Field optical clocks and sensitivity to mass anomalies for geoscience applications

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Overview

ROYMAGE (hoRloge Optique à Ytterbium Mobile Appliquée à l'exploration GEodésique) is a project dedicated to develop a transportable ytterbium (Yb) optical lattice clock. Connected to the fiber network REFIMEVE+, the clock will allow remote clock comparisons to perform chronometric geodesy and geoscience applications. With a relative frequency uncertainty targeted at a first step in the low 10^{-17} (or 10 cm height variation), the clock will provide geopotential difference measurements which are not directly available with traditional techniques (e.g. GNSS/levelling, gravimetry).

In this work, we focus on the contribution of chronometric observables for the detection and monitoring of geophysical processes (volcanic, hydrological, tectonic deformations, etc.). To this end, we have developed digital tools to model the gravitational response of mass anomalies and the associated vertical displacement of the surface (and thus frequency shift observed by the clock) due to the elastic deformation induced by buried geophysical structures, as well as the signal needs correcting for different effects, such as solid Earth tides, oceanic tidal loading, polar motion, and the centrifugal effect.

These synthetic simulations allow us to identify which types of structures can be detected by clock comparison measurements with a relative frequency uncertainty fixed at $10^{-17-18-19}$ (i.e. a vertical sensitivity at 10 cm - 1 cm - 1 mm respectively). We also present an application for an aquifer undergoing groundwater fluctuations due to anthropogenic exploitation and causing detectable gravitational signals.

What to do with chronometry ?

- General relativity predicts that time flows differently for clocks located in different gravitational potentials
- In practice, we compare the frequency of the field clock wrt a clock reference to measure the geopotential variations ΔW (or height variations Δh) between them

Link with geodesy

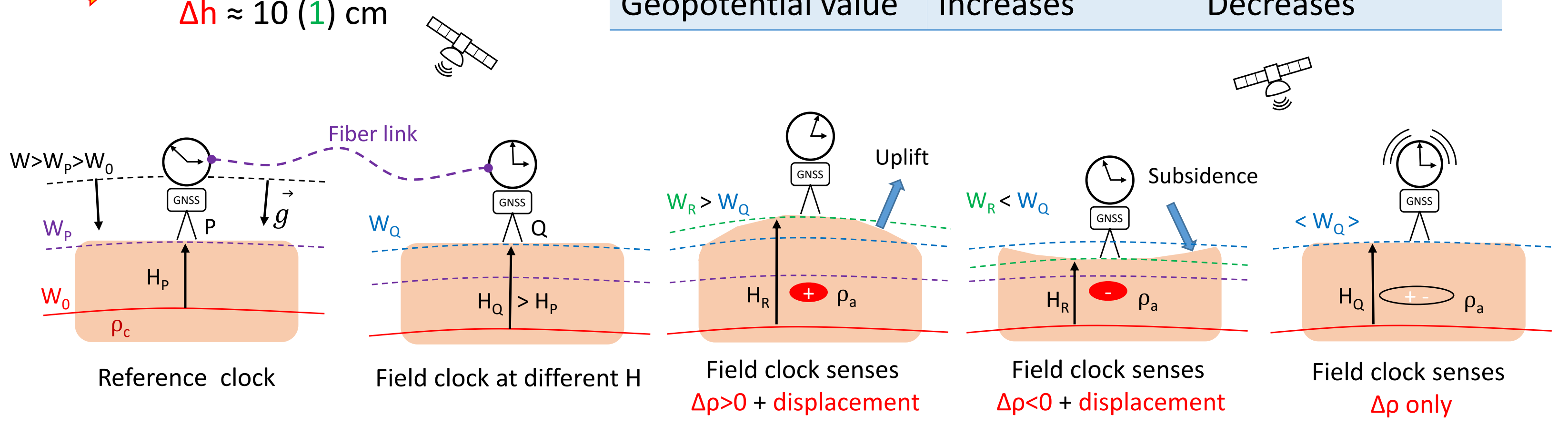
Clock frequency shift
$\Delta f/f = 10^{-17\ (-18)}$

Geopotential variations
$\Delta W \approx 1\ (0.1)\ m^2/s^2$

Height variations
$\Delta h \approx 10\ (1)$ cm

Link with geoscience

Effects	Near a mass or mass density contrast > 0	Away from a mass or mass density contrast < 0
Time flow	Speeds down	Speeds up
Clock frequency	Decreases	Increases
Geopotential value	Increases	Decreases



