Network geometries influence responses of alluvial river systems to external forcing

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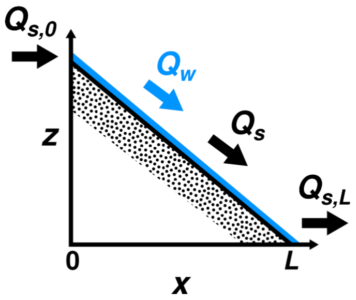
Motivation

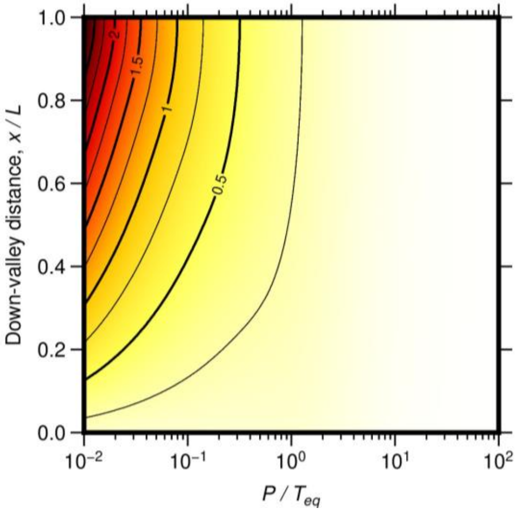
Alluvial river networks, source to sink processes, sedimentary & geomorphic archives.

Background

Modelling transport-limited river networks: set up & assumptions.

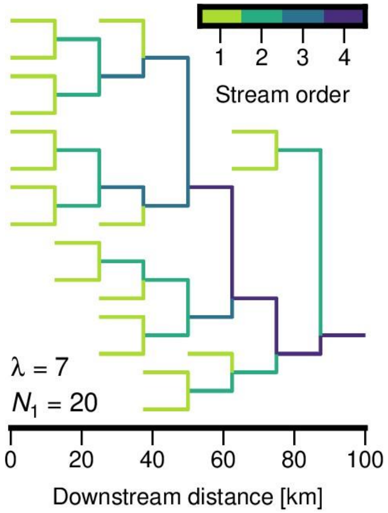


Linear case



Simple, (relatively) straightforward to characterize & understand.

Network case



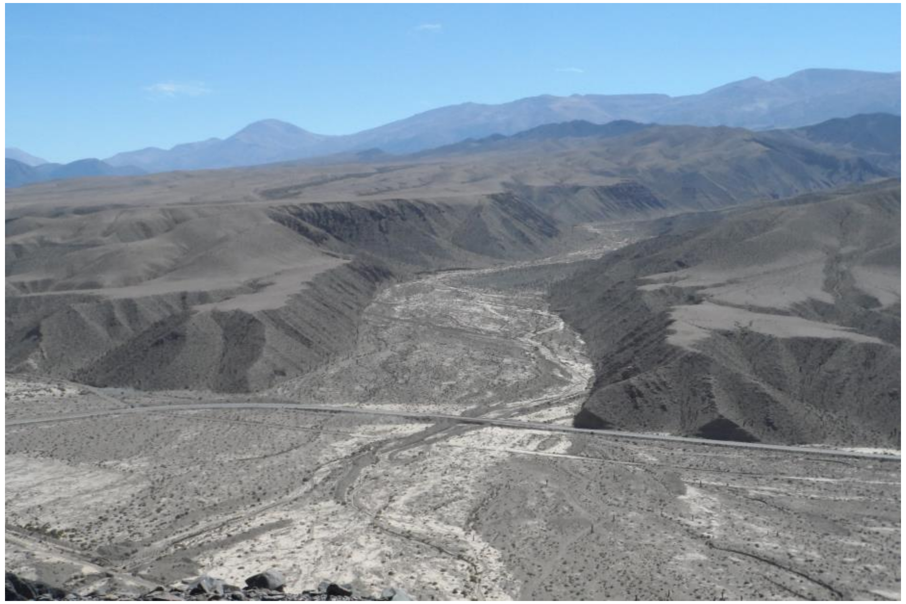
How is behaviour affected by tributary sediment & water delivery?

Alluvial rivers, source to sink processes, geomorphic and sedimentary archives

- Alluvial river transport sediment from upstream sources to downstream sinks
- Sediment transport sensitive to climatic and tectonic forcing
- Fluvial landforms and sedimentary sequences may record information about past climatic and tectonic conditions
- We want to interpret these records in a quantitative way

Toro Basin, NW Argentina. A sequence of cut-and-fill terraces records 100 kyr eccentricity-driven climate cyclicity (Tofelde et al., 2019). *Photo: Courtesy of S. Tofelde*

Network effects

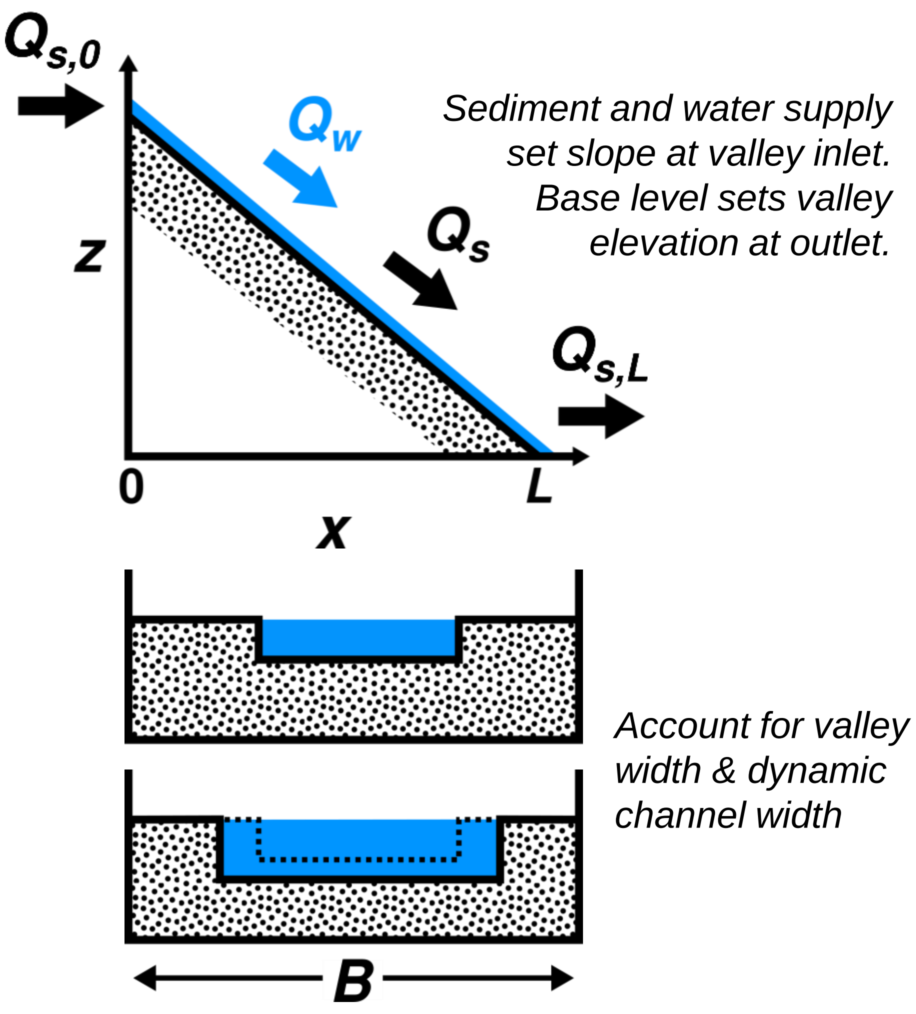


Toro Basin, NW Argentina. Terrace surfaces are continuous along trunk and tributary streams. Photograph courtesy of S. Tofelde.

- Intuition about these systems mostly based on theoretical studies with simplified geometry (e.g. Paola et al., 1992)
- In real catchments, sediment and water accumulate at discrete intervals downstream
- Previous studies highlight importance of system response time, often defined in terms of system length
- ‘Length’ of a river network is not clearly defined

- Key questions:
  - What complications are introduced by network configurations?
  - Can we apply predictions from simplified, one-dimensional models to real-world networks?

Modelling transport-limited gravel-bed rivers (Wickert & Schildgen, 2019)



- Conservation of mass

$$\frac{\partial z}{\partial t} = -\frac{1}{B\left(1-\lambda_p\right)}\frac{\partial Q_s}{\partial x} + U$$

- Sediment transport

$$Q_s = -\frac{k_{Q_s} I Q_w}{\mathbb{S}^{7/6}}\frac{\partial z}{\partial x}\left|\frac{\partial z}{\partial x}\right|^{1/6}$$

- Non-linear diffusion

$$\frac{\partial z}{\partial t} = \frac{k_{Q_s} I Q_w}{\mathbb{S}^{7/6} B\left(1-\lambda_p\right)}\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\left|\frac{\partial z}{\partial x}\right|^{1/6}\right) + U$$

Perturbation analysis: A linearised diffusion equation

- Assume signal is composed of small perturbations about some mean

$$z(x,t) = \bar{z}(x) + \delta z(x,t)$$

- Dropping small terms leads to linear diffusion equation describing perturbation’s evolution

$$\frac{\partial\ \delta z}{\partial t} = \kappa \frac{\partial^2 \delta z}{\partial x^2}$$

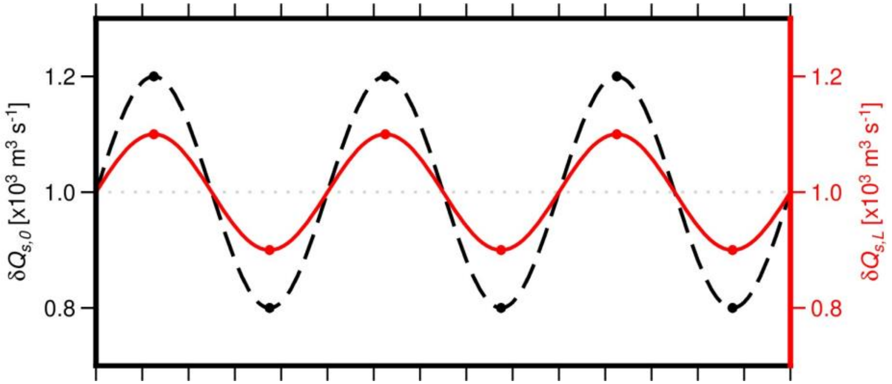
- Define system diffusivity

$$\kappa = \frac{7}{6} \frac{k_{Q_s} I \bar{Q}_w}{\mathbb{S}^{7/6} B \left(1 - \lambda_p\right)} \left| \frac{\partial \bar{z}}{\partial x} \right|^{1/6}$$

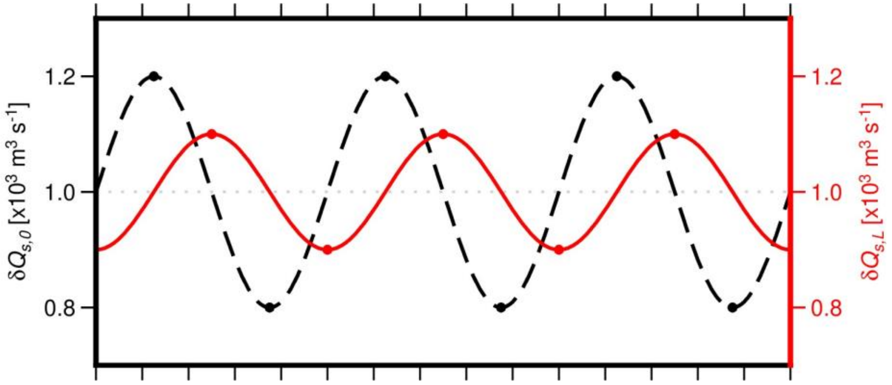
- Define system equilibration time

$$T_{eq} = {L^2}/{\kappa}$$

Perturbation analysis: Solving the linearised equation



G ('gain') describes amplitude of response perturbation relative to the imposed forcing. Above, the response (red) has half the amplitude of the forcing, hence G = 0.5.



