



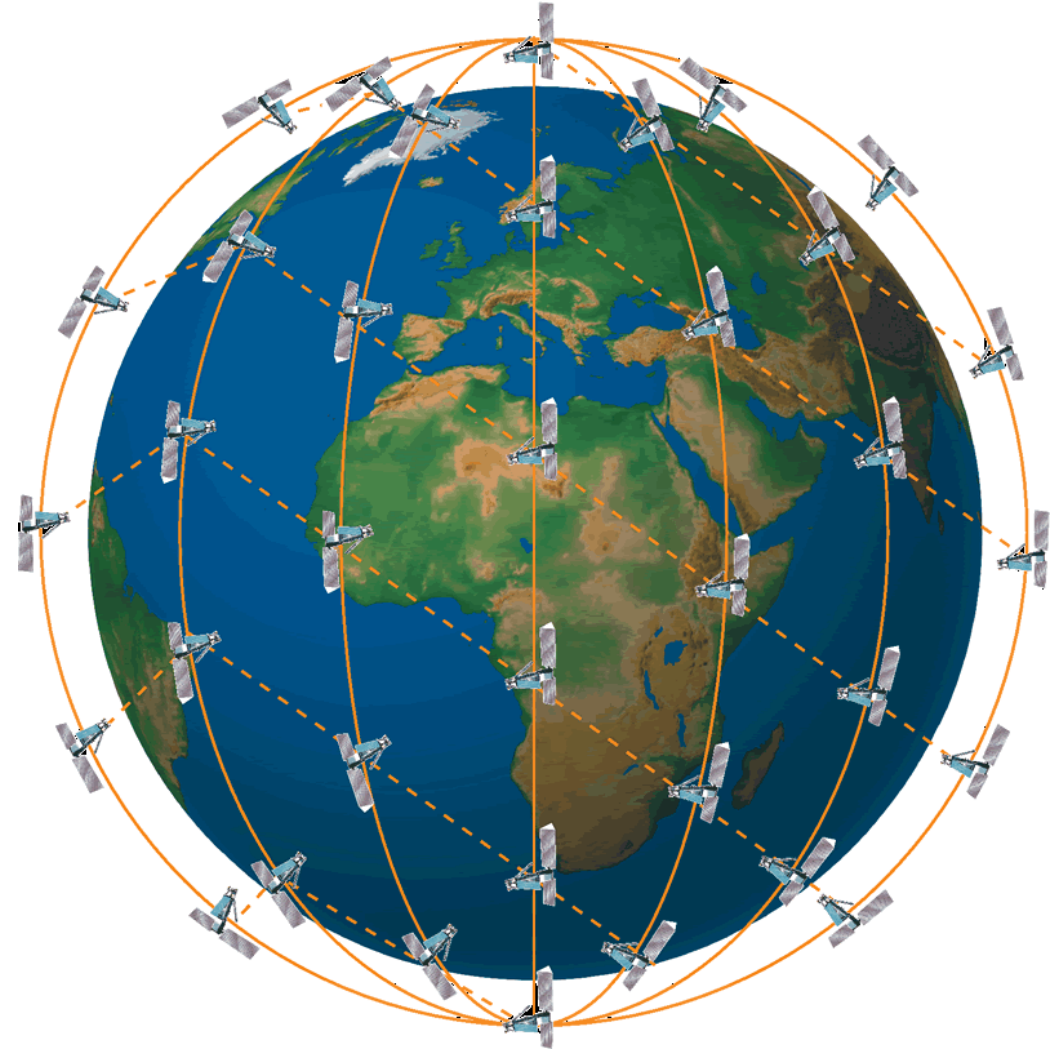
# Undifferenced and uncombined GNSS approach for absolute and relative POD of LEO satellites in formation flying

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# Outline

- Background: LEO formation flying
- UDUC Approach for LEO POD
- Results with the UDUC Approach
- Conclusions