- Since the clocks are on different sites, they undergo different effects (geodetic, geophysical and astronomical) that will have to be corrected to characterize a regional or local (10 - 100 km spatial resolution) and/or deep geophysical process
- **Main signal components**: $\Delta W = \Delta V_g + \Delta W_v + \Delta \Phi$ + (other effects)

  ΔΦ: Centrifugal variations depending on **latitude** and **height**

  ΔV_g: Gravitational variations depending on a **gravitational signal** (mass anomalies, solid earth tides (SET), ocean tide loading (OTL), polar motion, etc.)
  ΔW_v: Geopotential variations depending on **vertical displacements** (surface elastic deformation, tide displacements effect, etc.)
- Displacements and gravity effects can be computed according the IERS conventions or tidal models, but potential differences are rarely considered

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Gravitational signal of a mass anomaly

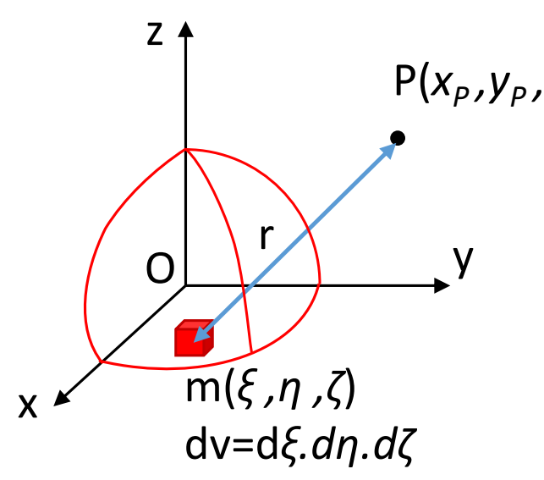
We have developed a Matlab package MASS-tools (Mass Anomaly Signal Simulation) to compute the gravitational signal (V, g and T) of a mass anomaly with different methods

Analytic solutions by calculating the integrals

Potential $V = G\int \frac{\rho(\xi,\eta,\zeta)\,dv}{r(\xi,\eta,\zeta;x_P,y_P,z_P)}$

Acceleration $\nabla V = g_i$

Tensor $\Delta V = T_{ij} = 0$ → outside mass

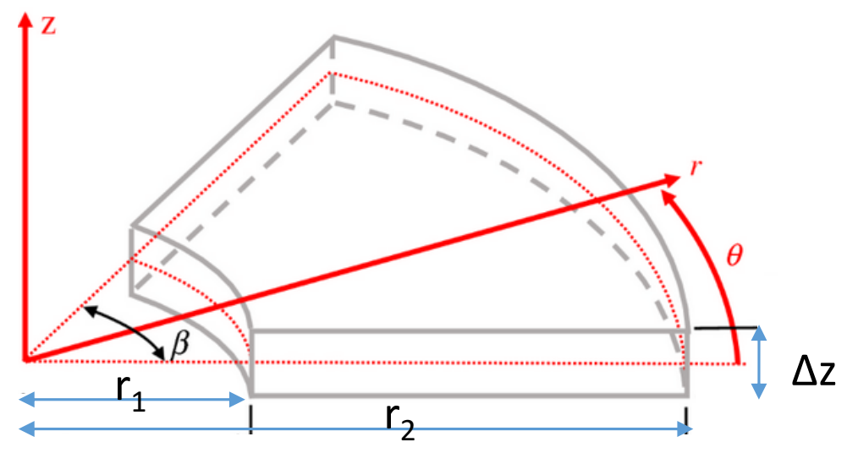


→ Very tricky or impossible
→ Calculations in the anomaly frame with appropriate coordinates
→ Simple and interesting structures with analytical solutions
Sphere, right rectangular prism (useful for discretizing a complex structure), horizontal cylinder and vertical cylinder (non exact solutions)

Numerical solutions using quadrature schemes

$$V = G\int_{r_1}^{r_2}\int_{\theta_1}^{\theta_2}\int_{z_1}^{z_2}\frac{\rho r\,dr\,d\theta\,dz}{R(x,y,z;x_P,y_P,z_P)} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} w_i w_j w_k f(x_i,y_j,z_k)$$

