Ensemble sensitivity localization

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1. Introduction

Ensemble sensitivity is a tool to quantitatively determine which initial conditions influence a forecast quantity of choice from the covariances of the initial ensemble to the forecast ensemble. Unfortunately, these covariances are prone to sampling errors due to the limited ensemble size, which can be mitigated by a distance-based damping, i.e., localization.

Research question:

How can localization can be applied to the ensemble sensitivity, and what are it's benefits and drawbacks?

Methodology:

Setup a 1D toymodel for which we know what the sensitivity should be, and compare the sensitivities computed from the ensemble to the expected solution.

2. Ensemble sensitivity

Basic Assumption: The ensemble deviations of a forecast metric $\boldsymbol{\delta j}$ are a linear approximation of the initial state ensemble deviations $\boldsymbol{\delta X}$. For example, the ensemble differences in forecasted rain ($\boldsymbol{\delta j}$) are linearly related to the humidity differences at the start of the forecast ($\boldsymbol{\delta X}$).

The ensemble sensitivity is a vector $\boldsymbol{s}$, so that:

$$\boldsymbol{s}\boldsymbol{\delta X} \approx \boldsymbol{\delta j}$$

The sensitivity can then be calculated from the covariances:

$$\boldsymbol{s} \approx \boldsymbol{\delta j}\boldsymbol{\delta X}^T \left[\boldsymbol{\delta X}\boldsymbol{\delta X}^T\right]^{-1}$$

Beware that the covariance matrix inversion will in practice require some form of regularization.

3. Localized ensemble sensitivity

To minimize the effects of spurious correlations in $\boldsymbol{\delta j}\boldsymbol{\delta X}^T$ and $\boldsymbol{\delta X}\boldsymbol{\delta X}^T$, a localization matrix $\mathbf{C}$ is multiplied element wise that dampens correlations based on their distance. To achieve this the forecast metric needs to be split into i subcomponents, one for each grid point.

$$\boldsymbol{s}_{loc} = \sum_i \left[\vec{C}_i \circ \boldsymbol{\delta j}_i \boldsymbol{\delta X}^T\right] \left[C \circ \boldsymbol{\delta X}\boldsymbol{\delta X}^T\right]^{-1}$$

The vector over the first $\mathbf{C}$ marks that it needs to be modified to account for signal propagation, as $\boldsymbol{\delta j}$ is in the future, and $\boldsymbol{\delta X}$ is at the beginning of the forecast

- **The smaller the ensemble size, the more localization helps compute the ensemble sensitivity.**
- **But localizing the sensitivity requires knowing how signals propagate over time.**