φ (phase shift, 'lag') describes timing of response perturbation relative to the imposed forcing. Above, the response (red) is delayed by a quarter period relative to the forcing, hence φ/P = 0.25.

- Impose sinusoidal variation in sediment and water supply

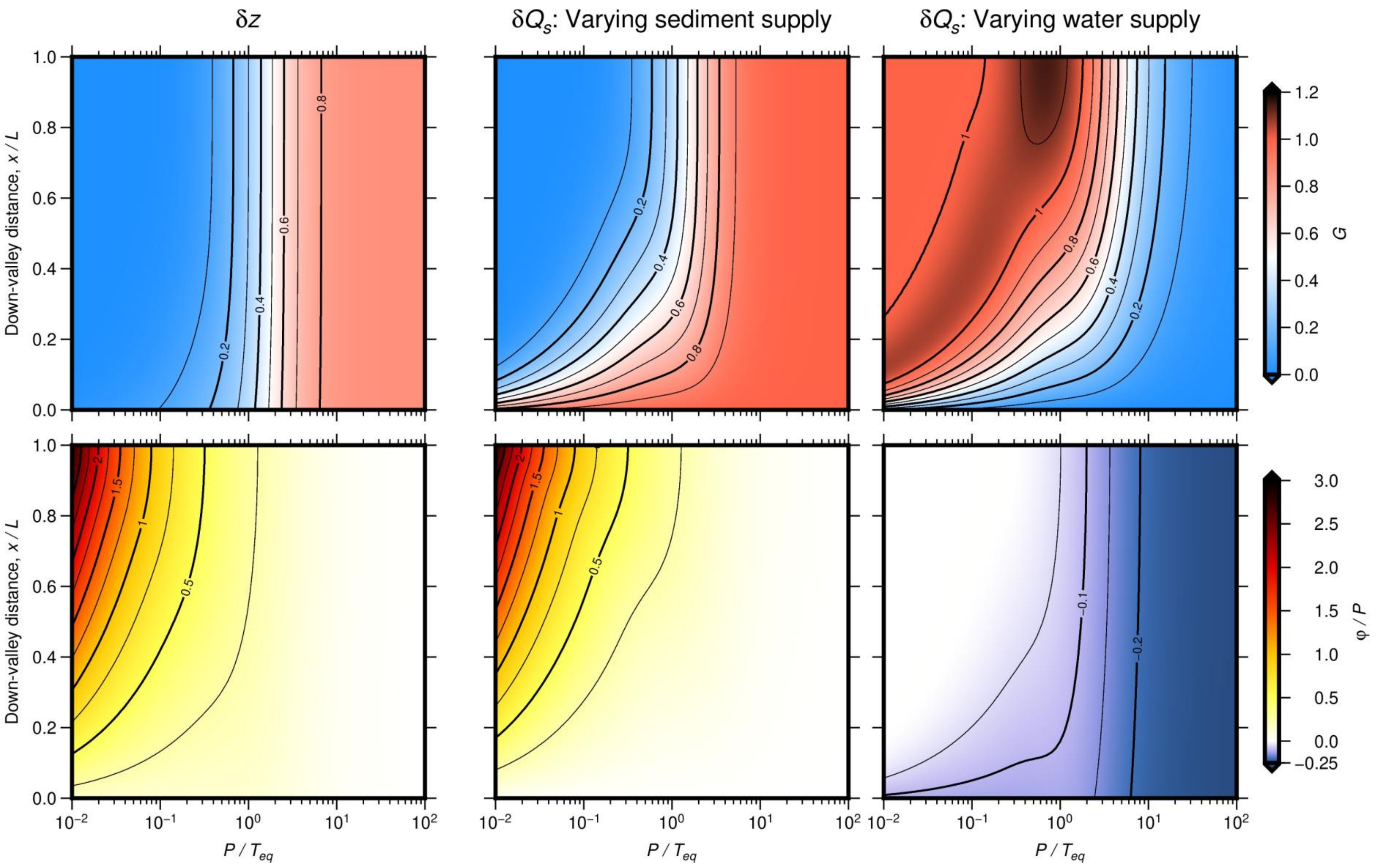
$$\delta Q_{s,0}(t) = A_{Q_{s,0}} \overline{Q_{s,0}} \sin\left(\frac{2\pi t}{P}\right)$$

$$\delta Q_w(t) = A_{Q_w} \overline{Q_w} \sin\left(\frac{2\pi t}{P}\right)$$

- Obtain solutions for perturbation to valley-floor elevation and sediment discharge

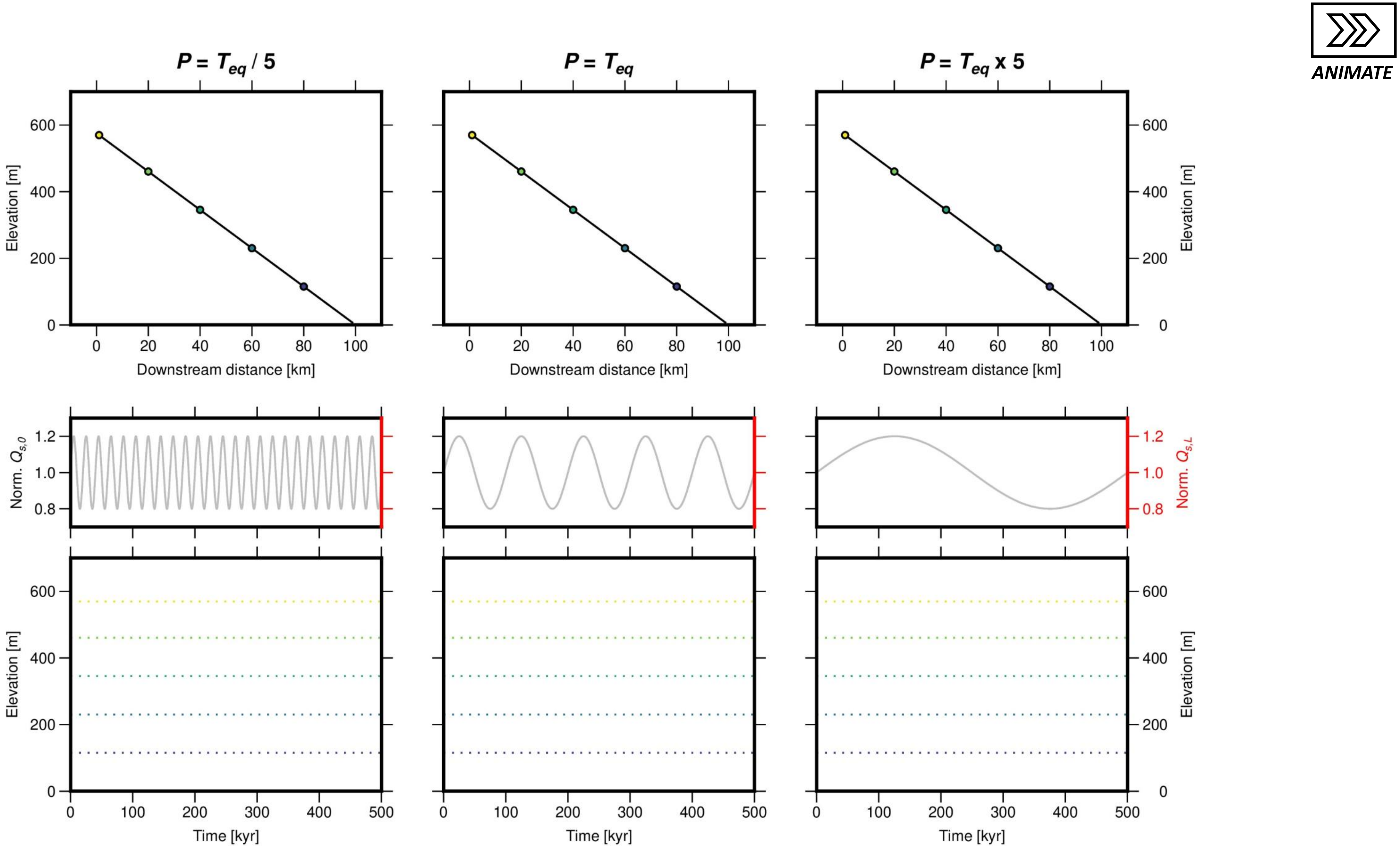
$$\delta z(x,t) = (A_{Q_s} - A_{Q_w})\ \bar{z}\ G_z\ \sin\left(\frac{2\pi}{P}(t - \varphi_z)\right)$$

$$\delta Q_s(x,t) = (A_{Q_s} - A_{Q_w})\ \overline{Q_s}\ G_{Q_s}\ \sin\left(\frac{2\pi}{P}(t - \varphi_{Q_s})\right)$$



Key controls on G and φ are forcing period (relative to system equilibration time) and position downstream. Variation in sediment discharge also depends on whether sediment or water supply is varied.

These patterns are highlighted in example simulations on the following slide.



ANIMATE
P = T_eq / 5
P = T_eq
P = T_eq x 5
Elevation [m]
Downstream distance [km]
Norm. Q_s,0
Norm. Q_s,L
Time [kyr]

Exploring effects of network geometry

- Key questions:
  - What complications are introduced by network configurations?
  - Can we apply predictions from simplified, one-dimensional models to real-world networks?

