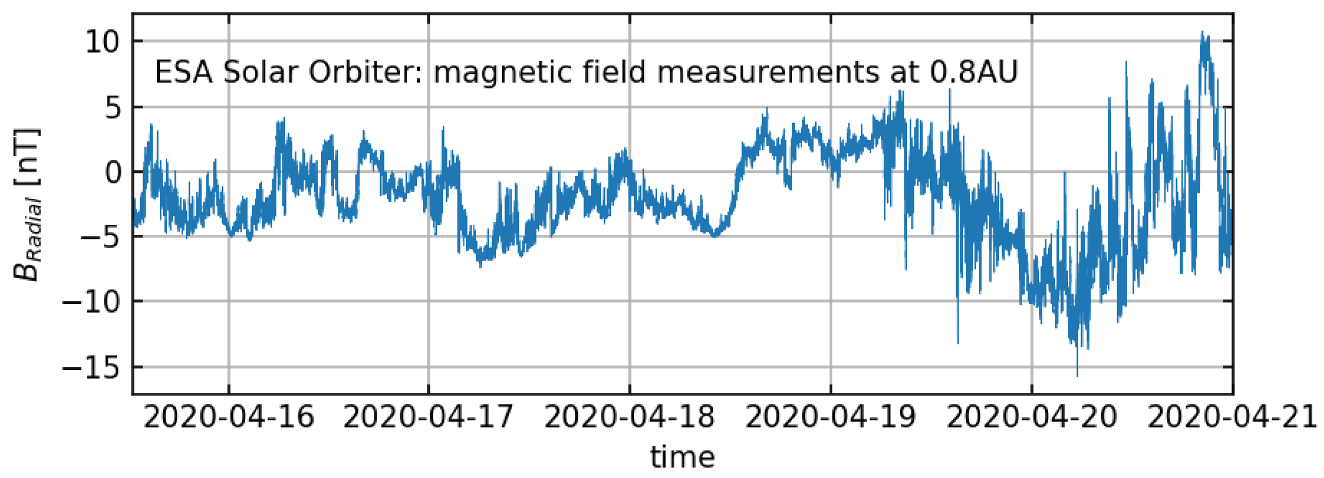
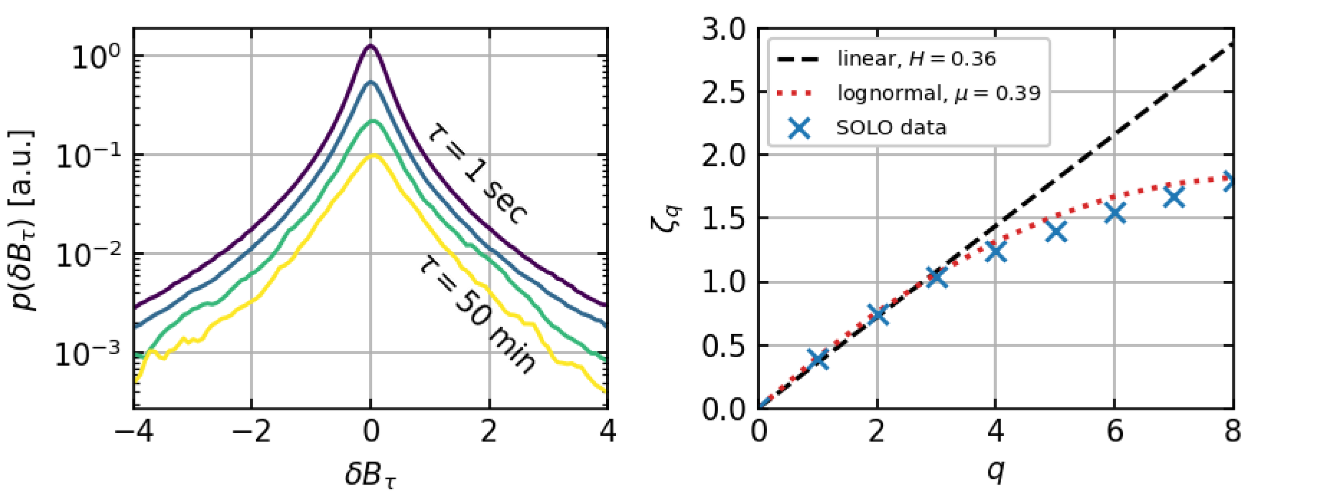
Motivation