Credit: Journal of Communications

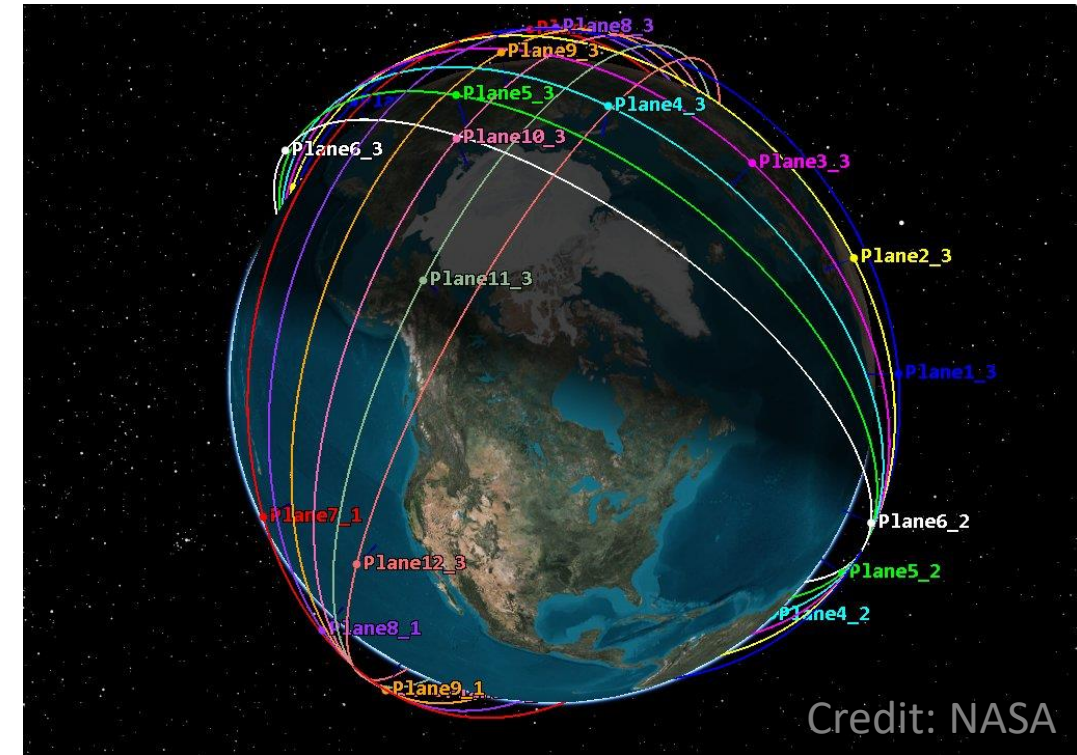
# Background: LEO formation flying

## Satellite formation flying

- Remote sensing, Environment
- Communications

## Precise Orbit Determination (POD) of LEO satellites in formation flying

- The precise orbit of each satellite is needed, i.e., **absolute POD**
- The relative positions between satellites is needed, i.e., **relative POD**



Credit: NASA

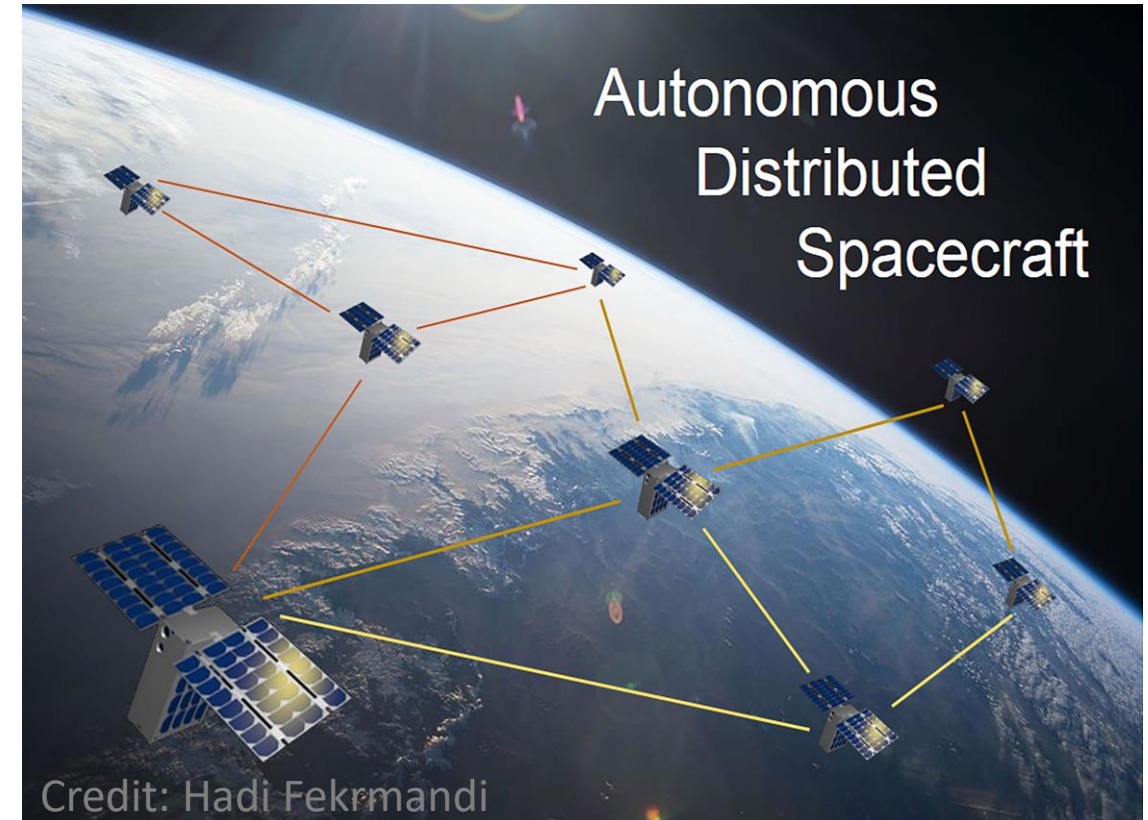
# Background: Kinematic POD of LEO satellites