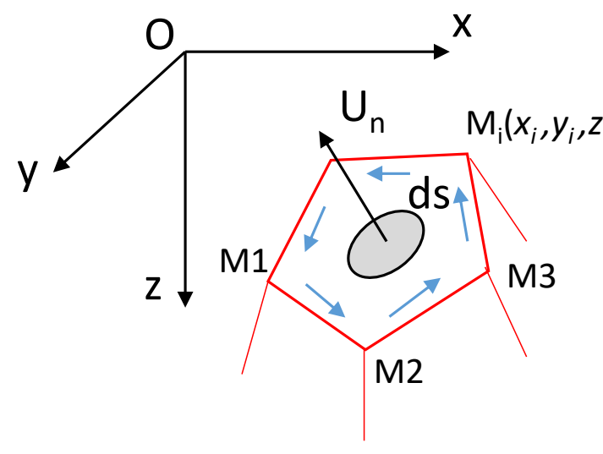
w_i: weights, x_i: nodes



→ More time consuming
→ Computations in the anomaly frame with appropriate coordinates
→ Excellent approximation for all primary structures
→ Can easily model hollow section (shell, tube, ring, …)
→ Can avoid mathematical singularities (ex. cuboid corners)

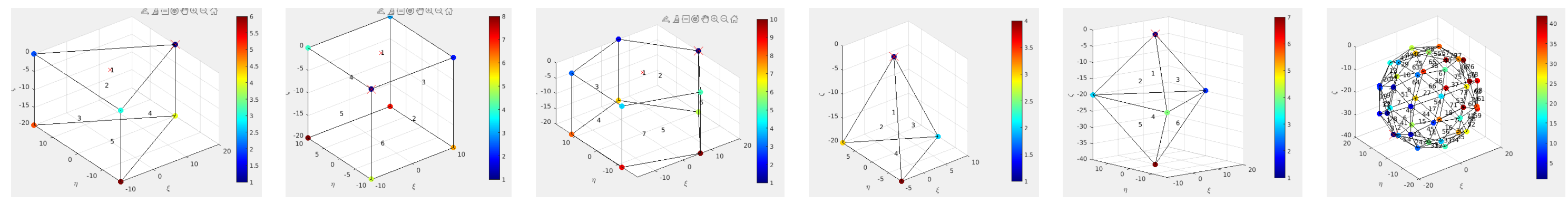
Analytic solution using line integrals: Tsoulis & Gavriilidou (2021)

Look at the flow of the gravity field through a curvilinear and oriented line/surface



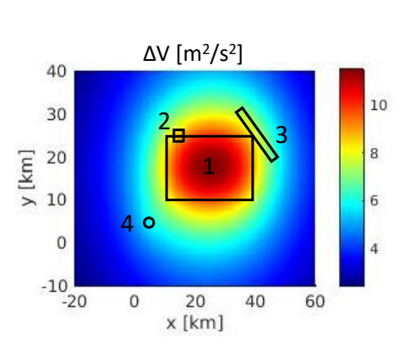
→ Very time consuming
→ Need to mesh the structure into polyhedral form (edges and faces coordinates) and orientate the faces
→ Allows any real structure to be modelled as a polyhedron
→ Computations made wrt any observer

This code has been interfaced in is MASS-tools by giving the possibility to generate, modify (shearing, stretching, orientation) a polyhedral structure