- Approach:
  - Generate random network topologies
  - Fix trunk length, mean sediment and water discharges
  - Measure gain and lag numerically
  - Compare to predictions from simplifies models

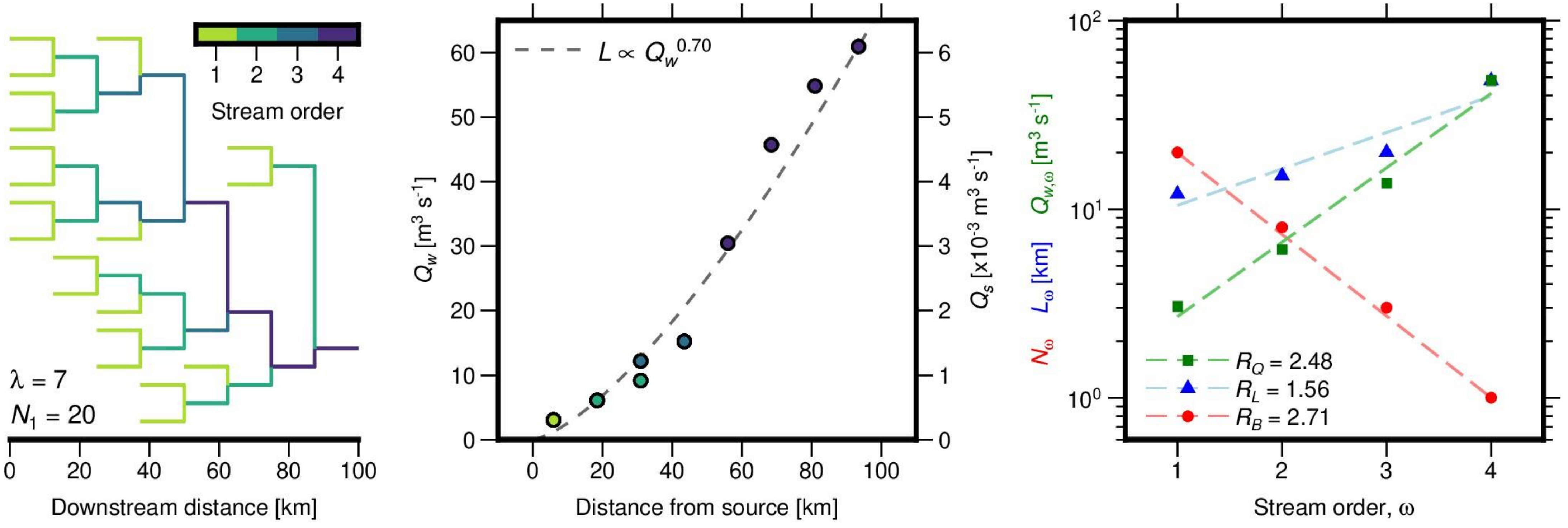
Example network set up

- Repeat for large number of network configurations:

Run 1: 200x with 20 source streams

Run 2: 200x with 40 source streams

Run 3: 200x with 2-60 source streams

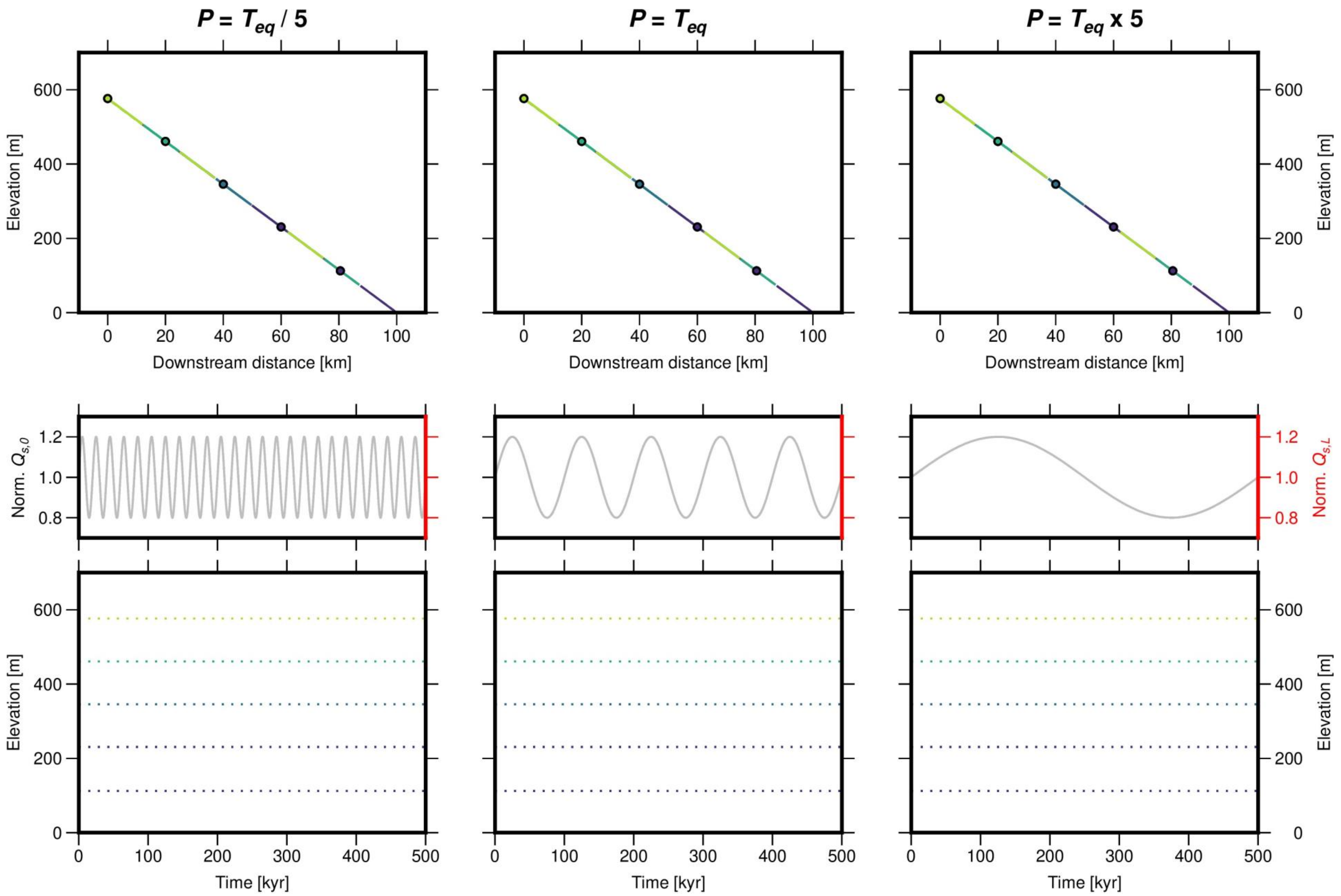


Random network topologies constructed using algorithm described by Shreve (1974).

Sediment and water supplied at source streams with means across network held constant.

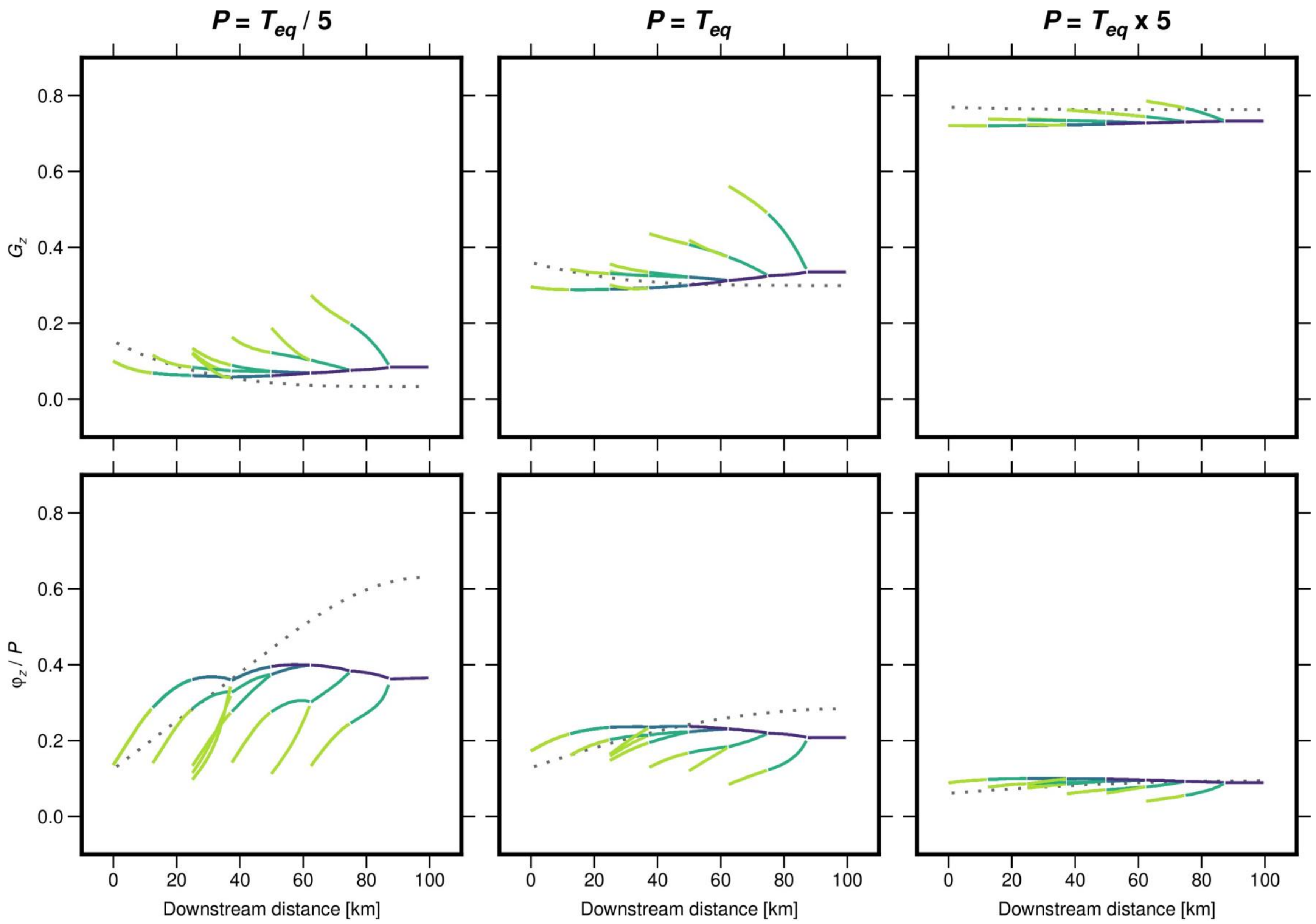
Network geometries described by metrics such as the Hack exponent and Horton's ratios.

Sinusoidal variation in sediment supply imposed at each source stream. Gain and lag measured numerically.