The solar wind is highly intermittent as shown by numerous observations, yet charged particle transport is predominantly modelled as a simple Gaussian process. Magnetic turbulence is modelled in this context by a prescribed energy spectrum with random phases.
Intermittency is characterized by heavy-tailed distributions of field increments $\delta\mathbf{B}(\mathbf{d}) = \mathbf{B}(\mathbf{x}+\mathbf{d}) - \mathbf{B}(\mathbf{x})$, which scale anomalously with the increment distance. This anomalous scaling is encoded in the scaling exponents ζ_p of the structure functions

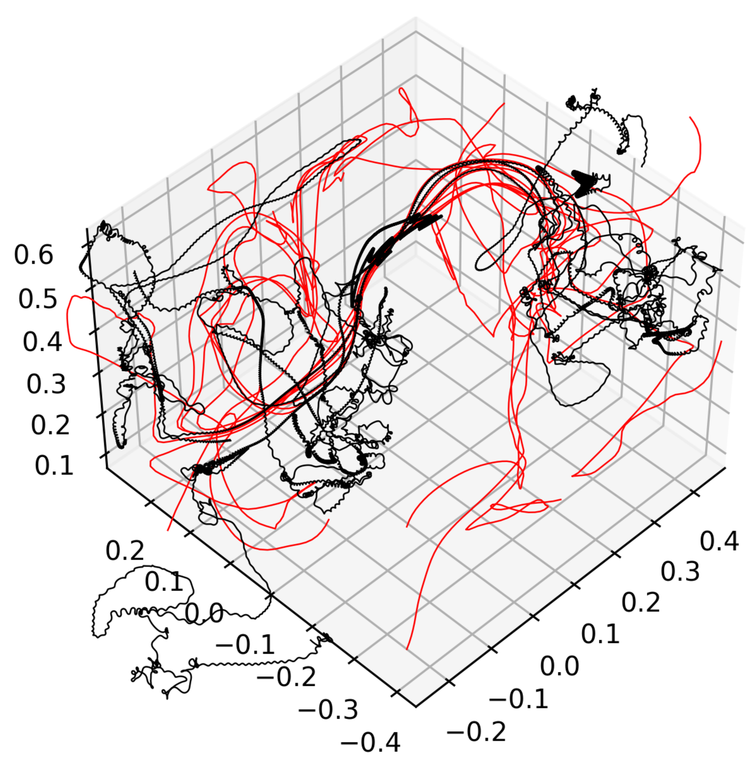
$$S_p(\mathbf{d}) = \langle [\delta\mathbf{B}(\mathbf{d})\cdot\mathbf{d}]^p \rangle \propto d^{\zeta_p}.$$

Additionally, magnetic turbulence exhibits distinct **coherent structures** in contrast to unstructured random phases models. Effective descriptions of charged particle transport in realistic magnetic turbulence are missing so far.