## Absolute POD: Ionosphere-free (IF) PPP

- Observation information is wasted
- Multi-frequency scenarios are not flexible
- Ambiguities are in float form

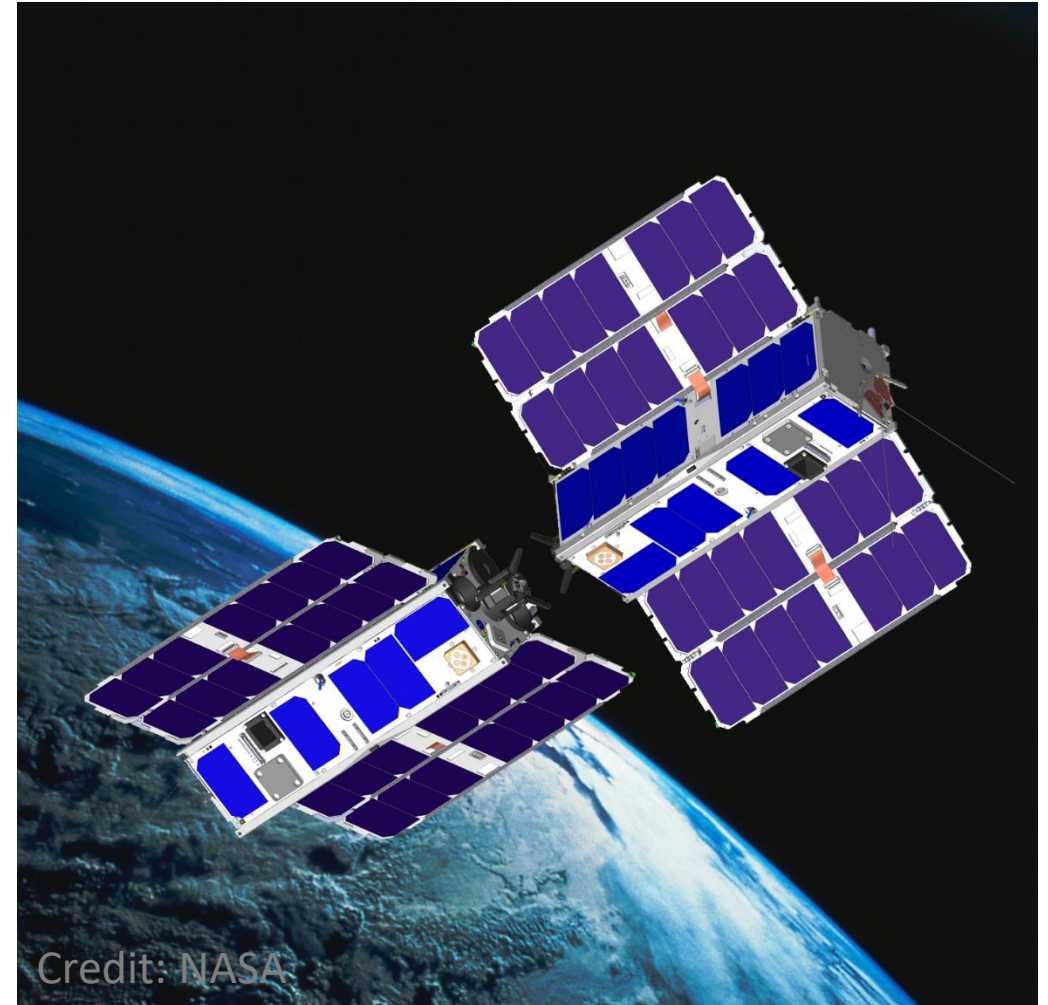
## Relative POD: Double-Differenced Method

- Observation information is wasted
- Use Common-in-view satellites
- DD observations are correlated
- Observation noise is amplified



# Advantages of the UDUC method

- The variance-covariance matrix is the simplest
- All parameters remain available for **possible model strengthening**
- Suitable for **any number of frequencies**
- **Atmospheric parameters** are estimable
- **Phase residuals of each frequency** can be separated