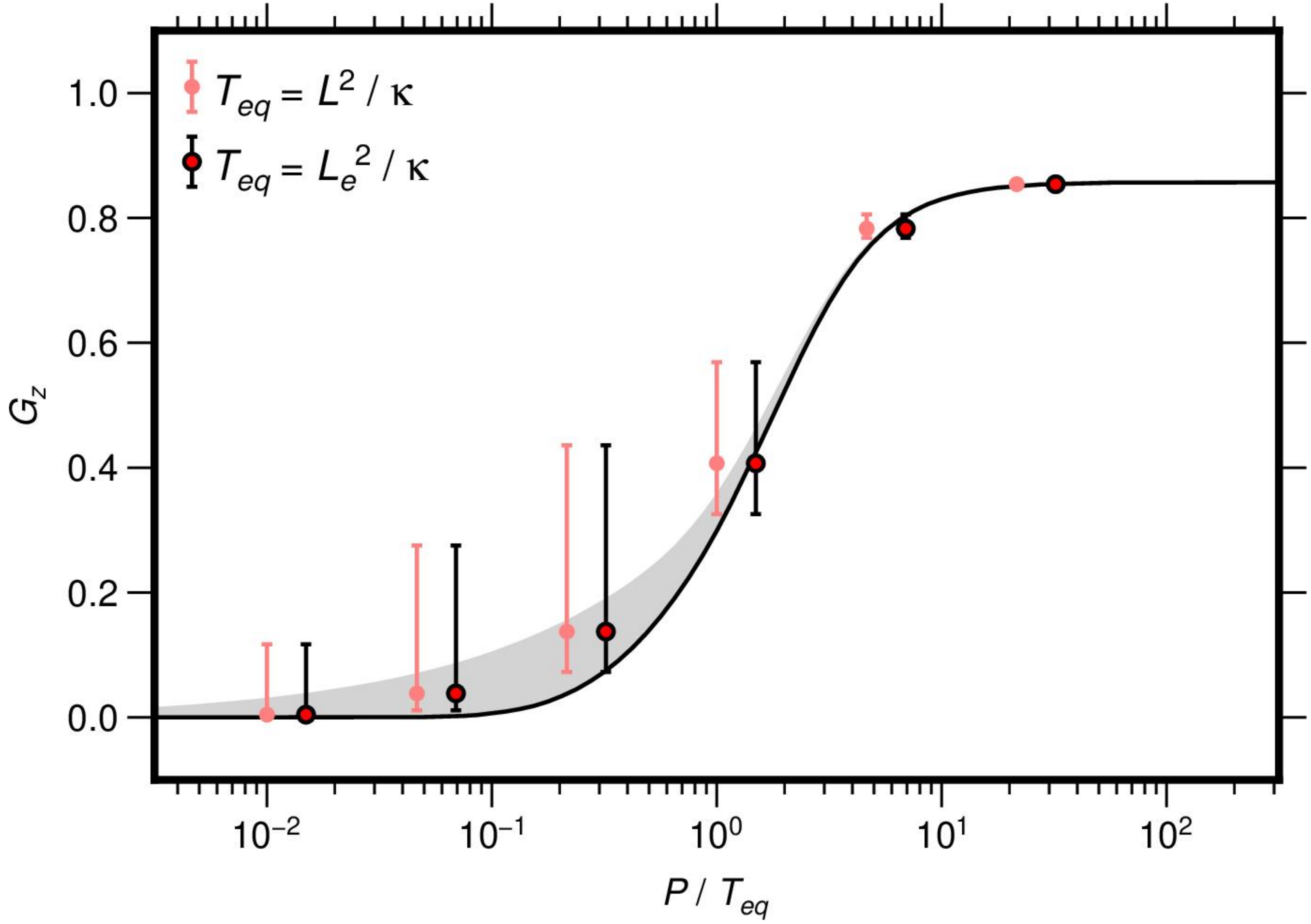
$P = T_{eq} / 5$
$P = T_{eq}$
$P = T_{eq}$ x 5
Elevation [m]
Downstream distance [km]
Norm. $Q_{s,0}$
Norm. $Q_{s,L}$
Time [kyr]
0
100
200
300
400
500
600
20
40
60
80
0.8
1.0
1.2

ANIMATE

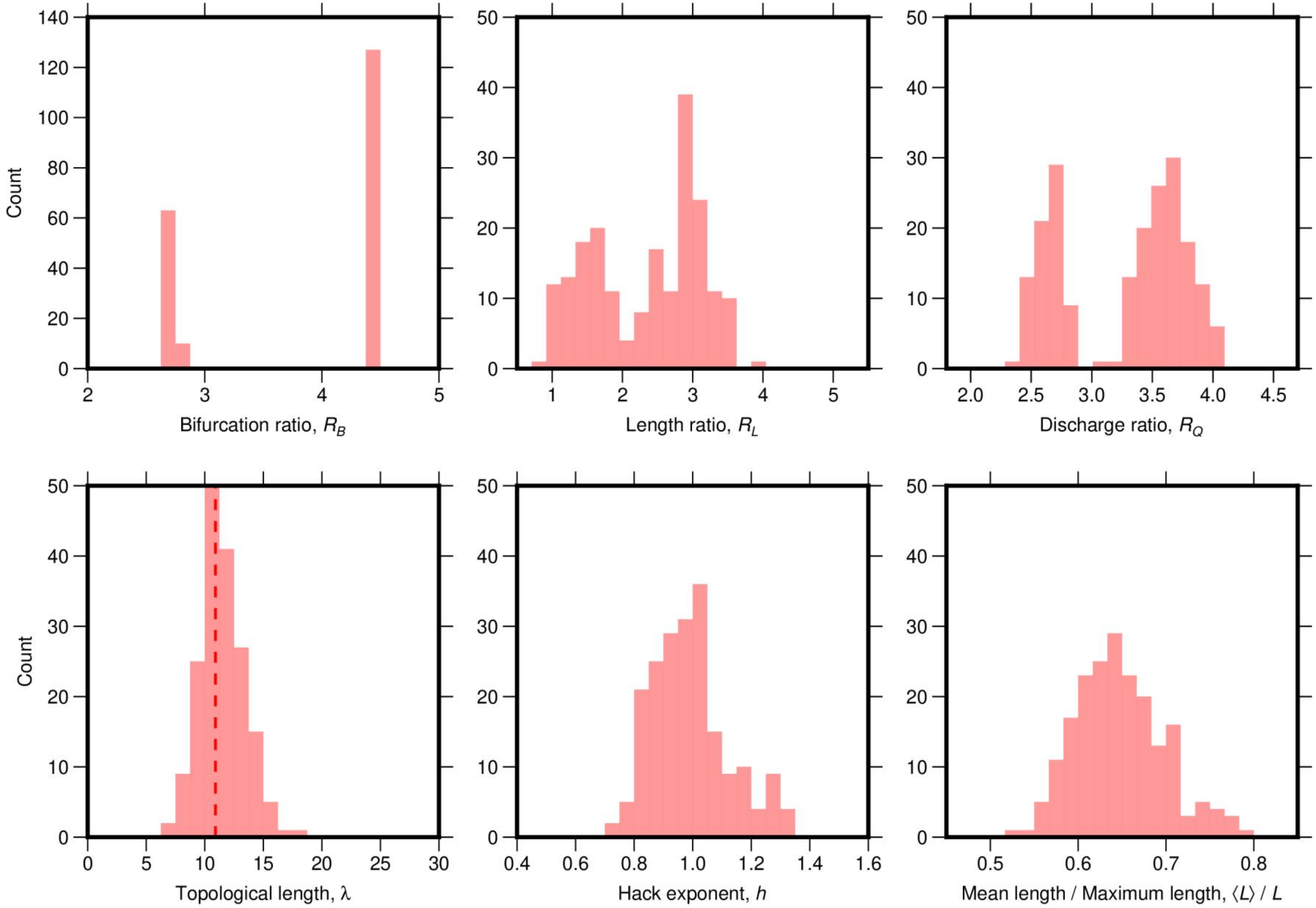


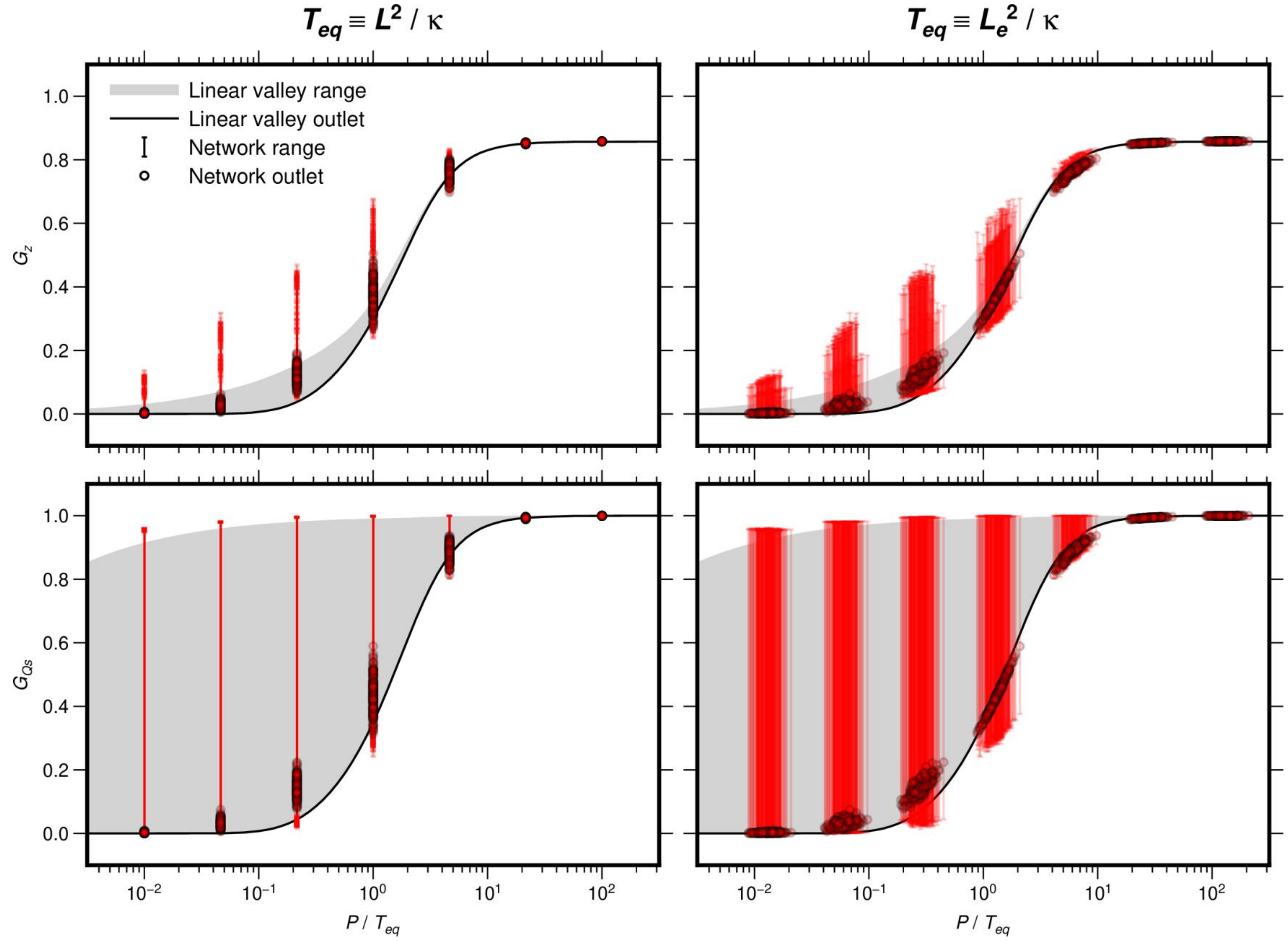
Broad patterns in gain and lag predicted for the linear case also arise for network case. Lower order tributaries characterised by higher gain and rapid downstream signal propagation. Signal can propagate upstream along trunk streams.