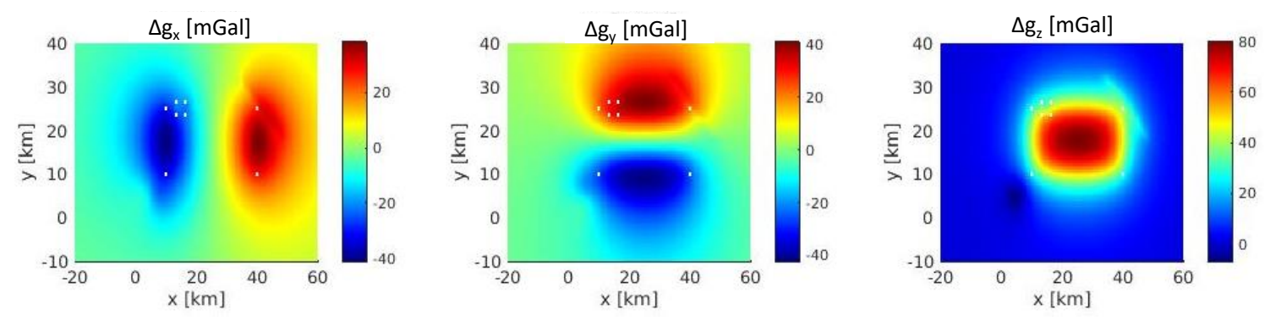
Example of signals

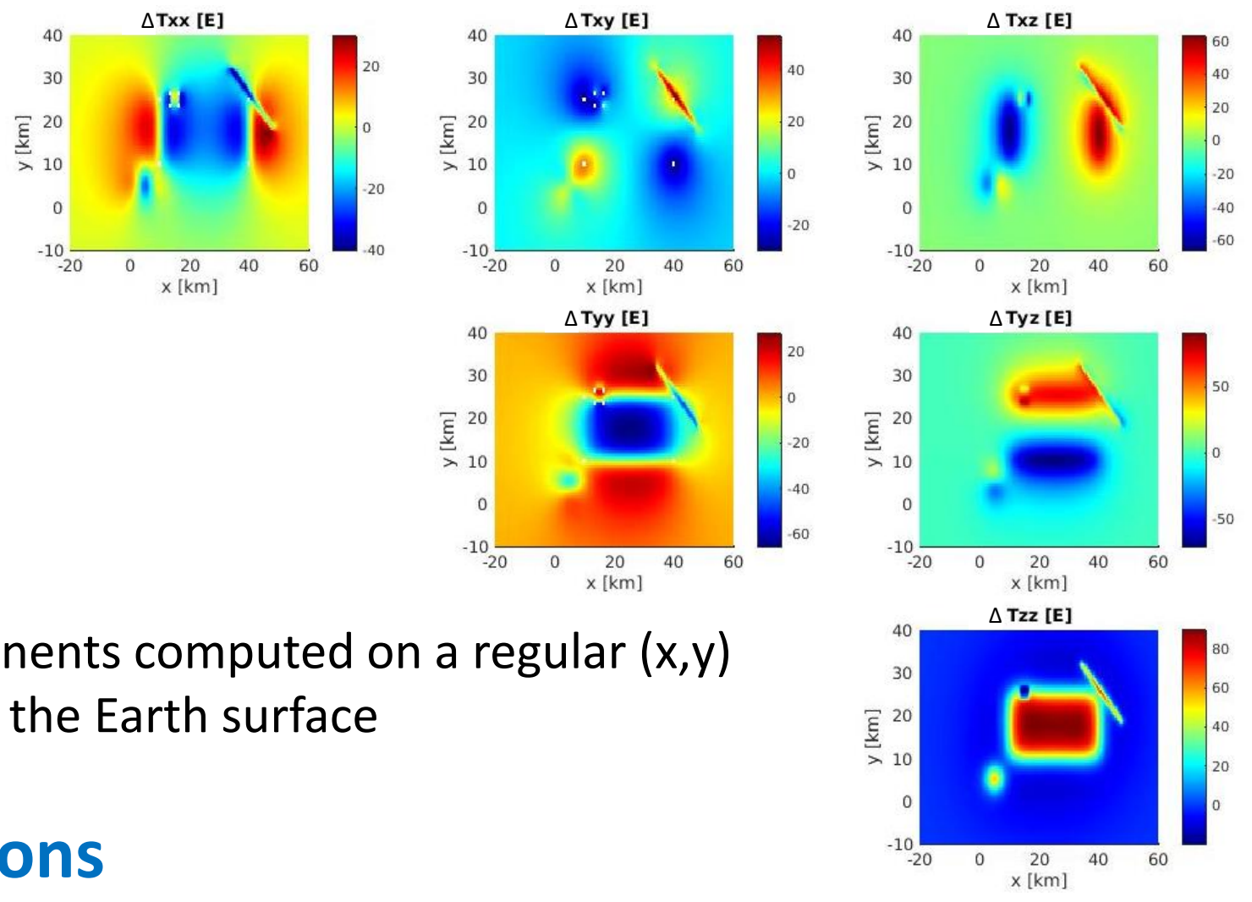
Gravitational signal from different structures



Characteristics of the prisms from Pajot et al. (2008)

