Analysis of Magnetic Helicity Generation in MHD-shell Model

Ilyas Abushzada [1], Egor Yushkov [2] and Dmitry Sokoloff [2]

[1] Baku State University, Baku, Azerbaijan (ilyasabushzade@gmail.com)

[2] Lomonosov Moscow State University, Moscow, Russia

The small-scale magnetic dynamo describes the generation of magnetic field energy at scales comparable to or smaller than the correlated velocity field length. Small-scale generation does not require any specific conditions, such as the mirror asymmetry of the turbulent flow or the presence of differential rotation. However, the small-scale dynamo is a threshold phenomenon with respect to the magnetic Reynolds number. Magnetic energy generation begins at a sufficiently large Rm: depending on the correlation function with a characteristic dimensionless correlation length, the threshold estimates are between Rm=10 and Rm=100. These thresholds were first obtained for Kazantsev-Kraichnan type models, which are not applicable to Kolmogorov turbulence. However, Kolmogorov MHD-turbulence is well described by shell (cascade) models. Here we just use the shell approach, comparing with the results obtained in the Kazantsev-Kraichnan approach, checking whether the small-scale generation is a threshold phenomenon for realistic turbulence or not.

1. A small-scale dynamo is described in a standard way by the Kazantsev-Kraichnan model, which is obtained from the magnetic induction equation by averaging over the velocity field

$$\partial_t \mathbf{B} = \mathrm{rot}(\mathbf{v}\times\mathbf{B}) + \eta\boldsymbol{\Delta}\mathbf{B}$$

However, in small-scale dynamo the average energy of the magnetic field growth, by the averaging we get the second moment of the field, that is, the correlation tensor

$$\langle b_i(\mathbf{x}_1,t) b_j(\mathbf{x}_2,t)\rangle = \left(M + \frac{r}{2}\frac{\partial M}{\partial r}\right)\delta_{ij} - \frac{r_i r_j}{2r}\frac{\partial M}{\partial r}$$

Averaging of the magnetic induction equation, which determines the evolution of the magnetic field correlation function M(r), was carried out by A.P. Kazantsev and R.H. Kraichnan for a time-delta-correlated turbulent flow. In such case, the turbulent diffusion $\eta(r)$ was a main function responsible for generation. Function $\eta(r)$ is determined through the magnetic Reynolds number Rm, the correlation time τ, and the correlation function of velocity field F(r) [1967JETPh_Kazantsev]:

$$\frac{\partial M}{\partial t} = \frac{2}{r^4}\frac{\partial}{\partial r}\left(r^4\eta\frac{\partial M}{\partial r}\right) + \frac{2M}{r^4}\frac{\partial}{\partial r}\left(r^4\frac{\partial \eta}{\partial r}\right) \quad \text{where} \quad \eta(r) = \frac{1}{\mathrm{Rm}} + \tau\left(F(0) - F(r)\right)$$

2. The shell approach describes the transport of energy and helicity along a discrete spectrum, between a finite number of spectral shells. Each spectral shell is characterized by collective variables of velocity and magnetic field. The equations for them are the Fourier transforms of the equations of magnetohydrodynamics, where the nonlinear terms are chosen in such a way that the conservation laws are satisfied in the non-dissipative case [2013PhysRep_Plunian]:

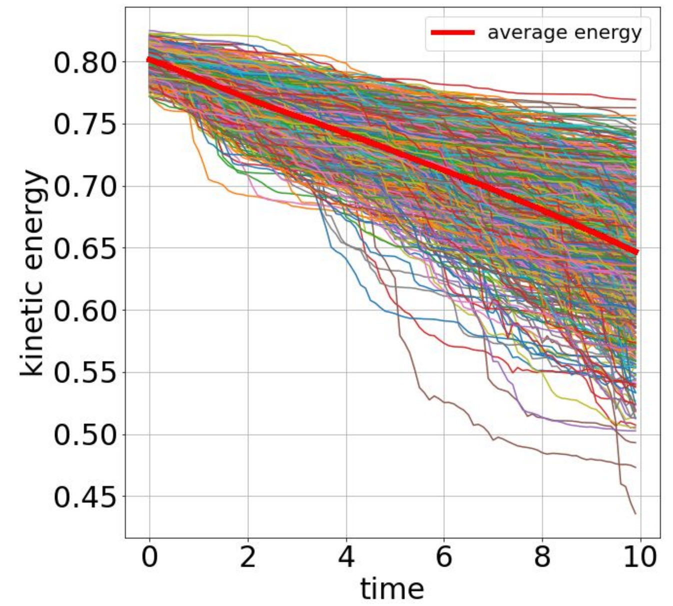
$$\frac{dU_n}{dt} = W_n(\mathbf{U},\mathbf{U}) - W_n(\mathbf{B},\mathbf{B}) - \frac{k_n^2}{\mathrm{Re}}U_n$$