Define empirically a network's 'effective length', L_e, as that which minimises the difference between G_z at the network outlet and G_z at the outlet of a simple linear valley with the same total length and mean diffusivity.

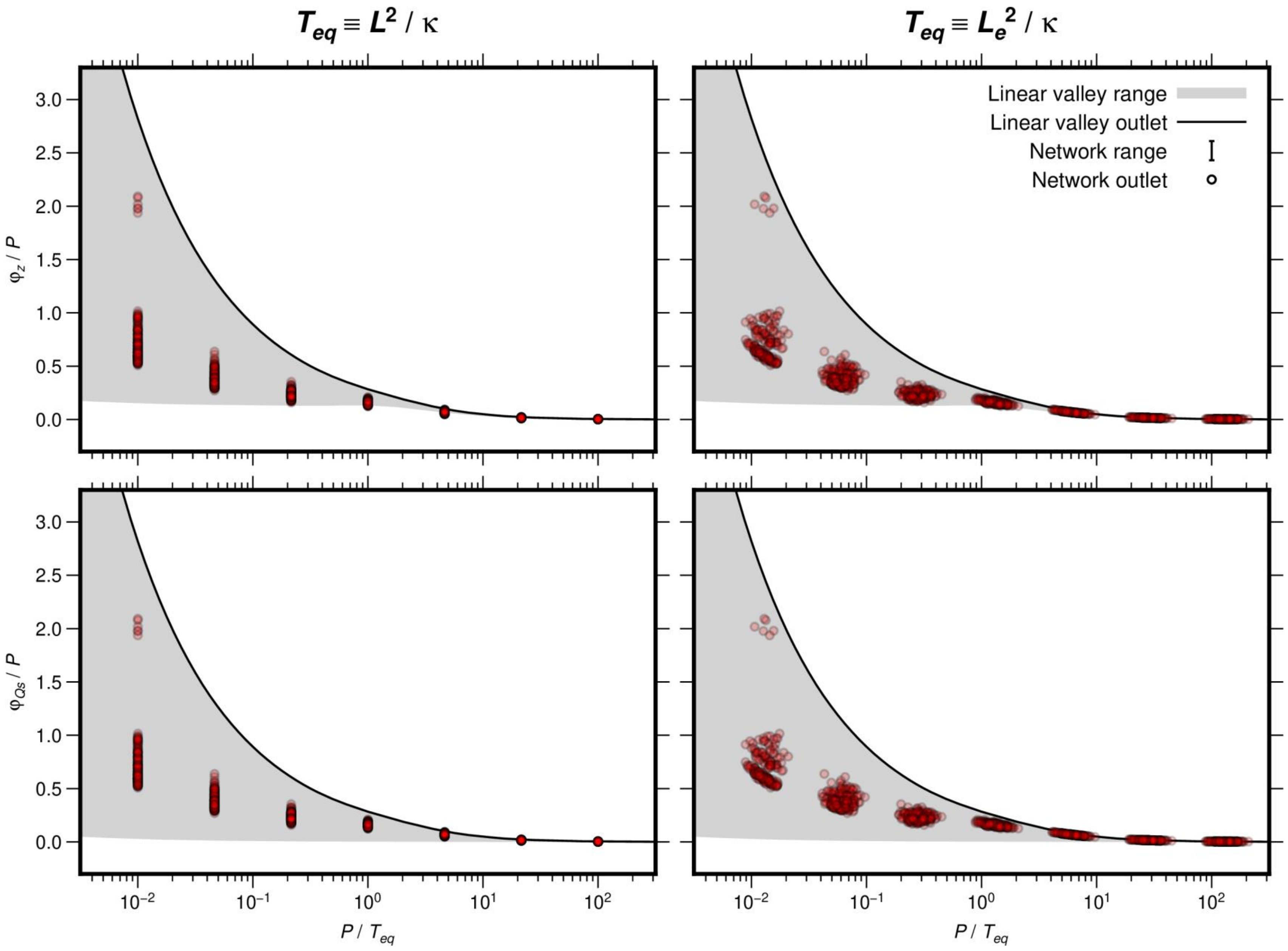


Properties of 200 randomly generated networks with 20 source streams.



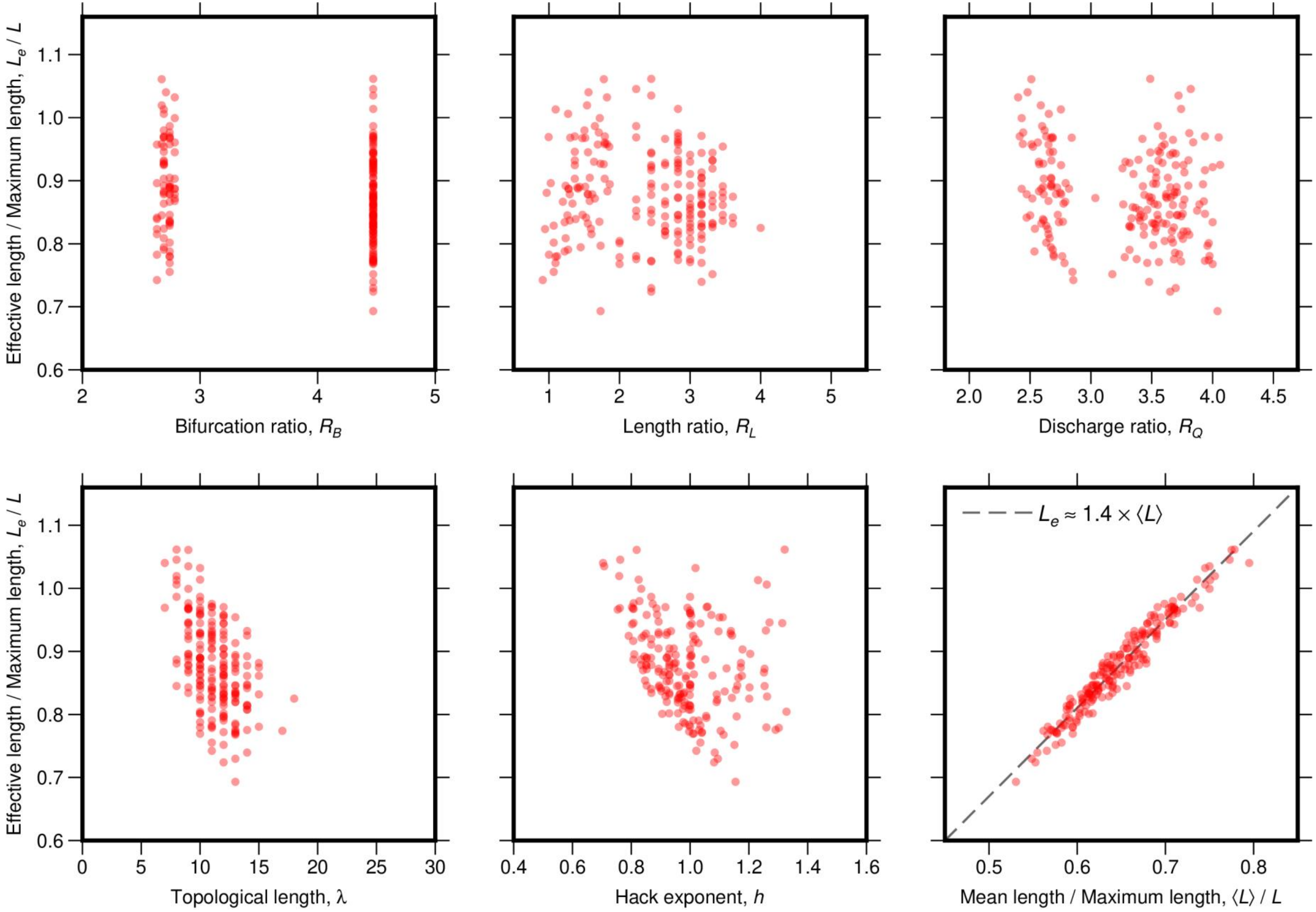


Variation due to network geometry captured by empirically determined 'effective length'.