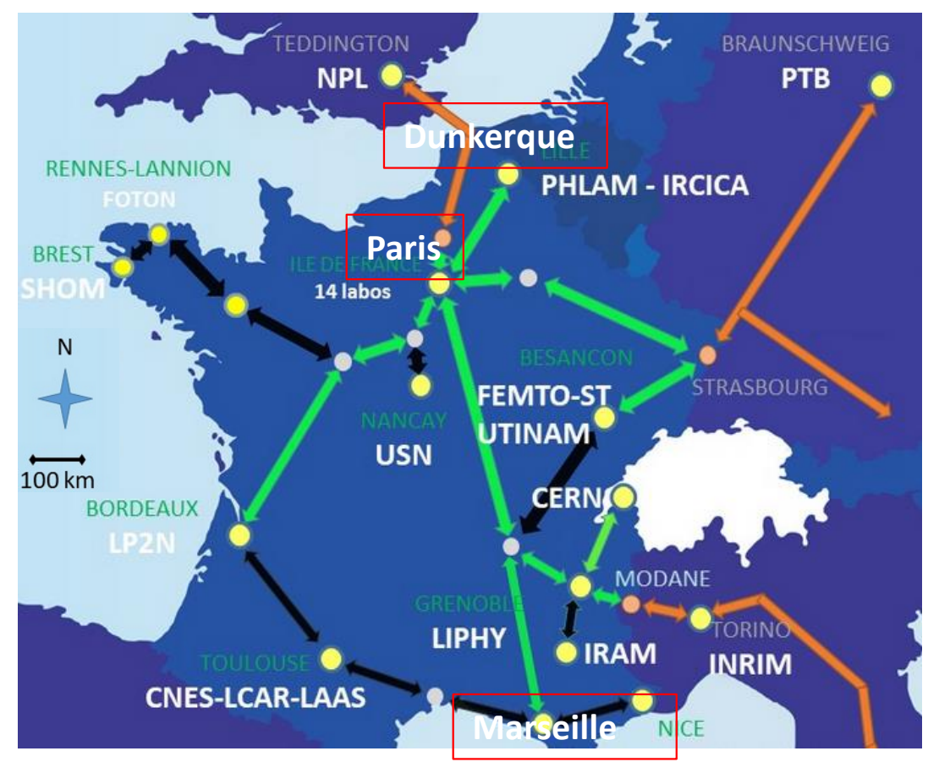
Prism	Dimensions $x\times y\times z$ (km)×(km)×(km)	Top side center (x,y) (km, km)	Top side depth (km)	Rotation angle (radians)	Density contrast (kg/m³)
1	30×5×8	(25.0, 17.5)	3.0	0	+500
2	3×3×1	(15.0, 25.0)	0.5	0	−300
3	1×20×7.5	(40.8, 25.1)	0.5	−π/4	+300
4 (sphere)	3 km radius	(5, 5)	5	0	+500





Components computed on a regular (x,y) grid on the Earth surface

Evaluation of some effects for clock comparisons



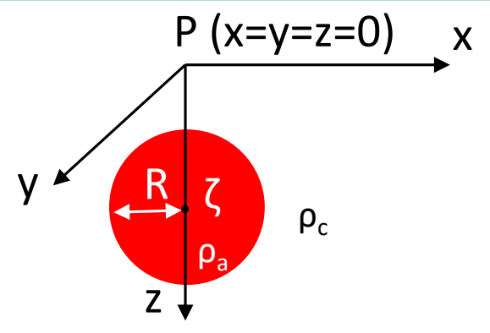
REFIMEVE+ network: operational, future, international