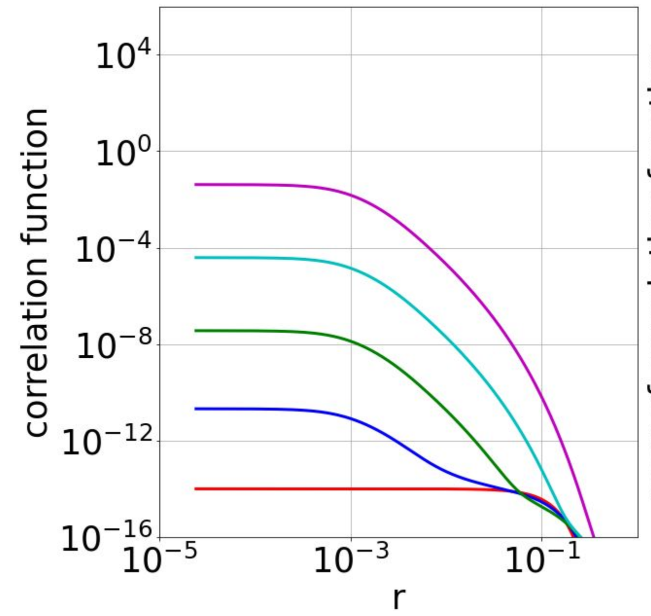
$$\frac{dB_n}{dt} = W_n(\mathbf{U},\mathbf{B}) - W_n(\mathbf{B},\mathbf{U}) - \frac{k_n^2}{\mathrm{Rm}}B_n$$

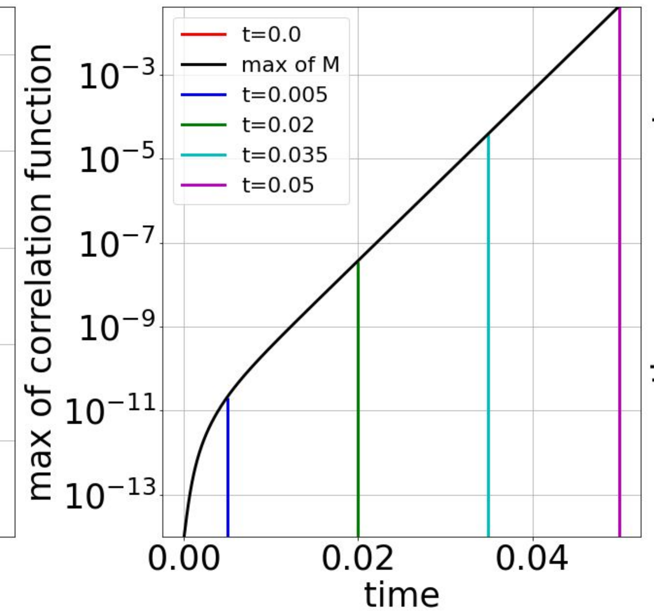
$$\begin{aligned} W_n(\mathbf{X},\mathbf{Y}) = {} & ik_n[(X_{n-1}Y_{n-1} + X^*_{n-1}Y^*_{n-1}) - \lambda X^*_n Y^*_{n+1} - \frac{\lambda^2}{2}(X_nY_{n+1} + X_{n+1}Y_n + X_nY^*_{n+1} + X^*_{n+1}Y_n) \\ & - \frac{\lambda}{2}(X^*_{n-1}Y_{n-1} - X_{n-1}Y^*_{n-1}) + \lambda X^* Y_{n+1}] - ik_n\lambda^{-5/2}[\frac{1}{2}(X_{n-1}Y_n + X_nY_{n-1}) + \lambda X^*_n Y^*_{n-1} \\ & -\lambda^2(X_{n+1}Y_{n+1} + X^*_{n+1}Y^*_{n+1}) + \frac{1}{2}(X_nY^*_{n-1} + X^*_{n-1}Y_n) - \lambda X^*_n Y_{n-1} + \frac{\lambda}{2}(X^*_{n+1}Y_{n+1} - X_{n+1}Y^*_{n+1})] \end{aligned}$$