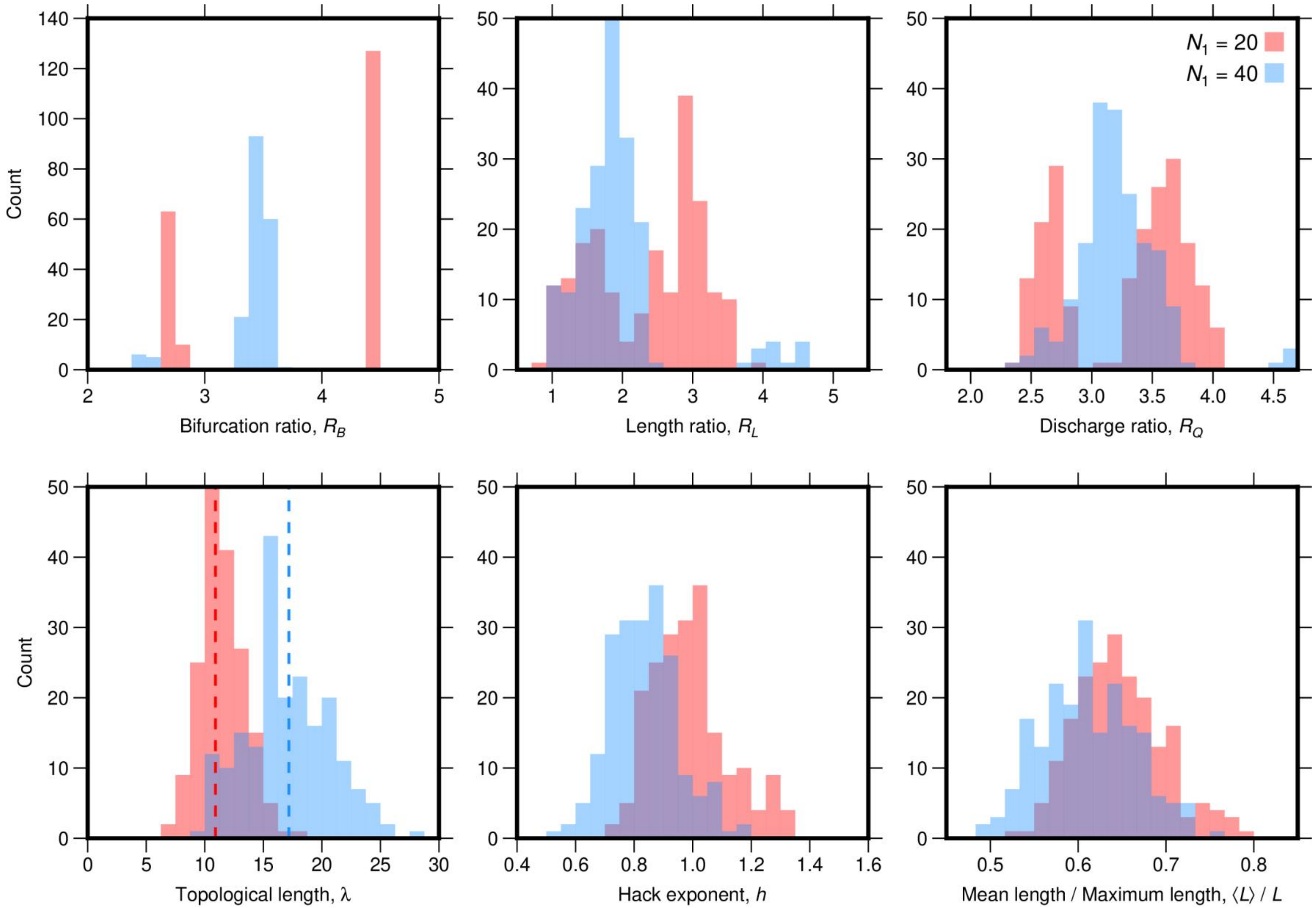


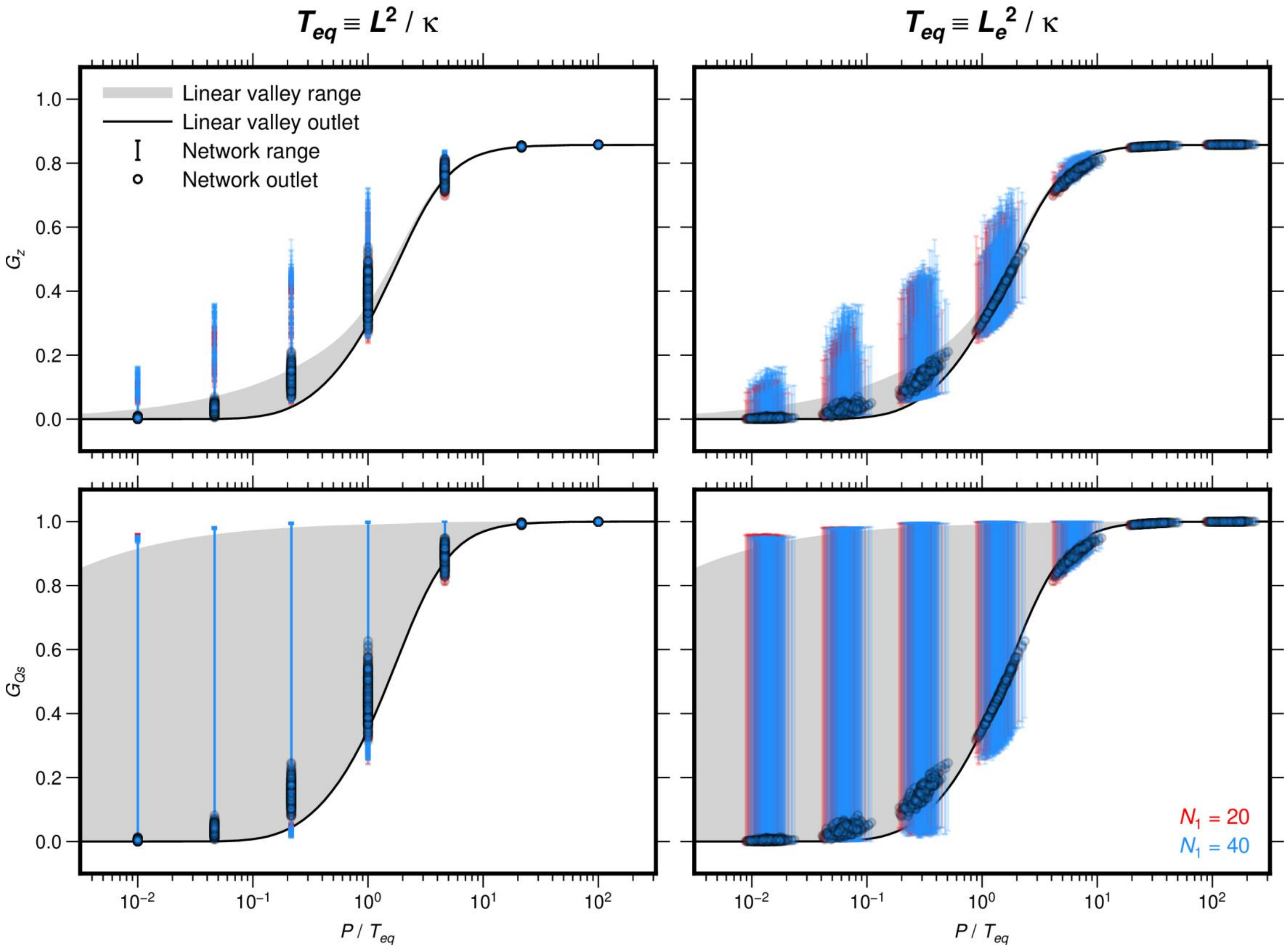
Effective length also captures some variation in lag, but scatter remains. Lag reduced relative to linear case.

Weak correlation between effective length and most network metrics – except mean length.



Properties of 200 randomly generated networks with 20 & 40 source streams.





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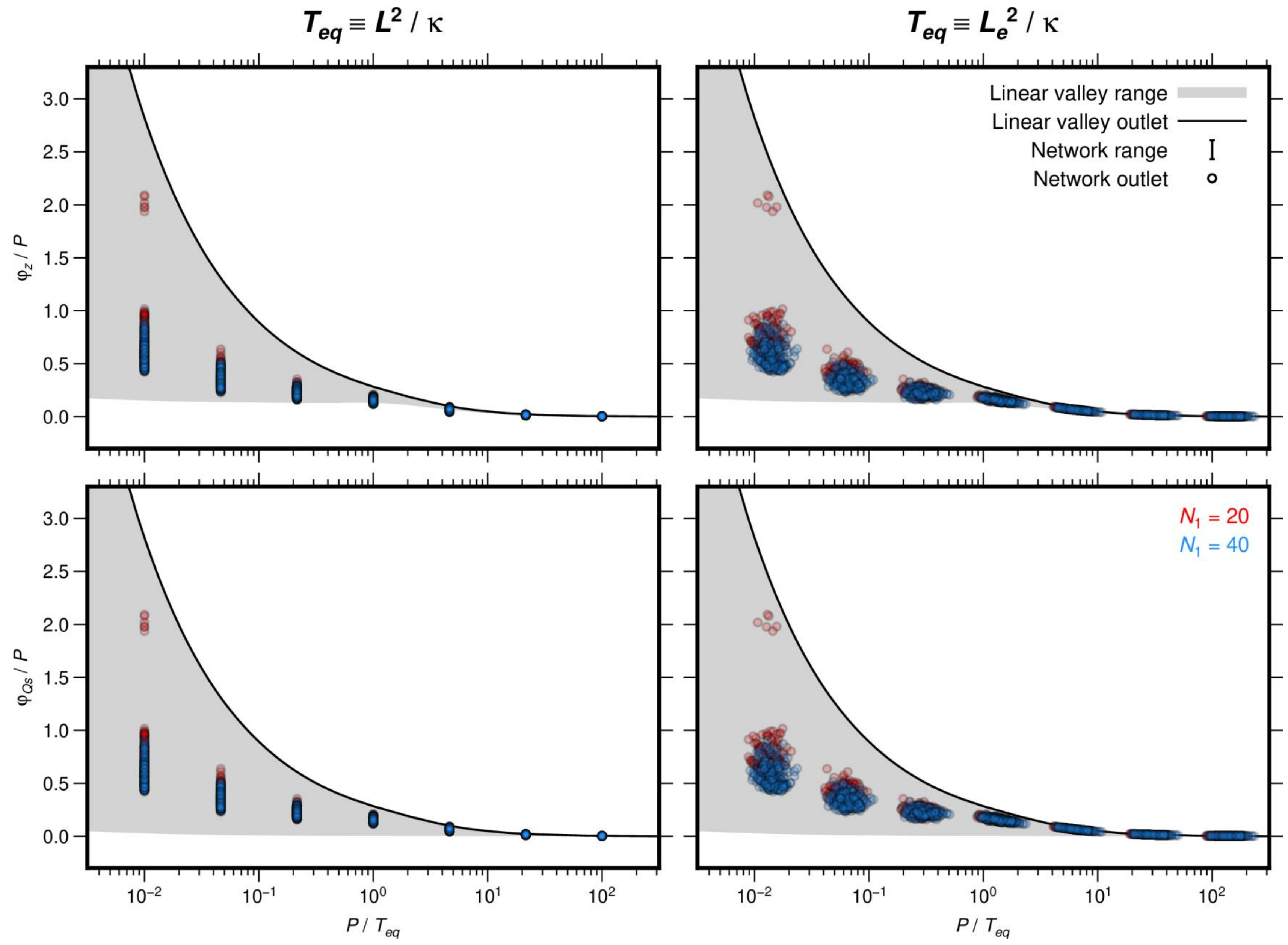