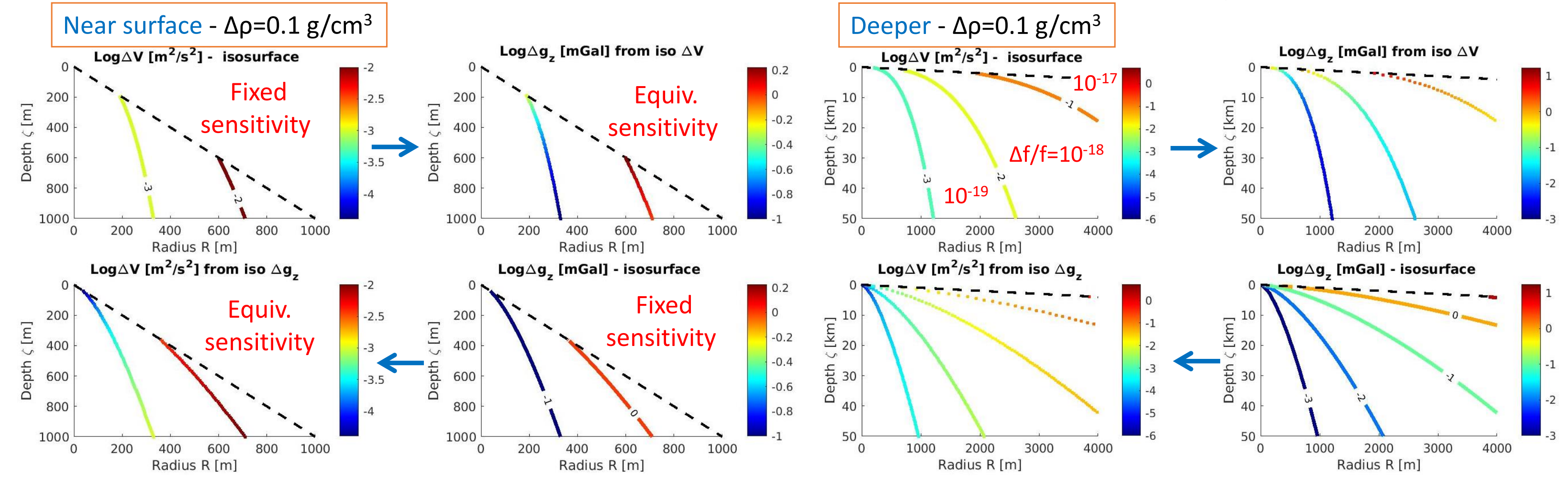
From 2023-04-19 to 2024-04-18	Paris		Dunkerque - Paris		Marseille - Paris	
	δg [μGal]	δV [m²/s²]	Δδg [μGal]	ΔδV [m²/s²]	Δδg [μGal]	ΔδV [m²/s²]
Centrifugal	2×10^6	48200	-121×10^3	-5023	219×10^3	9756
Solid Earth tide	249	6.84	14.1	0.39	43	1.17
Ocean tide loading	8.3	0.32	5.7	0.31	6.8	0.04
Polar motion	6	0.17	0.13	0.005	0.31	0.009

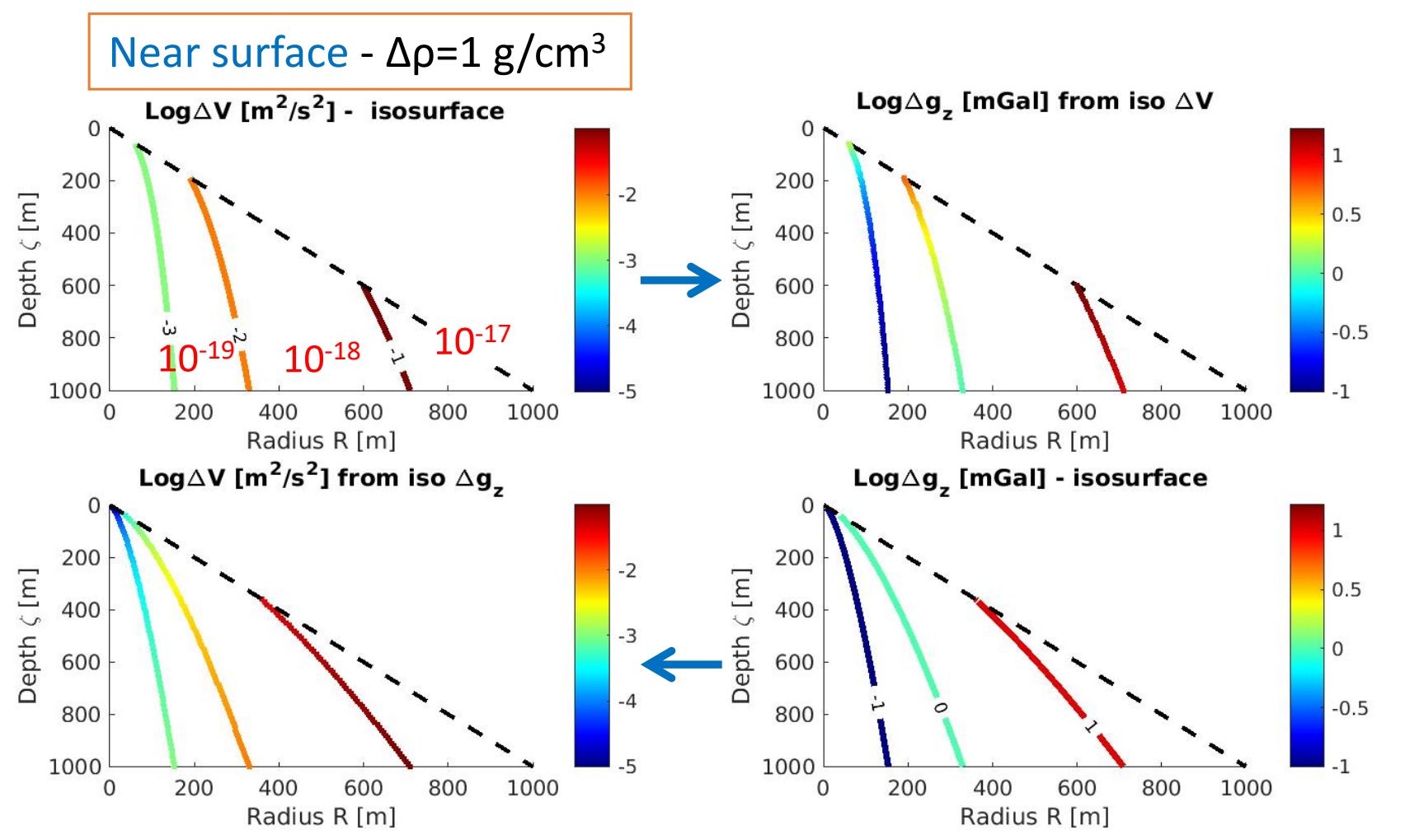
Max amplitude - Modeling from IERS conventions, Longman (1959 - upgraded), FES2014b