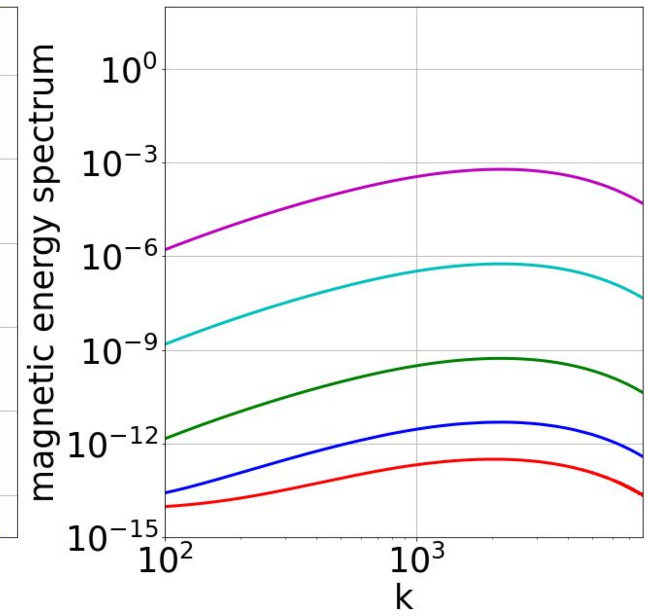
The cascade model allows us to analyze the growth of the magnetic field added at the initial moment in the turbulent conducting flow. The spectral density of the magnetic field demonstrates an increase of magnetic energy and helicity near the characteristic scale $10^{2.5}$ with characteristic growth rate 4.3. The Fourier transform makes it possible to relate r and k spaces such that the correlation function $F(r)$ is reconstructed from the hydrodynamic spectrum, and the turbulent diffusion is in turn calculated from this function. Then, the evolution of the magnetic correlation function is reconstructed in the Kazantsev-Kraichnan model. According to the function $M(r)$, the magnetic energy density is calculated, which was compared with the energy density obtained in the shell model. The calculation scheme can be roughly described as follows

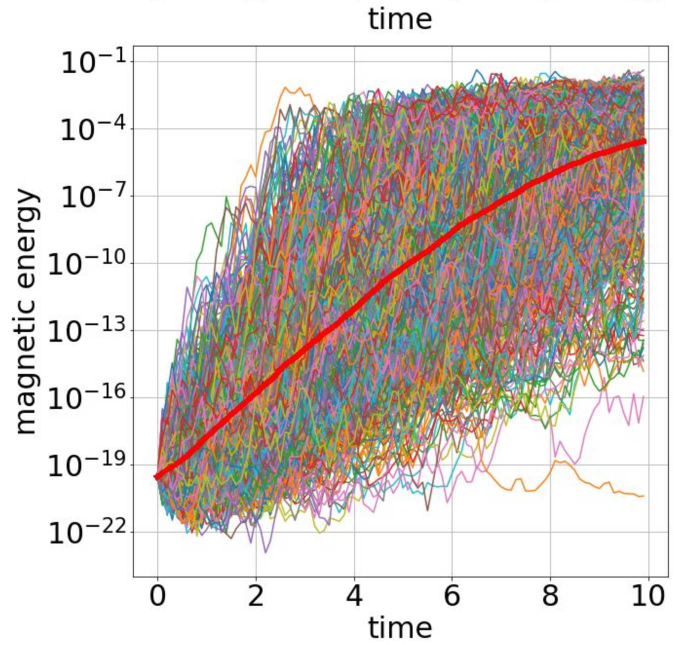
$$F(r) = \frac{1}{4\pi^3}\int_0^\infty E^u(k)\frac{\sin(kr) - kr\cos(kr)}{(kr)^3}dk \;\rightarrow\; \eta(r) \;\rightarrow\; M(r) \;\rightarrow\; E^b(k,t) = 8\pi^2\int_0^\infty M(r,t)kr\left(\sin(kr) - kr\cos(kr)\right)dr$$