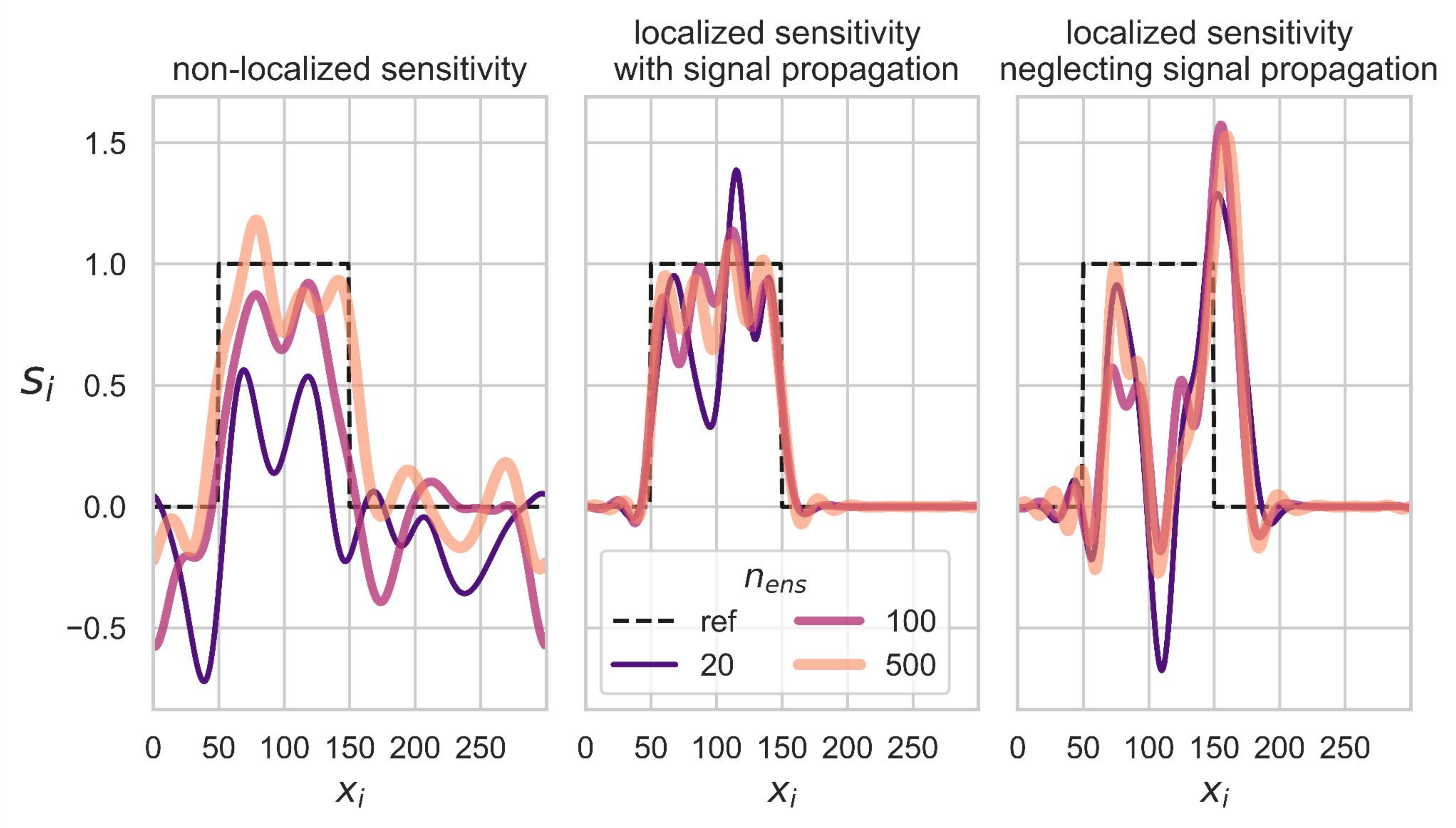
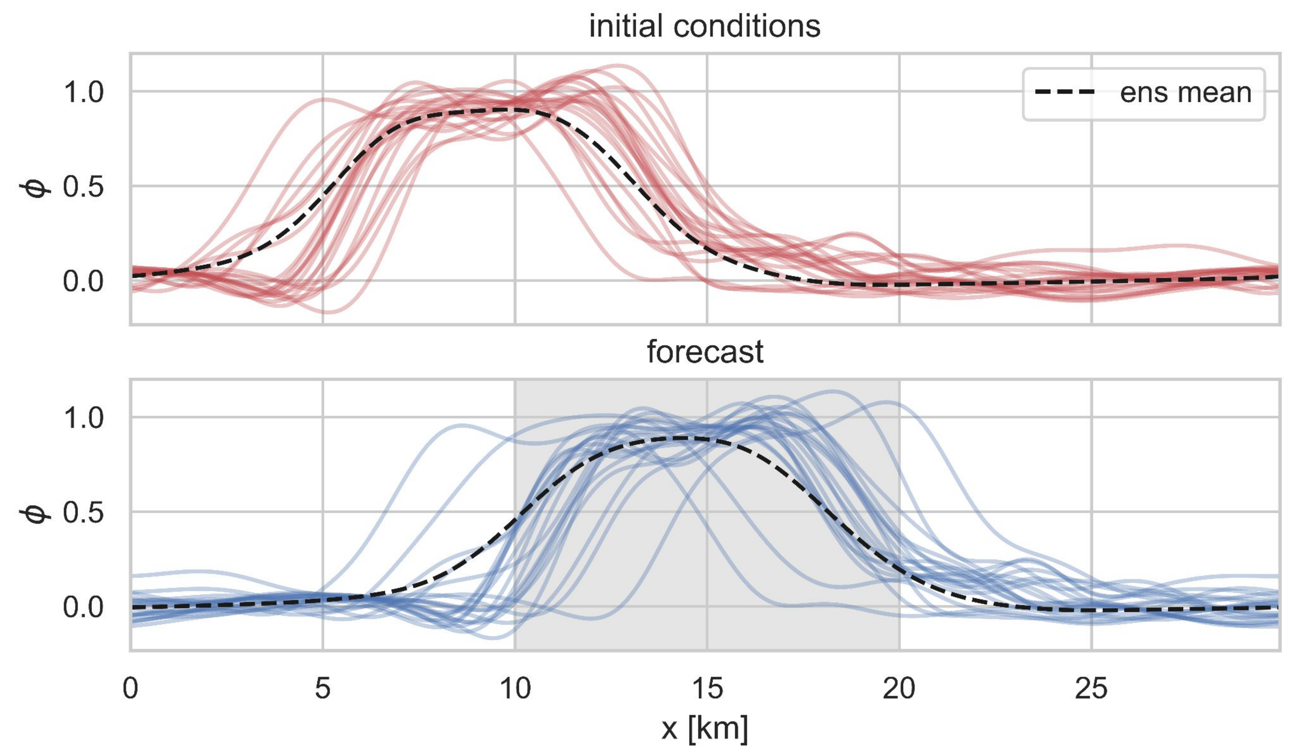
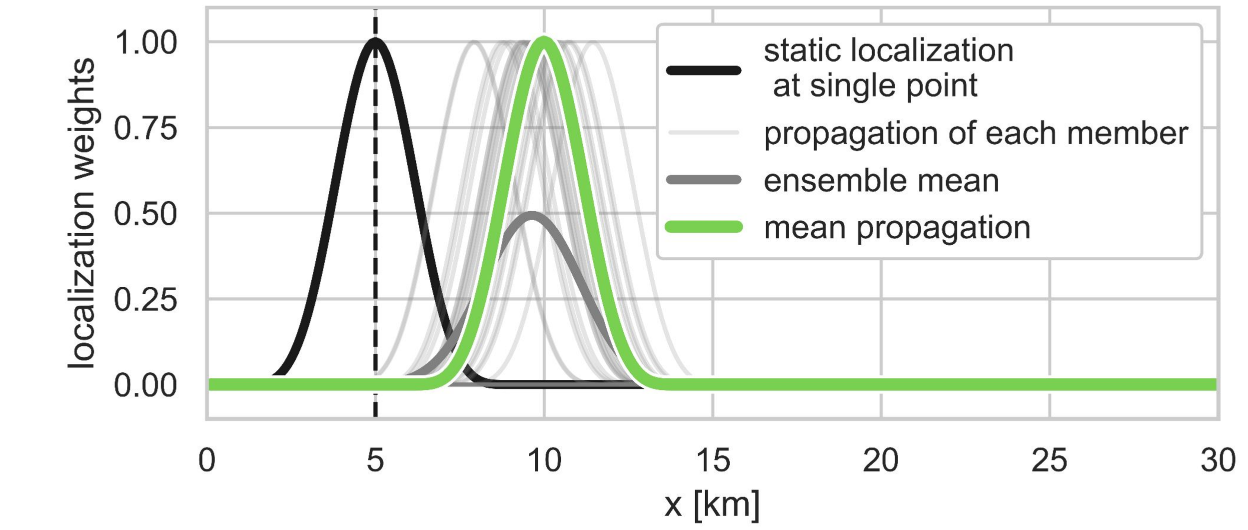