# Why UDUC approach for LEO POD

## UDUC observations

$$p_{r,j}^s = \rho_r^s + dt_r - dt^s + \mu_j I_r^s + d_{r,j} - d_{,j}^s + \varepsilon_{p,j}^s$$

$$\phi_{r,j}^s = \rho_r^s + dt_r - dt^s - \mu_j I_r^s + \lambda_j N_{r,j}^s + \delta_{r,j} - \delta_{,j}^s + \varepsilon_{\phi,j}^s$$

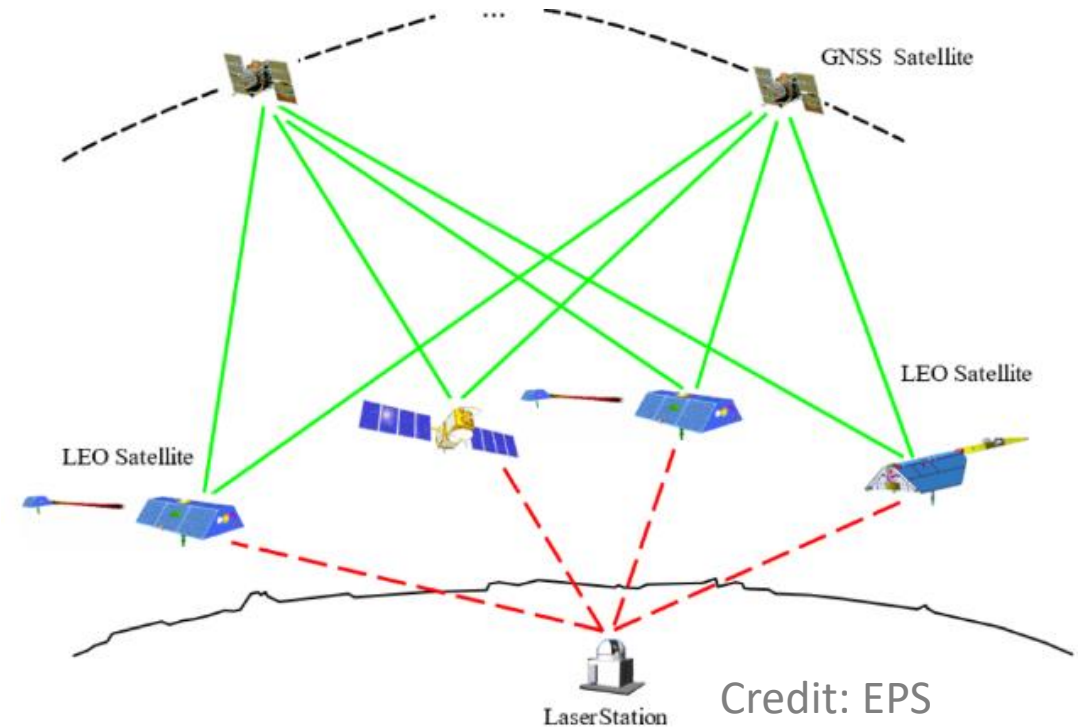
## Question

- **Multiple rank deficiencies** need to be addressed

## Solution

- Rank Deficiency Elimination Method:

### S-system theory



# Modelling the UDUC observations

## Long Baseline

$$\tilde{p}_{A,j}^s = \rho_A^s + d\tilde{t}_A + \mu_j \tilde{I}_A^s - \tilde{d}_{A,j}^s + \varepsilon_{p,j}^s$$

$$\tilde{\phi}_{A,j}^s = \rho_A^s + d\tilde{t}_A - \mu_j \tilde{I}_A^s - \tilde{\delta}_{A,j}^s + \varepsilon_{\phi,j}^s$$

$$\tilde{p}_{B,j}^s = \rho_B^s + d\tilde{t}_B + \mu_j \tilde{I}_B^s - \tilde{d}_{A,j}^s + \tilde{d}_{AB,j}^s + \varepsilon_{p,j}^s$$

$$\tilde{\phi}_{B,j}^s = \rho_B^s + d\tilde{t}_B - \mu_j \tilde{I}_B^s - \tilde{\delta}_{A,j}^s + \tilde{\delta}_{AB,j}^s + \lambda_j N_{AB,j}^{1s} + \varepsilon_{\phi,j}^s$$