Sensitivity to a mass anomaly

Gravitational signal due to a buried sphere







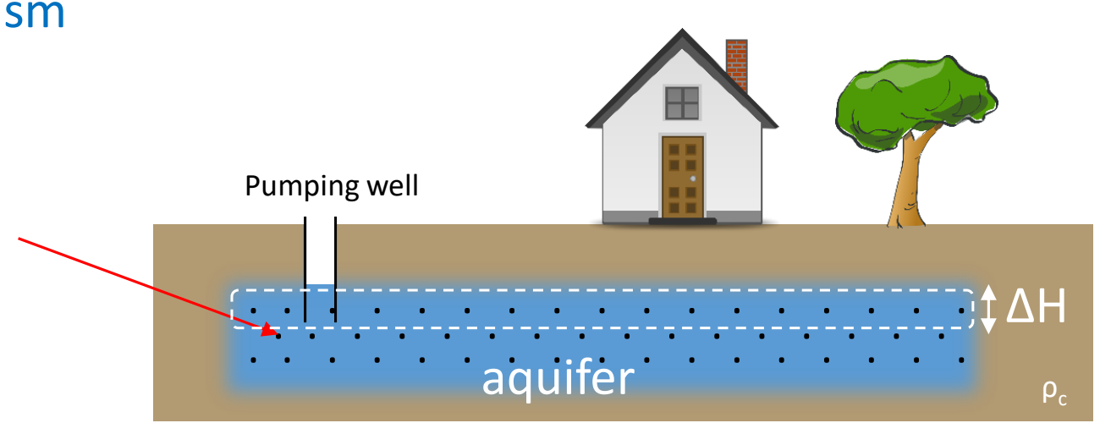
- ➔ Same signal can be generated by various anomalies at different depth, radius or density contrast
- ➔ Possible to determine what can be detected by a field clock with simple geometries
- ➔ Equivalent signal measured by clocks, gravimeters and gradiometers can be obtained
- ➔ Same work done with a right rectangular prism