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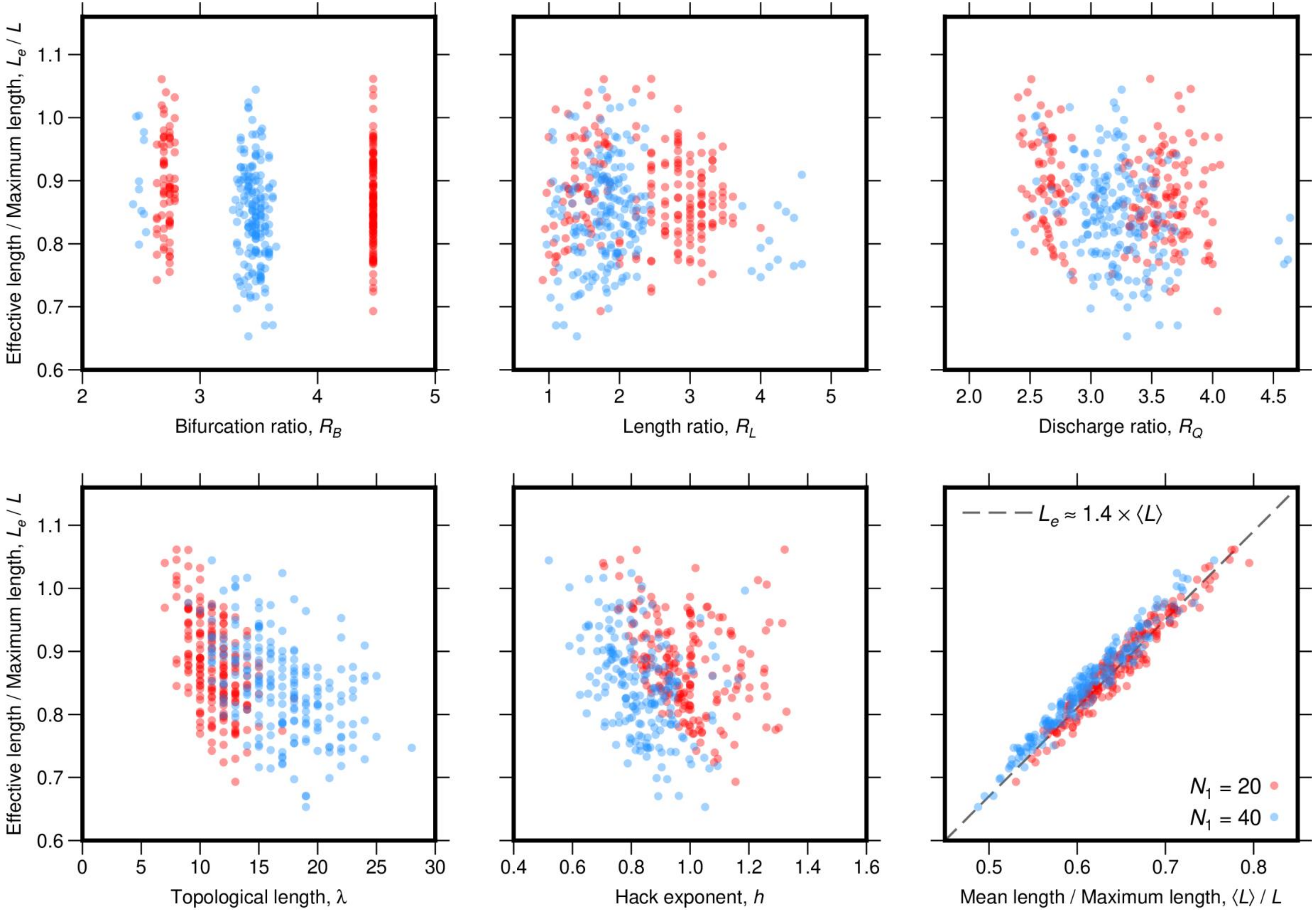

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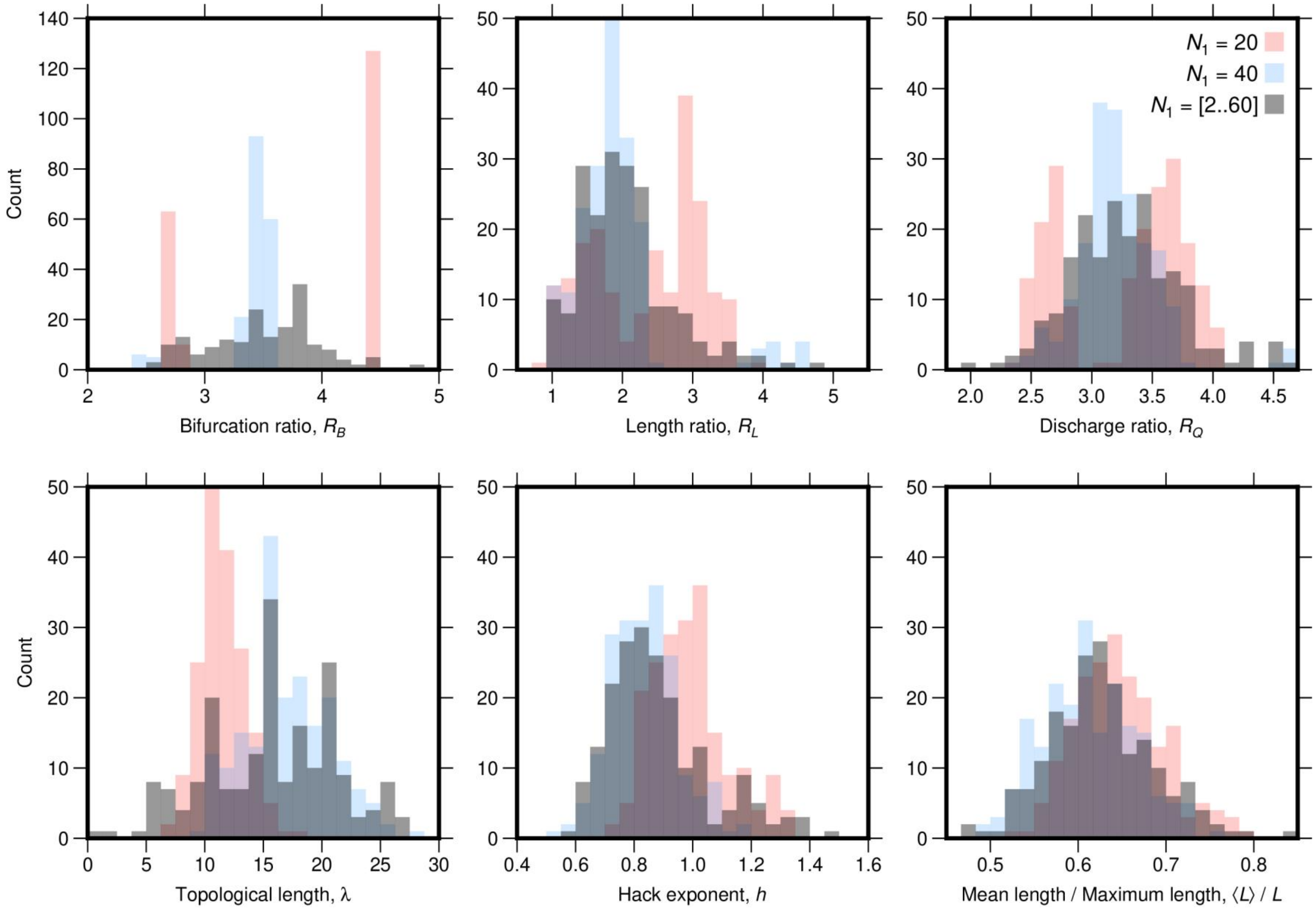
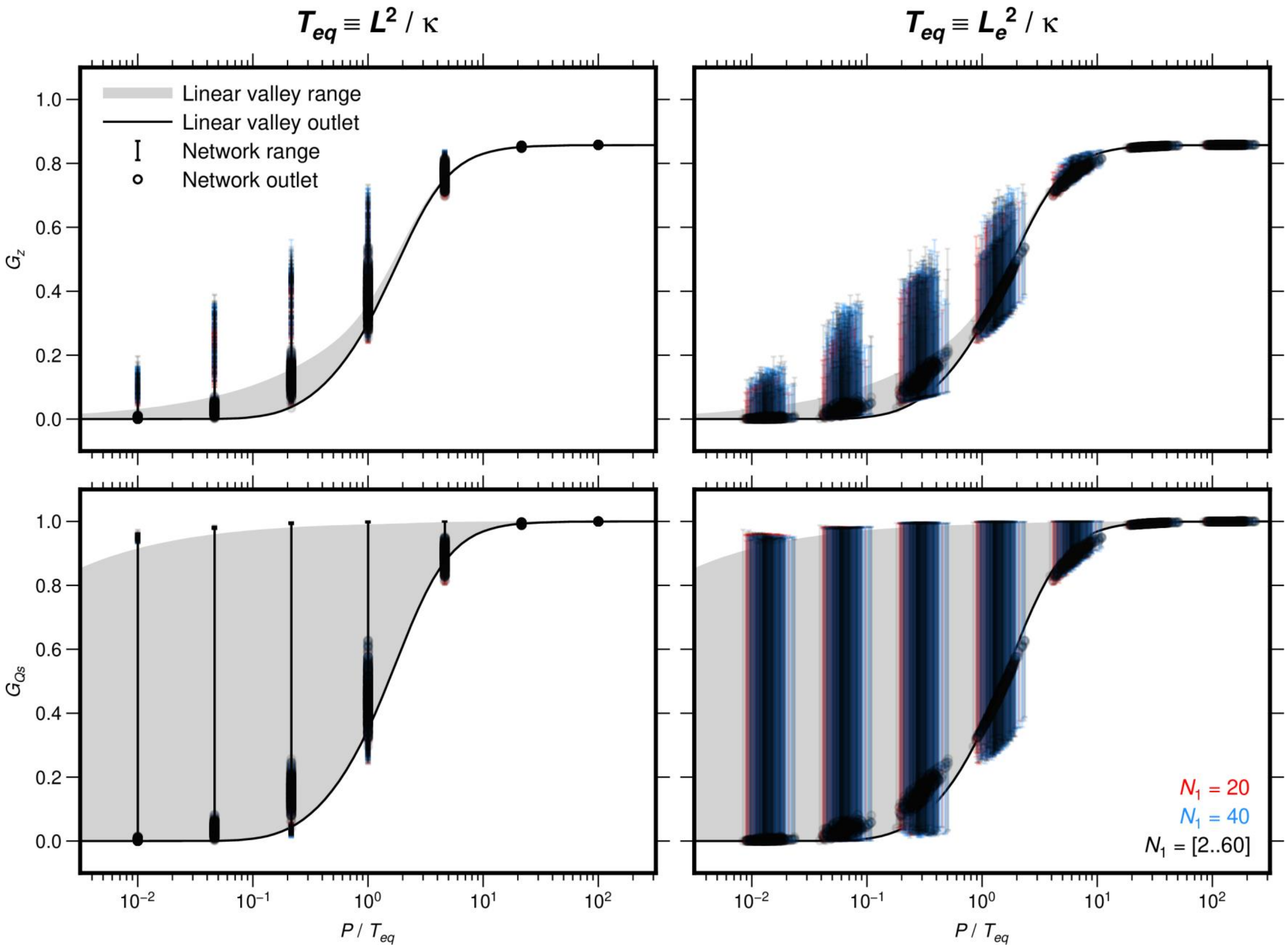
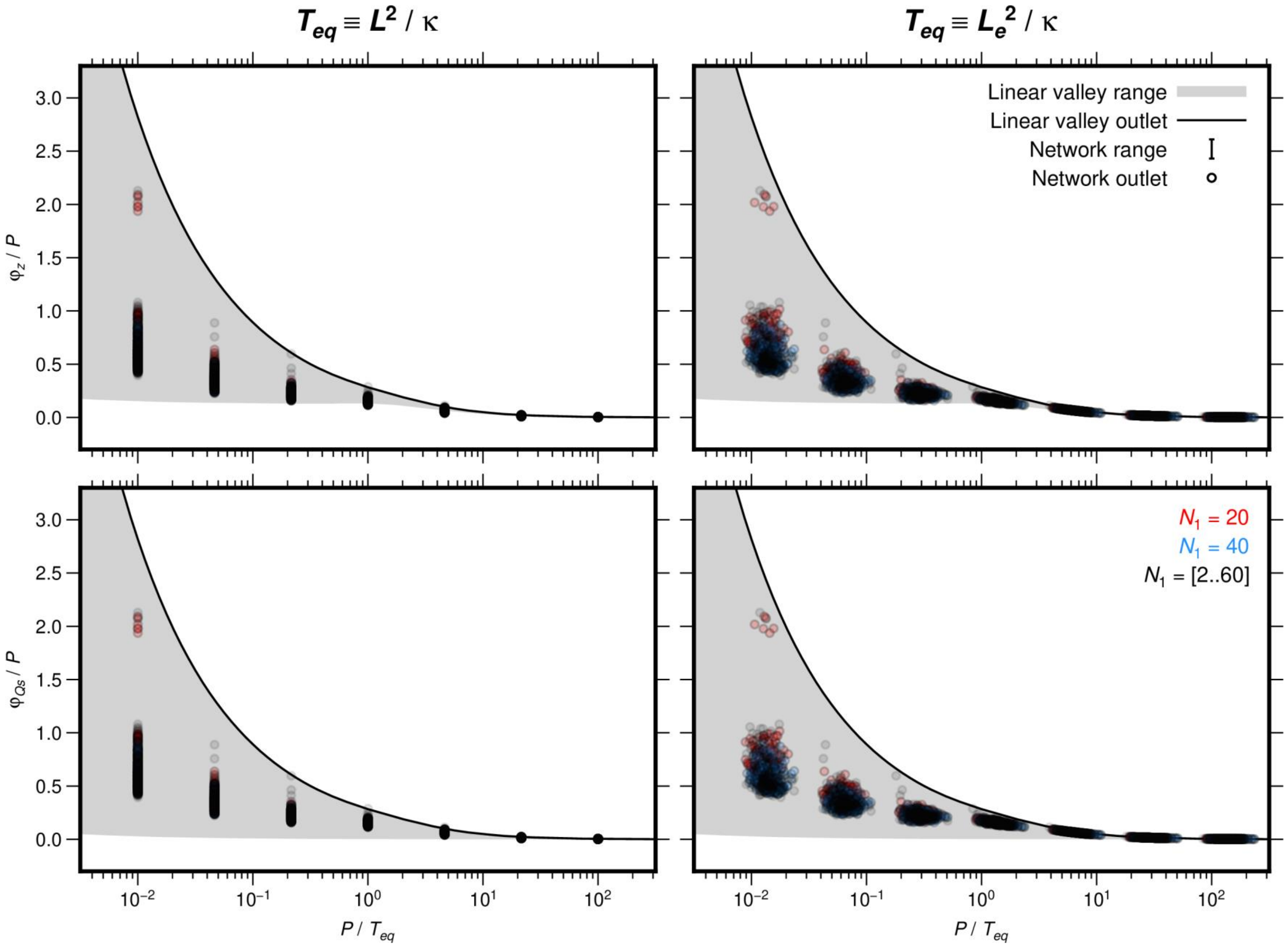