## Short Baseline

$$\tilde{p}_{A,j}^s = \rho_A^s + d\tilde{t}_A + \mu_j \tilde{I}_A^s - \tilde{d}_{A,j}^s + \varepsilon_{p,j}^s$$

$$\tilde{\phi}_{A,j}^s = \rho_A^s + d\tilde{t}_A - \mu_j \tilde{I}_A^s - \tilde{\delta}_{A,j}^s + \varepsilon_{\phi,j}^s$$

$$\tilde{p}_{B,j}^s = \rho_B^s + d\tilde{t}_B + \mu_j \tilde{I}_A^s + \mu_j d_{AB,GF} - \tilde{d}_{A,j}^s + \tilde{d}_{AB,j}^s + \varepsilon_{p,j}^s$$

$$\tilde{\phi}_{B,j}^s = \rho_B^s + d\tilde{t}_B - \mu_j \tilde{I}_A^s - \mu_j d_{AB,GF} - \tilde{\delta}_{A,j}^s + \tilde{\delta}_{AB,j}^s + \lambda_j N_{AB,j}^{1s} + \varepsilon_{\phi,j}^s$$

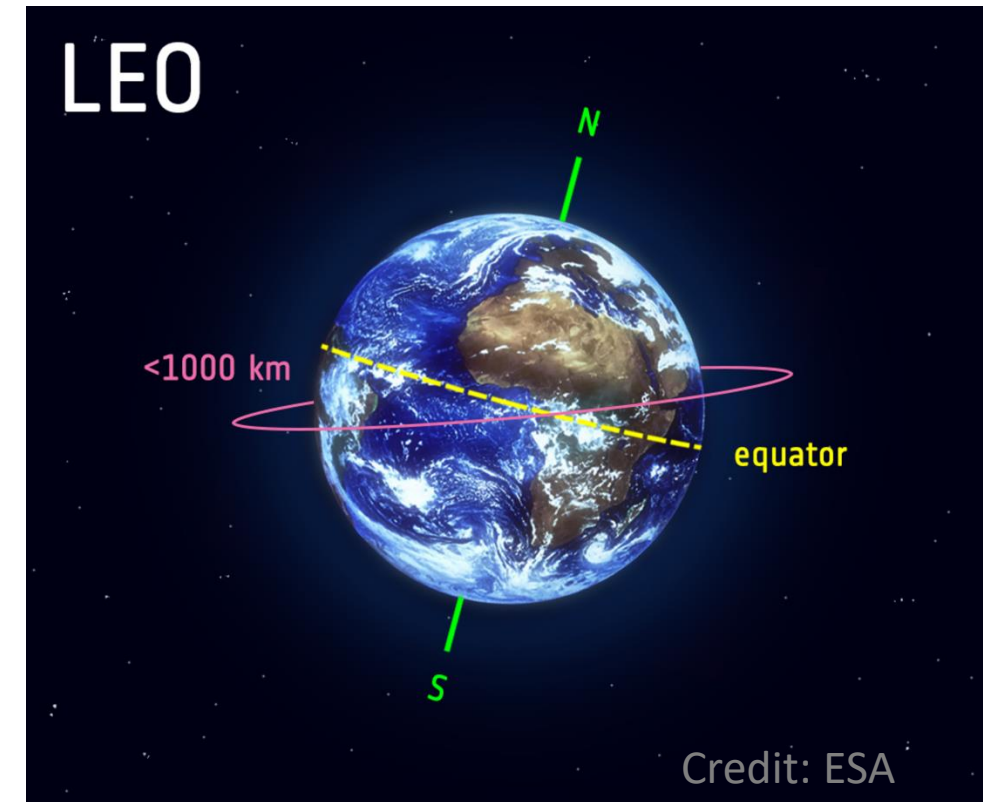
## Estimable unknowns and their interpretation

Estimable parameter	Notation and interpretation
Receiver clock	$d\tilde{t}_r = dt_r + d_{r,IF}$
Satellite code bias	$\tilde{d}_{r,j}^s = d_{r,j}^s - d_{r,IF}^s - \mu_j d_{r,GF}^s - d_{r,j} + d_{r,IF} + \mu_j d_{r,GF} \quad j \geq 3$
Satellite phase bias	$\tilde{\delta}_{r,j}^s = \delta_{r,j}^s - d_{r,IF}^s + \mu_j d_{r,GF}^s - \lambda_j N_{r,j}^s - \delta_{r,j} + d_{r,IF} - \mu_j d_{r,GF} \quad j \geq 1$
Ionospheric delay	$\tilde{I}_r^s = I_r^s + d_{r,GF} - d_{GF}^s$
Between-receiver code bias	$\tilde{d}_{AB,j} = d_{B,j} - d_{A,j} - d_{AB,IF} - \mu_j d_{AB,GF} ; j \geq 3$
Between-receiver phase bias	$\tilde{\delta}_{AB,j} = \delta_{B,j} - \delta_{A,j} - d_{AB,IF} + \mu_j d_{AB,GF} + \lambda_j N_{AB,j}^1 ; j \geq 1$
DD phase ambiguity	$N_{AB,j}^{1s} = N_{AB,j}^s - N_{AB,j}^1$
S-basis parameters	$d_{r,j}, \delta_{r,j}, d_{r,IF}, N_{r,j}^s, d_{r,GF}, d_{r,GF}^s, \tilde{d}_{A,j}, \tilde{\delta}_{A,j}, N_{AB,j}^1$