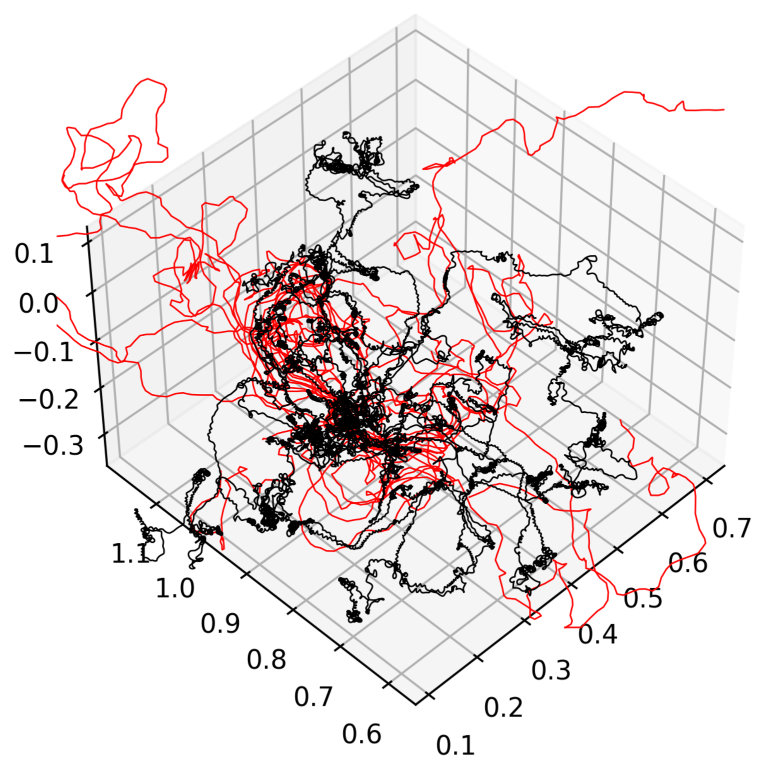




Magnetic field measurement by SolO, exhibting intermittency as seen by the increment distributions and scaling exponents



(i) MHD turbulence



(ii) Random phases turbulence

The structural differences between MHD simulations and random phases model illustrated by field line tracers (red) and charged particle trajectories (black)

Cascade Algorithm

Intermittency is accounted for by an **infinitely divisible stochastic process** in scale $\omega_{s\cdot s_{\max}}(s\mathbf{x}) \overset{\text{law}}{=} \Omega_s + \omega_{s_{\max}}(\mathbf{x})$ with $\Omega_s \sim \mathcal{N}(-\sigma_s^2/2, \sigma_s^2)$, $\sigma_s^2 \propto \mu/s^{d+1}$. This leads to log-normal multifractal scaling with intensity μ

$$\langle \exp p\,\omega_s(\mathbf{x})\rangle \propto s^{-\frac{1}{2}\mu(p-p^2)}.$$

A divergence-free vector field in dimension d with spectral index H is obtained by a **weighted wavelet transform** and application of curl

$$\mathbf{v}(\mathbf{x}) \propto \nabla \times \int_{s_{\min}}^{s_{\max}} s^{H-d}\left(e^{\omega_s} * \psi_s\right)(\mathbf{x})\,\mathrm{d}s\,.$$

Prototypes of coherent structures are introduced by considering the MHD equations in Elsässer variables with neglected RHS, i.e. $\nabla P_{\text{total}} \to 0$

$$(\partial_t + \mathbf{z}_\mp\cdot\nabla)\,\mathbf{z}_\pm = -\nabla P_{\text{total}}$$

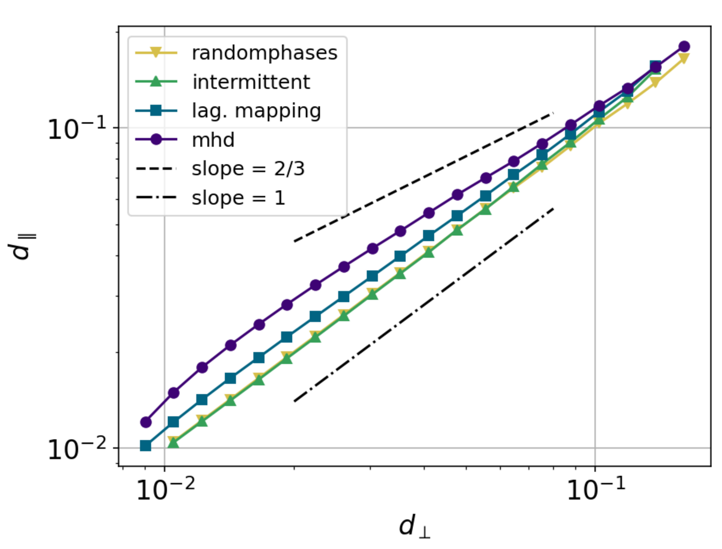
with the **Lagrangian solution**

$$\mathbf{z}_\pm(\mathbf{x}_t, t) = \mathbf{z}_\pm(\mathbf{x}_0, 0),$$
$$\mathbf{x}_t = \mathbf{x}_0 + t\,\mathbf{z}_\mp(\mathbf{x}_0, 0).$$