Variation due to network geometry captured by empirically determined ‘effective length’.



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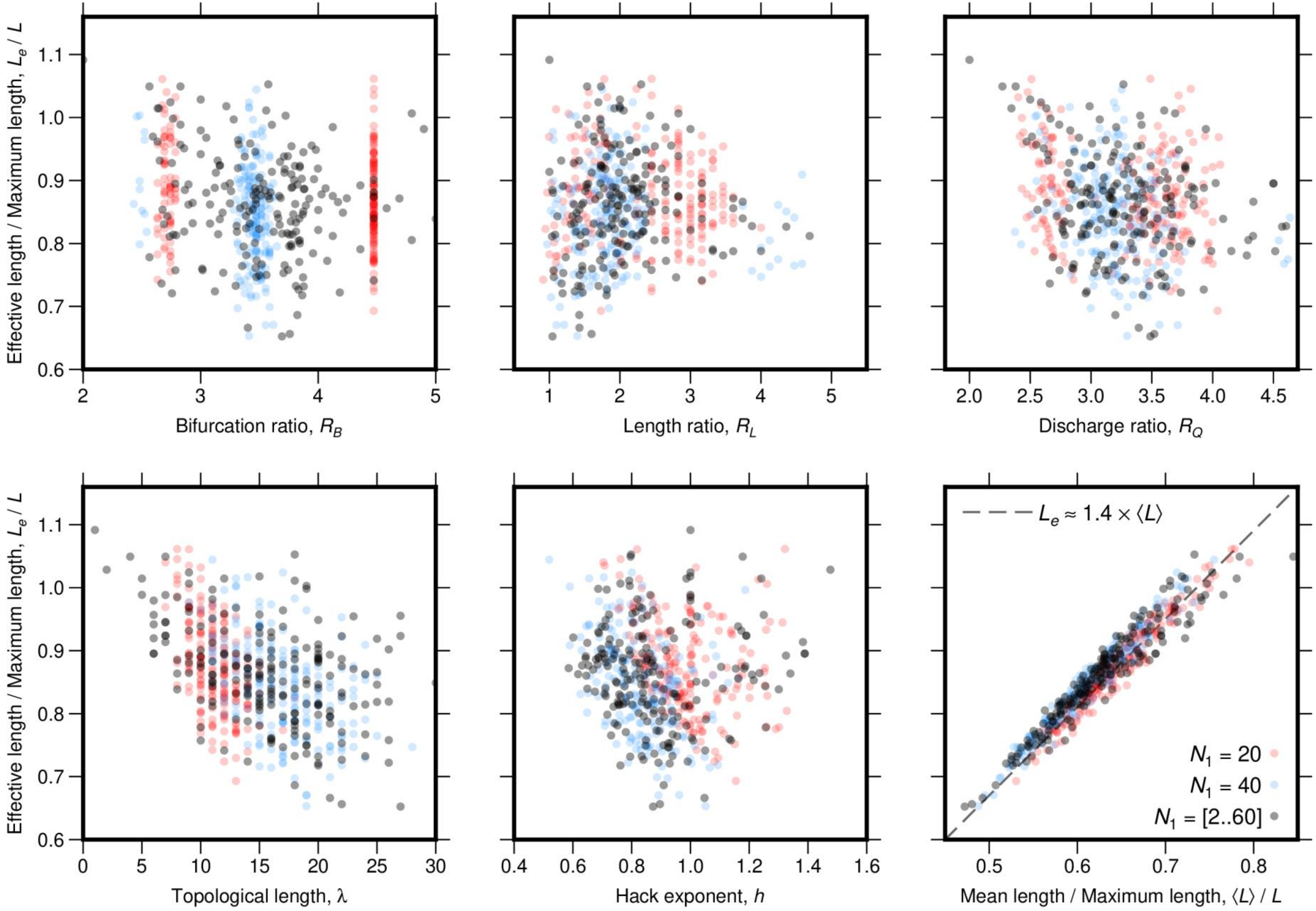
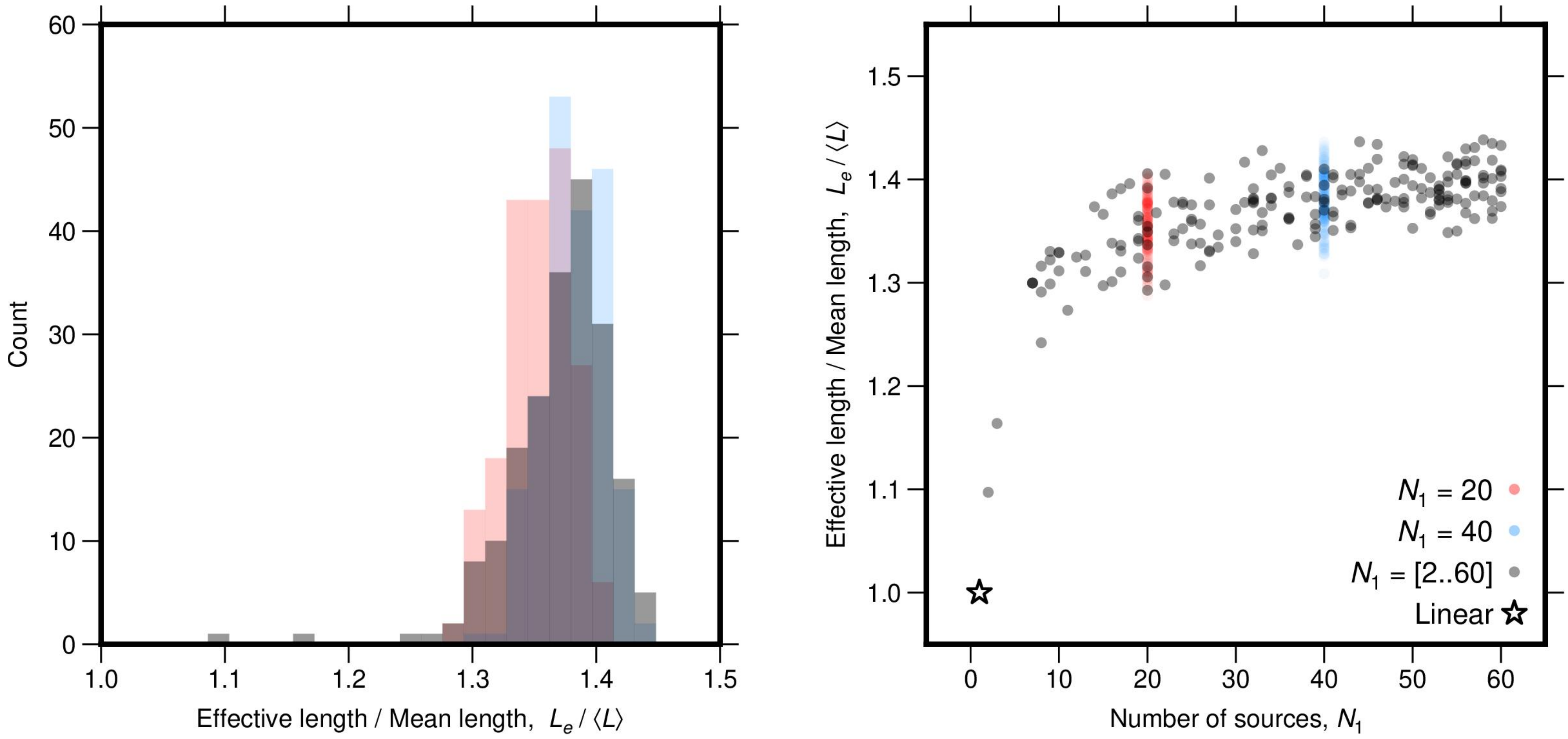


Average ratio of effective to mean length approaches 1.4 for networks with ≳ 40 source streams.



Key points

- Alluvial river networks respond to external forcing in broadly similar ways to simple one-dimensional valleys
- Complications can arise around tributary junctions, depending on precise network geometries
- Low-order streams behave differently to trunk streams
- Simplified models can capture network behaviour – if appropriate lengthscale is chosen
- This ‘effective length’ is a constant factor of the mean distance from source to outlet