# Advantages of the UDUC approach for LEO POD

## UDUC model with DD ambiguity

- The model can serve **absolute and relative POD**
- **IAR can be performed without external SPB products**
- The relative POD of an LEO constellation **reduces the number of estimated parameters**
- **Code and phase biases at both LEO and GNSS-end remained** for further model strengthening
- **Non-common-in-view** GNSS satellites can contribute to POD with the UC POD model

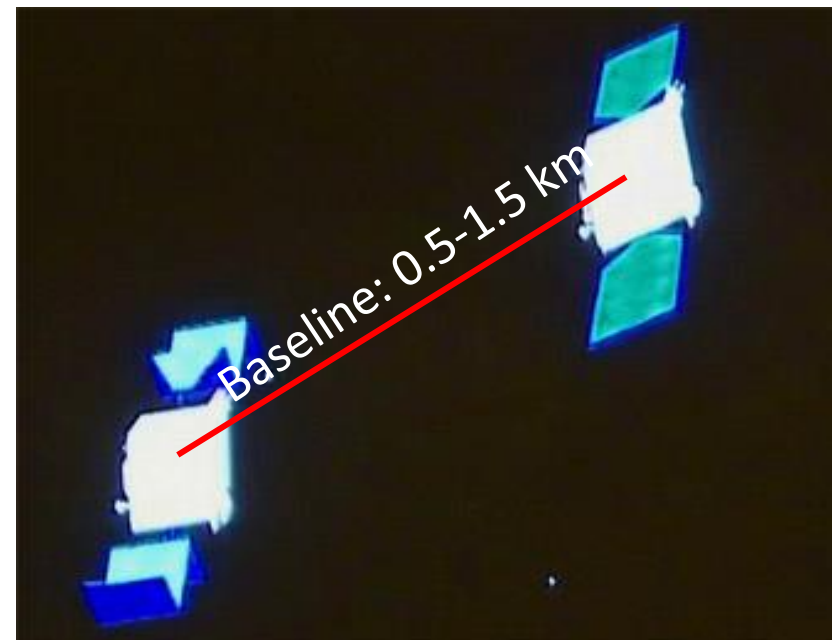




# Real LEO data and Processing Strategy

## Real LEO in formation flying

- T-A and T-B
- GPS L1+L2



T-A and T-B in formation flying

## Main data processing strategies

Item	Strategy
Observation	GPS L1+L2
GPS antenna offset	Corrected
LEO antenna offset	Corrected
LEO attitude	Quaternions from onboard star trackers
SPB	Estimated as time-constants
Between-receiver phase biases	Estimated as time-constants
Between-receiver DCB	Estimated as a time-constant
Slant ionospheric delays	Estimated as white noise
Parameter estimator	Kalman filter
IAR and validation	LAMBDA with ratio test
Outlier detection and elimination	DIA procedure