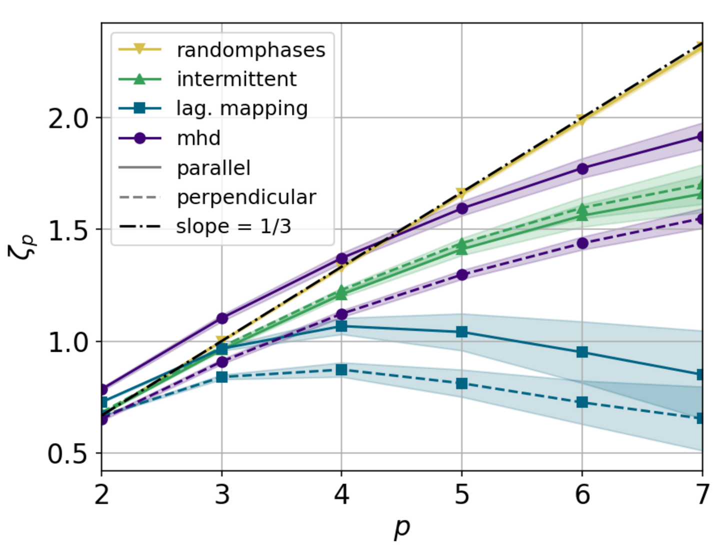
$\mathbf{z}_\pm$ are two independent realizations of the field $\mathbf{v}$.

Result: vector potentials $\mathbf{A}_\pm(\mathbf{x})$, multifractal fields $\boldsymbol{\omega}_\pm(\mathbf{x})$, coordinate arrays $\mathbf{x}_\pm$
for $s \in \{s_{\min}, \cdots, s_{\max}\}$ **do**
  $\sigma^2 \leftarrow c_d\mu\Delta s\Delta x^d s^{-d-1}$;
  generate $\Omega_\pm \sim \mathcal{N}\left(-\sigma^2/2, \sigma^2\mathbb{I}\right)$;
  $\boldsymbol{\omega}_\pm \leftarrow \boldsymbol{\omega}_\pm + \texttt{mollify}_s(\Omega_\pm)$;
  $\mathbf{A}_\pm \leftarrow \mathbf{A}_\pm + \Delta s\, s^{H-d}\left(\psi_s * \exp\boldsymbol{\omega}_\pm\right)$;
  $\mathbf{x}_\pm \leftarrow \mathbf{x}_\pm + \mathrm{cfl}\cdot s\cdot\nabla\times\mathbf{A}_\mp$;
end
$\mathbf{A}_\pm(\mathbf{x}), \boldsymbol{\omega}_\pm(\mathbf{x}) \leftarrow \texttt{interp}(\mathbf{A}_\pm(\mathbf{x}_\pm), \boldsymbol{\omega}_\pm(\mathbf{x}_\pm))$;