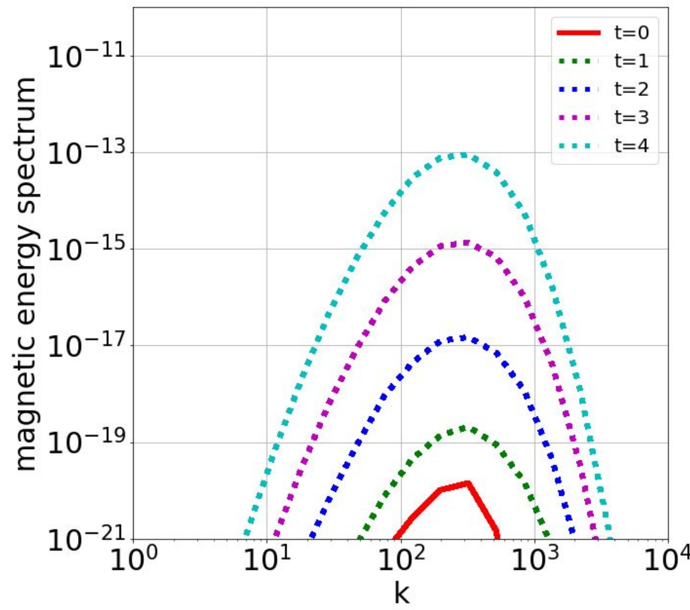












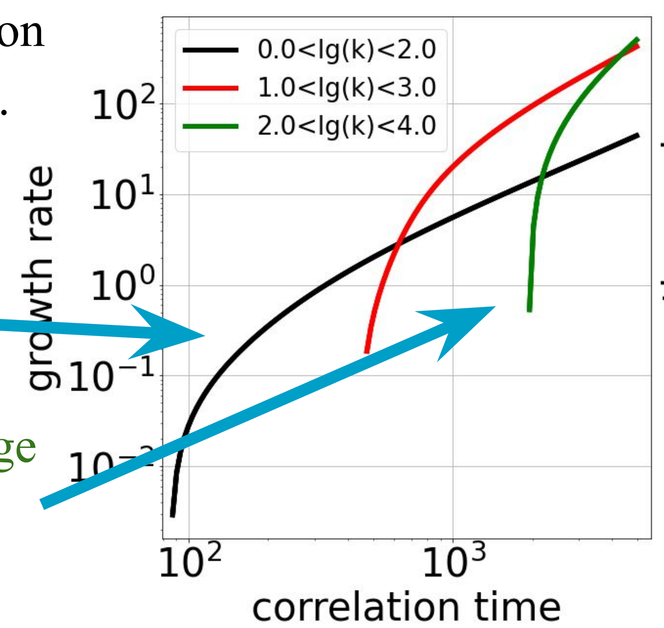
1. If we take the developed hydrodynamic spectrum up to the dissipative scale, the generation rates in the KK model turn out to be much higher than the generation rates in the shell model, the generation scales turn out to be much smaller than in the shell model, and the generation threshold turns out to be extremely high.

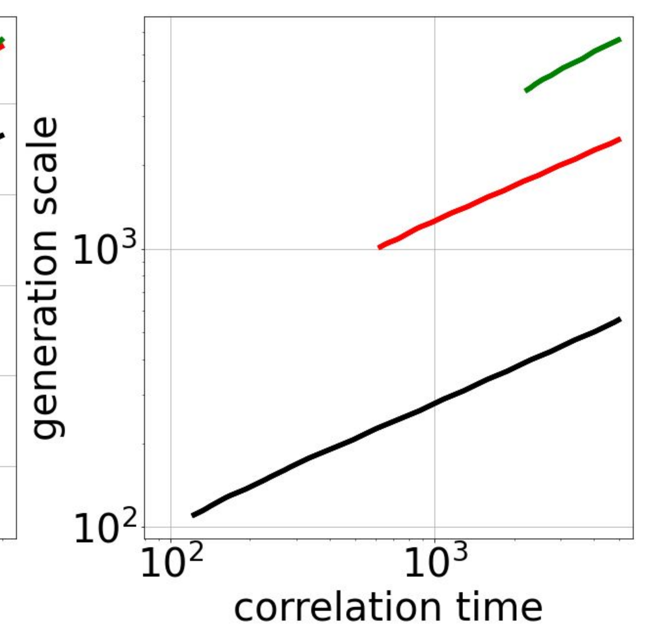
2. However, if we take the large-scale part of the hydrodynamic spectrum, it is possible to choose such a correlation time so that the scales of generation and generation rates in the two models coincide.