# UDUC approach for POD of LEO satellites

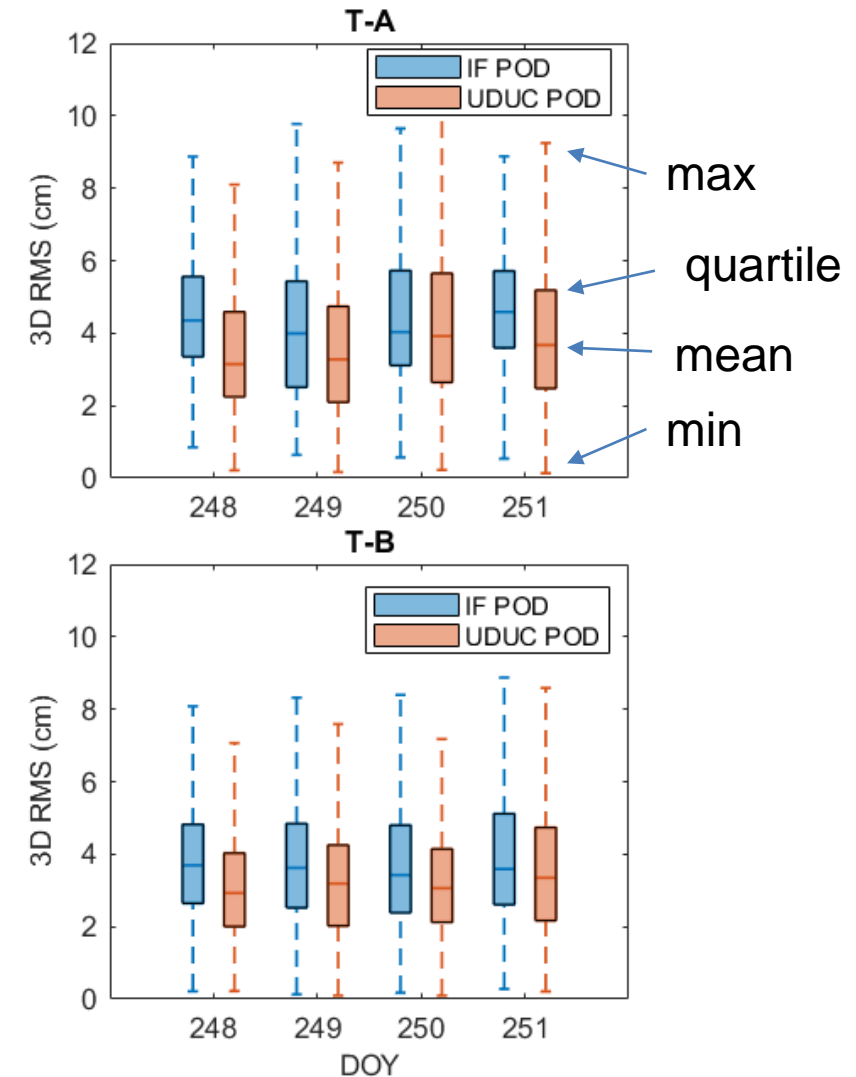
## Processing Strategy

- IF POD: Kinematic POD with fixed ambiguities using Bernese GNSS Software V5.4
- UDUC POD: Short baseline mode with fixed ambiguities

## Results

With the UDUC algorithm and IAR, the proposed model presented a discrepancy of **3-4 cm in 3D** with the reference orbits, and the orbit difference was thus:

reduced by **16.3% and 10.6%** for T-A and T-B compared with the classical IF POD



POD results of T-A and T-B for four days

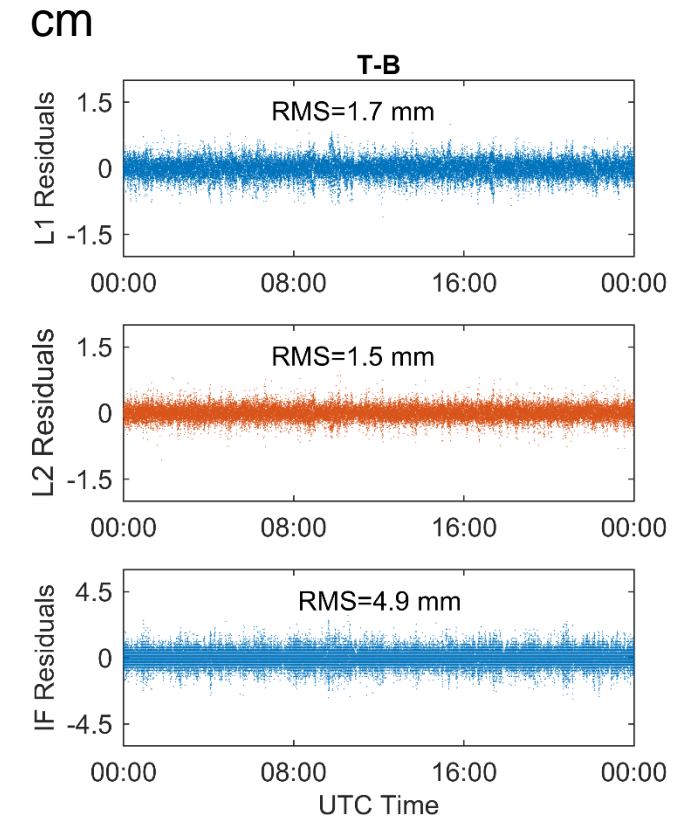
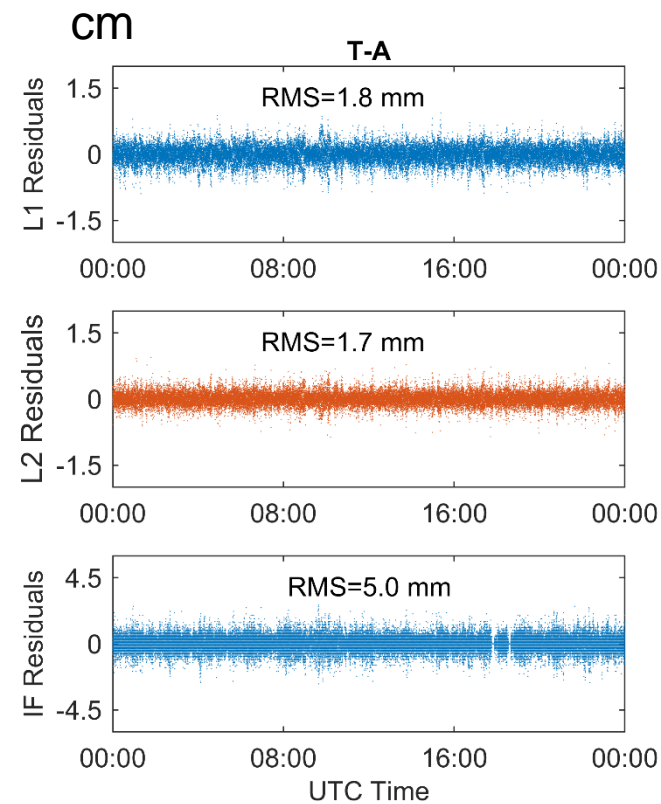
# UDUC approach for POD of LEO satellites

