Propagation And Dissipation Of Lee Wave Energy In A Single Column Model

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Introduction

A thorough understanding of the ocean circulation is one of the major challenges in oceanography. Climate change is rapidly changing the conditions and forcings one the global ocean. It is important to understand how these changes will affect the global circulation. For this end it is vital to understand the mechanisms driving the global ocean circulation.
The global ocean conveyor belt is a massive network of ocean currents spanning the whole globe, from surface currents to abyssal currents. These currents carry an enormous amount of kinetic energy. There are two main sources for this energy:

- Wind stress (e.g. in the Southern Ocean)
- Breaking of internal waves in the interior of the ocean

The role of wind stress is well understood, e.g. see Wunsch (1998), and supplies the majority of the energy. The other source is the breaking of internal waves. The breaking of these waves induces mixing which increases the diapycnal diffusivity. Essentially, the wave breaking increases the potential energy in the ocean. Observing internal waves is difficult, thus increasing the importance of theoretical and numerical models to understand their effect. In this study a particular type of internal waves, called Lee waves, are considered and a model for the propagation of their energy is developed and tested in a single column ocean model.

What are Lee waves

Roughly speaking there are two types of internal waves that are generated by flow over rough topography. Internal tides are generated by tidal flows and Lee waves are generated by geostrophic flows. As the currents flow over the uneven topography they are displaced upwards, this causes a wave to propagate upwards. When the wave reaches the surface it is reflected downwards. Since the topography is stationary, the resulting wave will also be stationary. Lee waves follow the same equations as other internal waves. Linear wave theory was used by Bell (1975) to calculate the energy flux due to lee waves. Using Bells theory it was estimated that Lee waves contribute around 0.2TW to the global overturning circulation, e.g. (Nikurashin and Ferrari 2011). However, estimates like that fail to account for the propagation of the waves in the watercolumn. Do the waves break somewhere? How do they interact with the mean flow? These questions are left unanswered.

Lee Wave Energy Model

In order to better understand the propagation of lee waves a model based on the radiative transfer equation was developed by Eden, Olbers, and Eriksen (2021). The radiative transfer equation is a general equation that describes the evolution and propagation of a wave spectrum. In the form presented here it depends on space, time and wavenumbers. The left-hand side represents the propagation and refraction of the wave while the right-hand side represents the sources and sinks. In order to reduce the complexity, the radiative transfer equation is integrated over the whole wavenumber space.

$$\partial_t\mathcal{E} + \partial_z(\dot{z}\mathcal{E}) + \partial_m(\dot{m}\mathcal{E}) = \dot{\omega}\mathcal{E}/\omega + S \qquad (1)$$

$$\downarrow$$

$$\partial_t E^+_{lee} + \underbrace{\partial_z(c_{lee}E^+_{lee})}_{\text{propagation}} = \underbrace{\tau^{-1}_{lee}E^+_{lee}}_{\text{mean flow interaction}} - \underbrace{\frac{\tau_s^{-1}}{2}\Delta E_{lee}}_{\text{damping}} - \underbrace{\sigma_c^-\tau_{cl}^{-1}E^+_{lee}}_{\text{critical layer dissipation}} - \underbrace{\alpha_w E_{gm}E_{lee}}_{\text{wave-wave interaction}} \qquad (2)$$

This energy equation can be introduced into ocean models to study how lee waves propagate and interact with the model.

Critical Layer Parameterization

For this study a critical layer parameterization was added to the model developed by Eden, Olbers, and Eriksen. Critical layers occur when the wave propagates into a region with a different current velocity. Due to the change in velocity the wave is shifted to larger wavenumers, i.e. smaller wavlengths. If this shift is strong enough the wavelength becomes too small and wave ends up breaking. A similar but weaker effect can be found when considering a varying stratification. Both effects are included here. In order to capture this behaviour with the energy equation, the vertical refraction term $\partial_m(\dot{m}\mathcal{E})$ is integrated over the wavenumbers. During the integration over m a cutoff wavenumber m_l is introduced. If there is energy flux through this limit, it is assumed that the energy is lost because the waves break.

$$\int_0^{2\pi}\int_{f/U}^{N/U}\int_{-m_l}^{m_l} \partial_m(\dot{m}\mathcal{E})dmdkd\phi = F_0 \max\left(0, \mp sign(\xi)\right)|\xi|E^{\pm} = \sigma_c^-\tau_{cl}^{-1}E^+_{lee} \qquad (3)$$

Results

The complete energy equation, including the boundary conditions (emission at the bottom, reflection at the surface) is now implemented in a single column ocean model using pyOM. The model was run in different configurations, testing the different effects from (2). The model included a very simple, constant velocity forcing.