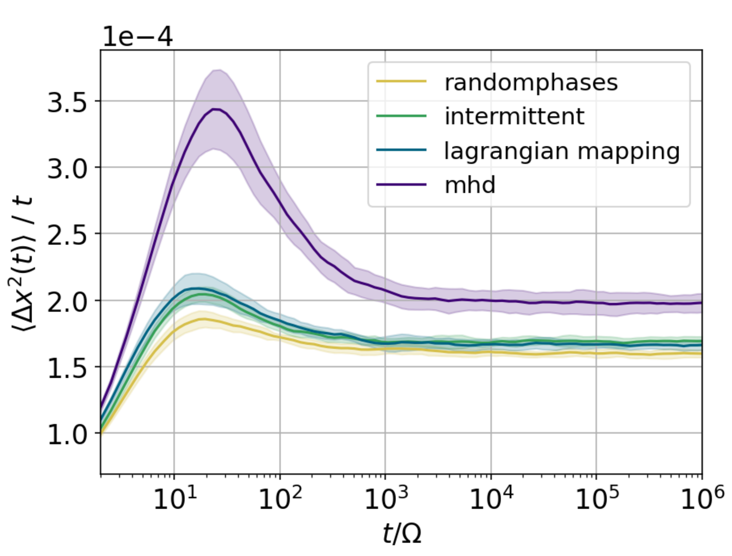
First Results



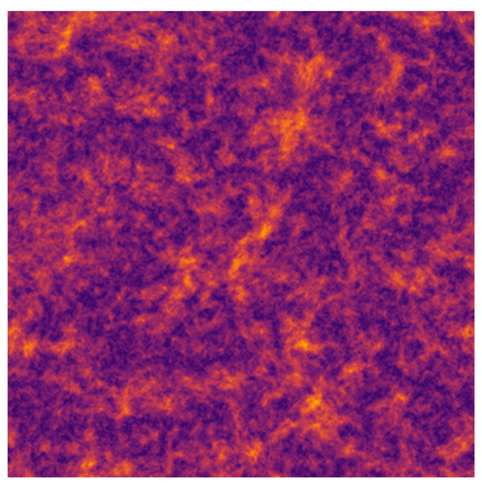
Comparing $S_2(\mathbf{d})$ for $\mathbf{d} \parallel \mathbf{B}_{\text{local}}$ and $\mathbf{d} \perp \mathbf{B}_{\text{local}}$. $d_\perp \propto d_\parallel^{2/3}$ indicates locally anisotropic structures, while $d_\perp \propto d_\parallel$ indicates no structures.



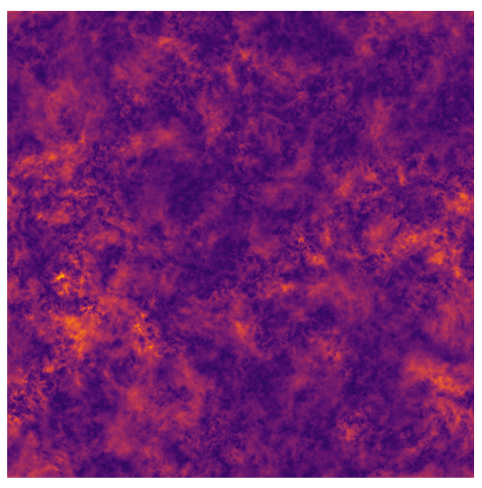
Scaling exponents ζ_p vs p. A linear relation indicates simple self-similar scaling, while sub-linear behavior indicates multifractal anomalous scaling.



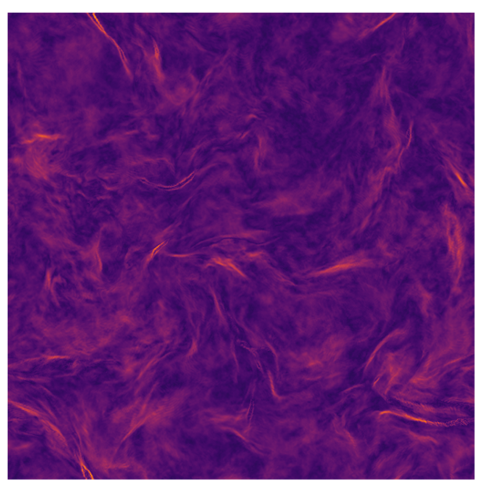
Running diffusion coefficient for charged particles with Lamor radius $r_L \sim 4\Delta x$. All cases eventually converge to diffusive behavior, but the MHD case shows an extended periode of superdiffusion.



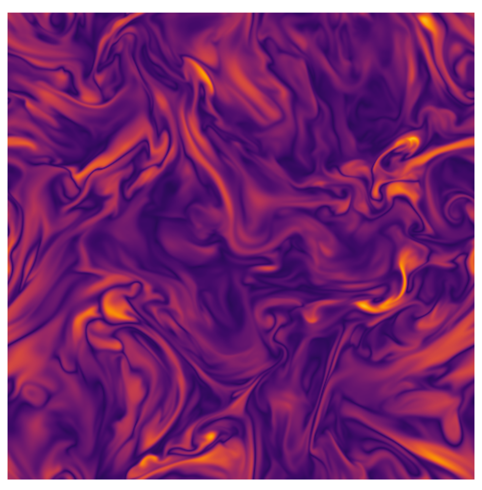
(i) Random phases



(ii) Intermittent



(iii) Lagrangian mapping



(iv) MHD

Slices through the field strength of the four considered models

Outlook

- Employ wavelets with scale-dependent anisotropy inspired by critical balance $k_\parallel \sim k_\perp^{2/3}$
- Implement the cascade algorithm in a grid-free fashion to achieve wider dynamical ranges
- Consider compressible turbulence

Acknowledgements & References

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