## Processing Strategy

- L1 and L2 residuals are obtained with the UDUC approach with fixed ambiguities
- Residuals are obtained with Bernese GNSS Software V5.4 with fixed ambiguities

## Results

- The original observation noise not amplified
- The residuals at each frequency can be separated



# UDUC approach for POD of LEO satellites

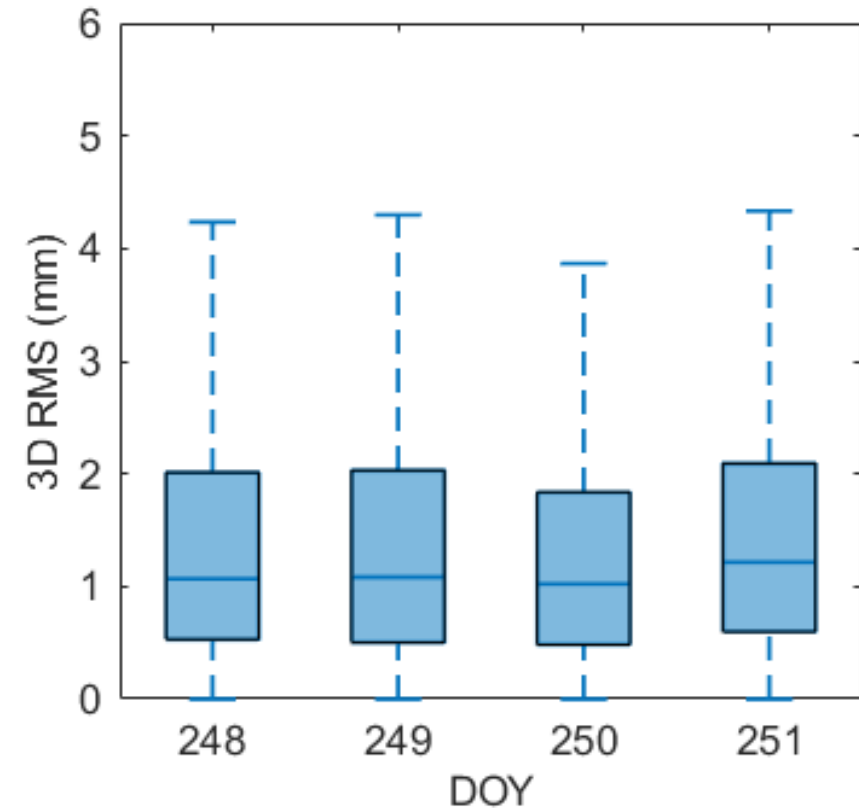
## Processing Strategy

- T-A with a reduce-dynamic method using Bernese GNSS Software V5.4
- Process baseline with the UDUC approach

## Results

The UDUC POD with DD ambiguity can achieve **mm-level relative POD**, which can be used for:

- *formation flying,*
- *space docking, and*
- *rendezvous missions*



Relative POD results for four days

# Conclusions

1. The UDUC approach was proposed for both **absolute and relative POD**
2. The difference between the UDUC POD with DD ambiguity solution and the reference orbit was smaller than when using the IF POD
3. The **phase residuals of L1 and L2 were obtained** with the proposed model, which are much smaller than the IF phase residuals with the IF POD. This shows the advantages of the UDUC model with DD ambiguity
4. The ability of the UDUC POD with DD ambiguity to achieve **millimeter-level relative POD** was demonstrated, proven that the model could be used for formation flying missions

# Questions

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# Thanks for your attention

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