Mean flow interaction

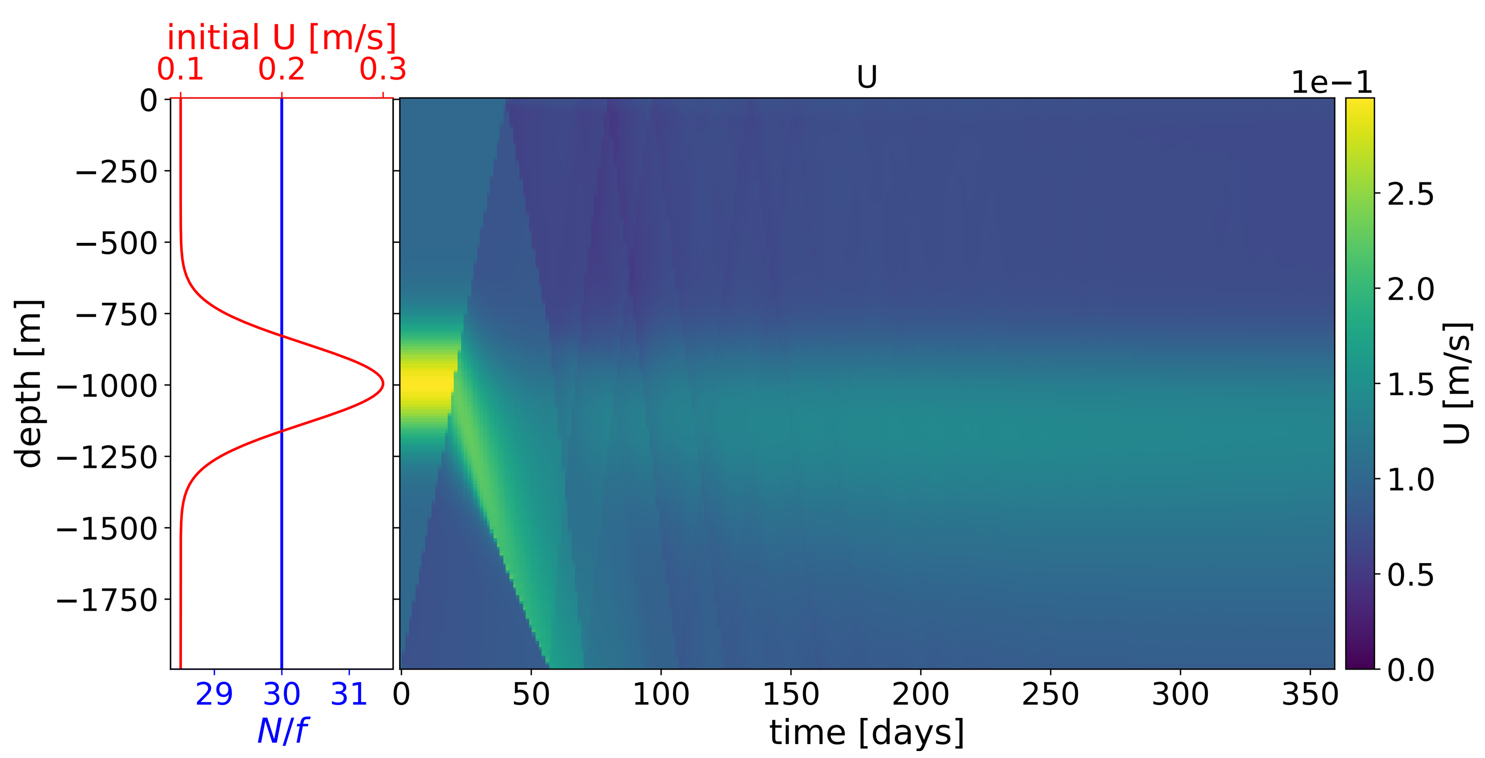

Figure 1. Evolution of a velocity jet

In figure 1 a downwards deflection and reduction of a velocity jet can be observed. This is a result of the interaction with the mean flow due to wave action conservation.

Critical Layer dissipation

The critical layer dissipation can be seen to be in agreement with the full non-integrated model. Furthermore it can be seen that the dissipation due to a varying stratification is at least one order of magnitude lower than the other critical layer dissipation to due velocity shear.
The amount of critical layer dissipation only depends on the strength of the velocity peak, not the steepness of the velocity shear.