Figure caption: Examples of sensitivity calculated from the randomly selected toymodel test shown in Box 4. Goal is to lie close to the dashed ref line. The different lines mark different ensemble sizes.

4. Linear-advection toymodel

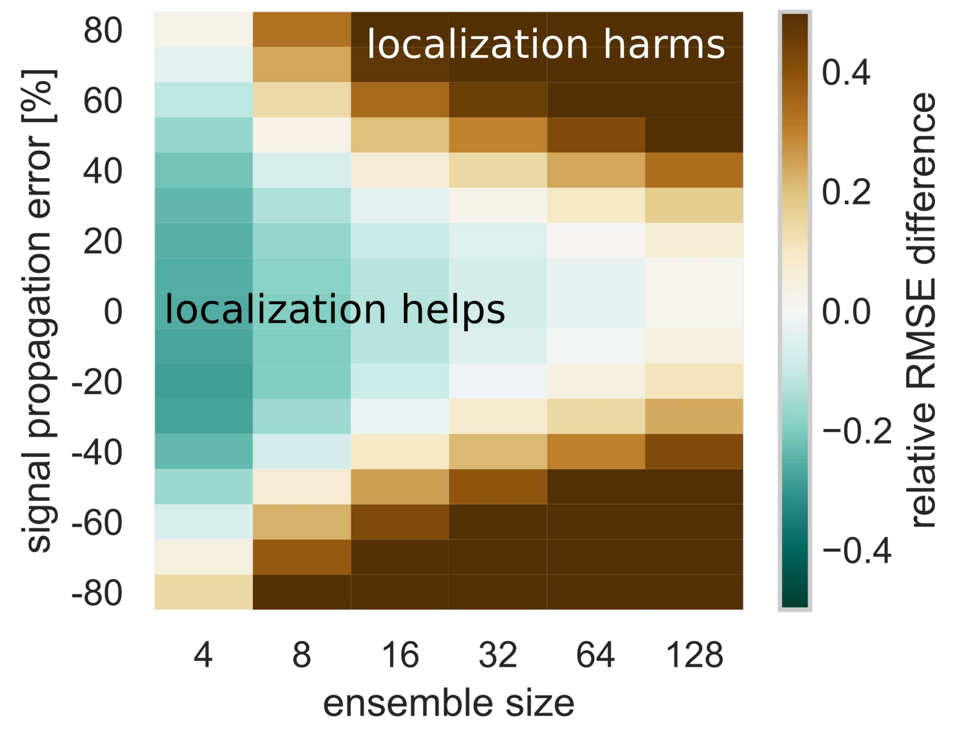


Each ensemble member travels to the right with a randomly perturbed velocity that is constant over time. The forecast metric of choice is the sum over the gray box. In this toymodel signal propagation is identical to the velocity times the forecast lead time.



5. Implications for observation impact studies

Griewank et al 2023 found that when estimating the impact of potential observations, the benefit of sensitivity localization varies strongly with ensemble size and the precision with which signal propagation can be estimated.



6. Reference

Griewank, Weissmann, Necker,, Nomokonova, and Löhnert (2023), Ensemble-based estimates of the impact of potential observations. Q J R Meteorol Soc. Accepted Author Manuscript. https://doi.org/10.1002/qj.4464