Gravitational signal and vertical displacement of an aquifer

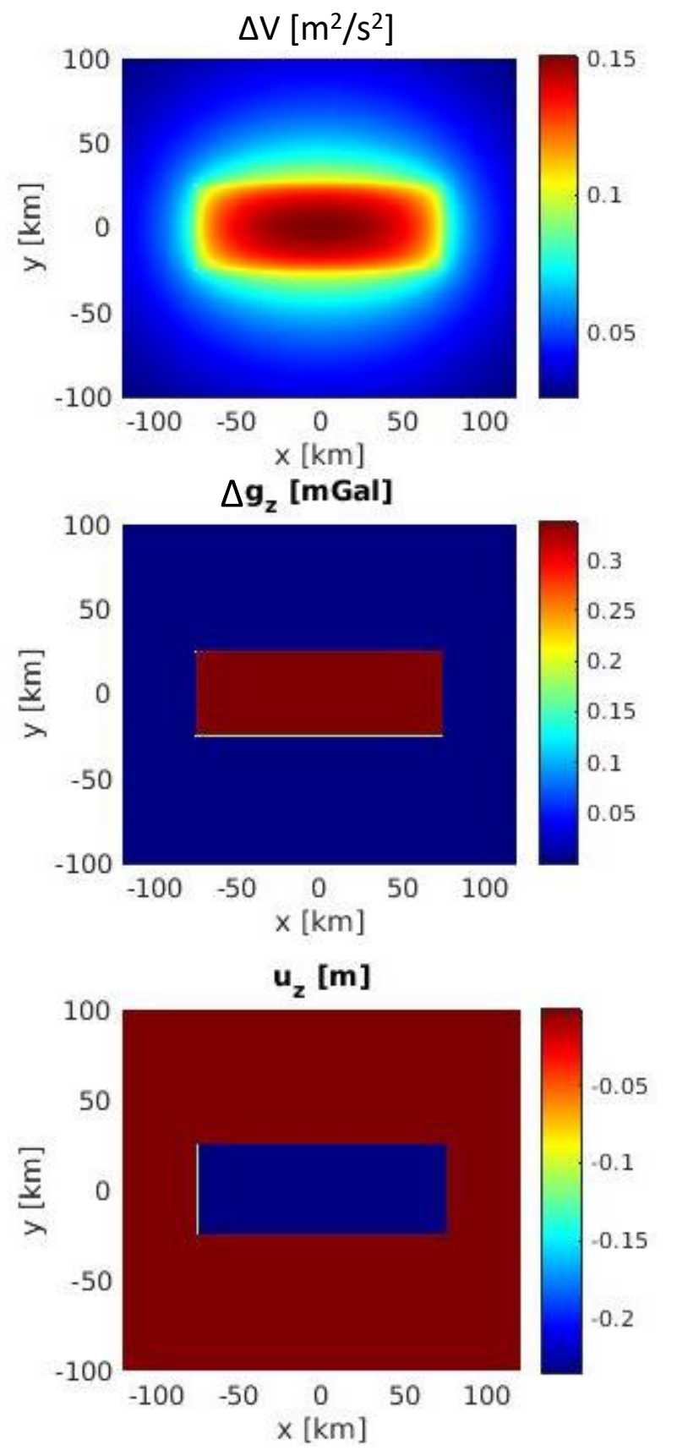
→ 3D displacement and stress fields are coded in MASS-tools

Simulation using a right rectangular prism
Dimension: 150 km x 50 km x ΔH
Depth: 30 m
ρ = 2700*0.85 kg/m³ → mean porosity

Rheological parameters
Young's modulus: E=20 GPa
Poisson's coeff: ν=0.25
Uniaxial compaction coeff: Cm=0.8/E
Pressure: $\Delta P = \rho_{water}\, g\, \Delta H$



ΔH=20 m



ΔH [m]	ΔV [m²/s²]	Δg_z [mGal]	ΔT_{zz} [E]	u_z [cm]
1	0.008	0.017	0.18	5×10^{-2}
5	0.038	0.085	0.85	1.5
10	0.075	0.17	1.8	6
20	0.15 (15 cm)	0.34	3.5	23
30	0.23	0.51	5	52

➡ Both effects can produce a geophysical signal (coherent with deformations observed by GNSSS and InSAR) in the sensitivity range of the ROYMAGE field clock

Perspectives

- ➢ Make realistic simulations with characteristic instrumental noise
- ➢ Study French aquifers to characterize vertical deformations of the levelling network
- ➢ Extend the applications to the case of volcanoes
- ➢ Invert the problem to identify anomalies with different data sources
- ➢ Operational strategy to monitor a process with field clocks around the REFIMEVE+ network and beyond

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