Wave-Wave dissipation dominates

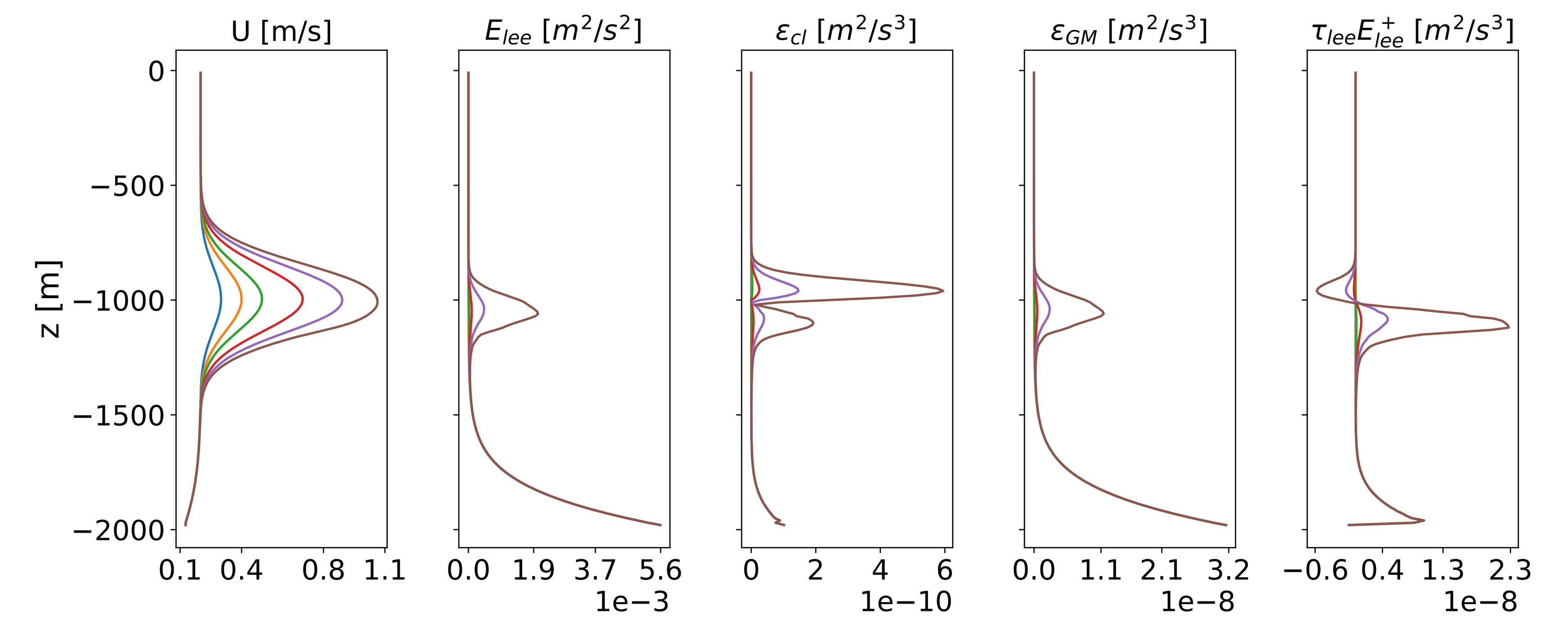

Figure 2. Final state of the model after spin-up, showing the different velocity profiles, wave Energy, critical layer dissipation, wave-wave dissipation and mean-flow interaction.

Figure 2 shows the final state of the model after equilibrium has been reached. All effects were included in this simulation and different velocity profiles were used. The dissipation due to interactions with the background wave-field dominates. Critical layer dissipation is much weaker because a lot of energy has dissipated before the critical layer is reached. Mean flow interaction only becomes relevant for strong velocity peaks.

Limitations

- the model assumes steady state, but it takes a while until the model reaches the steady state
- turning points are not yet included
- the parameterization for the wave-wave dissipation is taken from the GM-spectrum waves, perhaps changes need to be made for lee waves
- for velocities above 1m/s there are often numerical instabilities

Conclusions

- Values found are similar as in other models like Baker and Mashayek (2021) and ARGO float observations from Whalen, Talley, and J. MacKinnon (2012)
- Other observations e.g. by Waterhouse et al. (2014) and Waterman et al. (2014) measured lower values than this model
- the interaction with the background wave field dominates, critical layer dissipation is generally weaker
- dissipation due to changes in stratification is several orders of magnitude lower
- waves dissipate before they can reach the surface, except in shallower waters

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