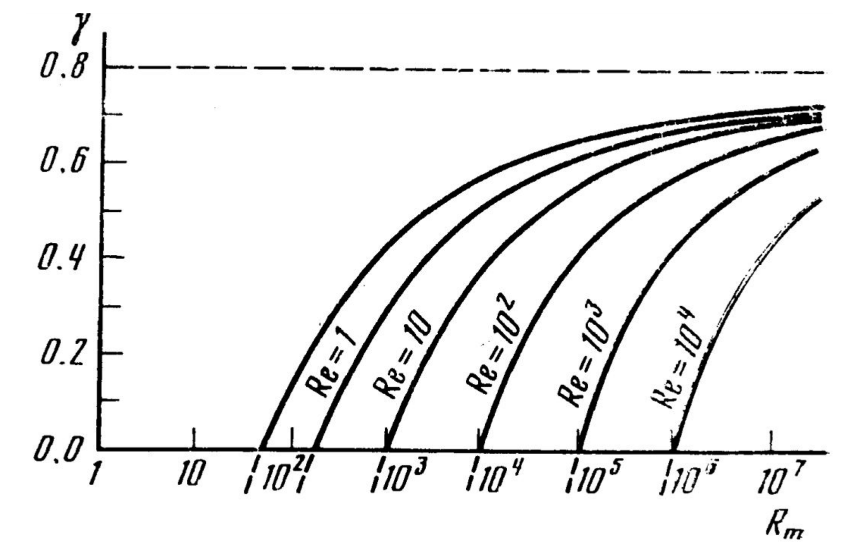
3. Thus, if we assume that both models describe similar phenomena, then we are forced to make the assumption that it is the large-scale part of the hydrodynamic spectrum that is responsible for the operation of the small-scale dynamo.

Large-scale part of kinetic spectrum inertial range

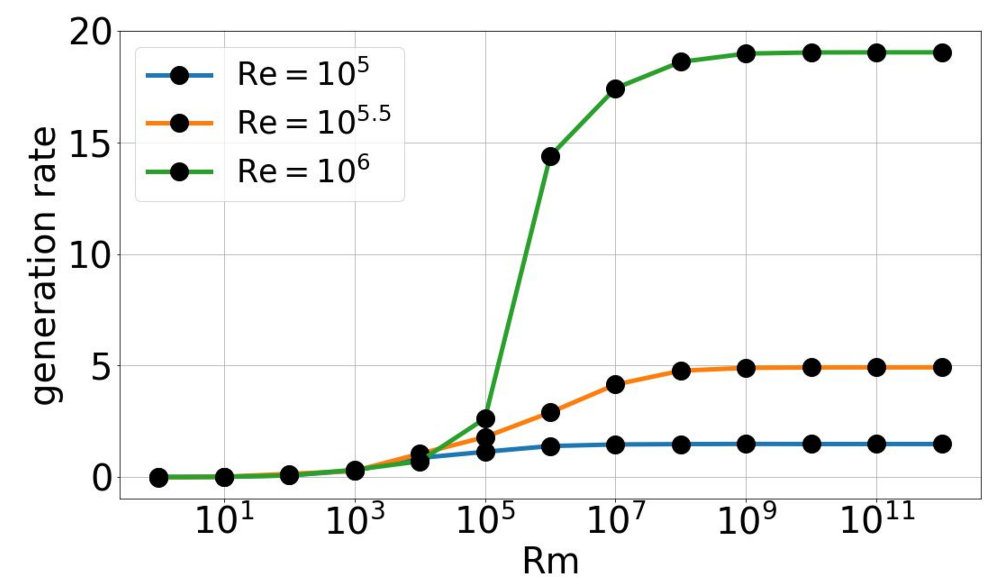
All Inertial range from large scales till dissipative one







Exponential growth rate of magnetic energy as a function of the magnetic Reynolds number, with different values of Re [1983JETPh_Novikov]. The dependence of the generation rate in the KK model for different parts of the hydrodynamic spectrum proves the threshold nature of small-scale generation.



A similar dependence is demonstrated by the cascade model, however, the generation thresholds in it are much higher. However, it also contains a generation tail at low Rm, which is apparently associated not with small-scale, but with large-scale generation.

Conclusions: KK model of the small-scale (s.-s.) dynamo in general terms repeats the process of s.-s. magnetic energy and helicity generation in the shell model. Full agreement can be achieved by assuming that the large-scale part of kinetic spectrum is responsible for the s.-s. generation. The growth rate also goes to the saturation at large Rm. However, the generation threshold has a smoothed character, the reason for which is apparently another type of dynamo generation. This work was supported by the BASIS Foundation grant no. 21-1-3-63-1