Test cases in moist shallow water models using compatible finite element methods

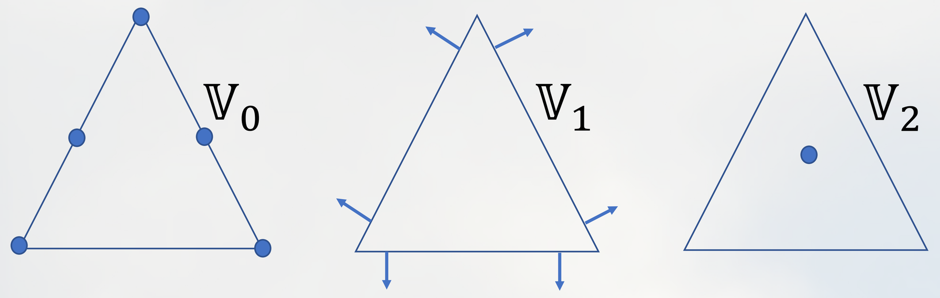
Nell Hartney and Jemma Shipton, University of Exeter, UK
Thomas Bendall, Met Office, UK
nh491@exeter.ac.uk

Background

- The shallow water equations are a widely-used simplified equation set for weather and climate modelling.
- Including moisture in the shallow water system introduces numerical complexities – new physics timescales and non-linear switch behaviour – that challenge time-stepping schemes.
- Test cases in moist shallow water models could be used to explore physics-dynamics coupling and how this is handled by time steppers.
- Aim: a suite of test cases in moist shallow water, using compatible finite elements.

Compatible Finite Elements

- The finite element method is a discretisation technique that seeks solutions in function spaces, suitable on non-orthogonal grids.
- Compatible discretisations make choices for spaces that preserve vector calculus identities and have desirable conservation and wave-propagation properties.



$$\mathbb{V}_0 \xrightarrow{\nabla^{\perp}} \mathbb{V}_1 \xrightarrow{\nabla\cdot} \mathbb{V}_2$$

We choose $u \in \mathbb{V}_1$ and h $\in \mathbb{V}_2$

- Gusto is a dynamical core toolkit, built in the Firedrake finite element library. It sets up compatible finite element spaces, and offers capabilities for different equation sets, different geometries and different time-stepping schemes.

Moist Shallow Water Equations

moist convective

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -f\hat{k}\times\boldsymbol{v} - g\nabla h$$
$$\frac{\partial h}{\partial t} + \nabla\cdot(\boldsymbol{v}h) = -\beta P$$
$$\frac{\partial Q}{\partial t} + (\boldsymbol{v}\cdot\nabla)Q = -P$$

moist thermal convective

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -f\hat{k}\times\boldsymbol{v} - b\nabla h - \frac{h}{2}\nabla b$$
$$\frac{\partial h}{\partial t} + \nabla\cdot(\boldsymbol{v}h) = -\beta_1 P$$
$$\frac{\partial b}{\partial t} + (\boldsymbol{v}\cdot\nabla)b = \beta_2 P$$
$$\frac{\partial Q}{\partial t} + \nabla\cdot(\boldsymbol{v}Q) = -P$$

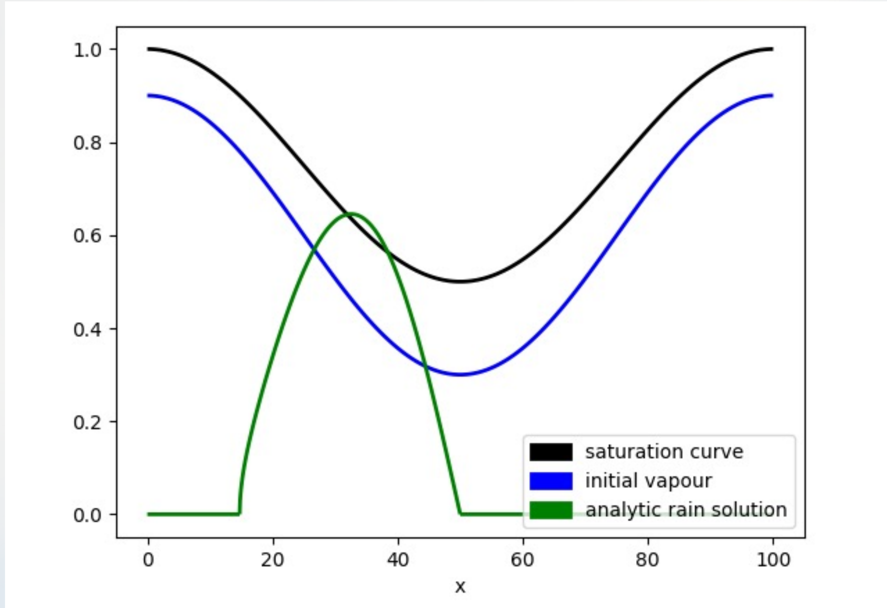
moist thermal

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -f\hat{k}\times\boldsymbol{v} - g\nabla h + \frac{g}{h}\nabla\left(\frac{1}{2}h^2\theta\right)$$
$$\frac{\partial h}{\partial t} + \nabla\cdot(\boldsymbol{v}h) = 0$$
$$\frac{\partial \theta}{\partial t} + (\boldsymbol{v}\cdot\nabla)\theta = S_\theta$$
$$\frac{\partial q^{(k)}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\, q^{(k)} = S_q^{(k)}$$

Our implementation of these equations sets in Gusto involves extending the compatible discretisation to include a DG space for the buoyancy and/or moisture fields.

Test 1: 1D Forced Advection

- Tests the moist physics capability.
- 1D transport of water vapour v_m by a constant velocity u_0. Where the vapour exceeds a saturation function it is converted to rain, which can be compared to an analytic solution.



$$\frac{\partial v_m}{\partial t} + u_0\frac{\partial v_m}{\partial x} = S$$

Figure 1: Saturation curve, initial water vapour profile and analytic rain solution for a forced advection test. The vapour profile is advected through the saturation profile to produce the rain solution.

Test 2: Reversible Moist Advection

- Tests the advective component of the model with moist physics.
- A cosine bell is advected around the sphere, as in the first test of the Williamson *et al.* test suite. A prescribed saturation function causes conversions between water vapour and cloud and is designed so that the initial water vapour is recovered by the final timestep.

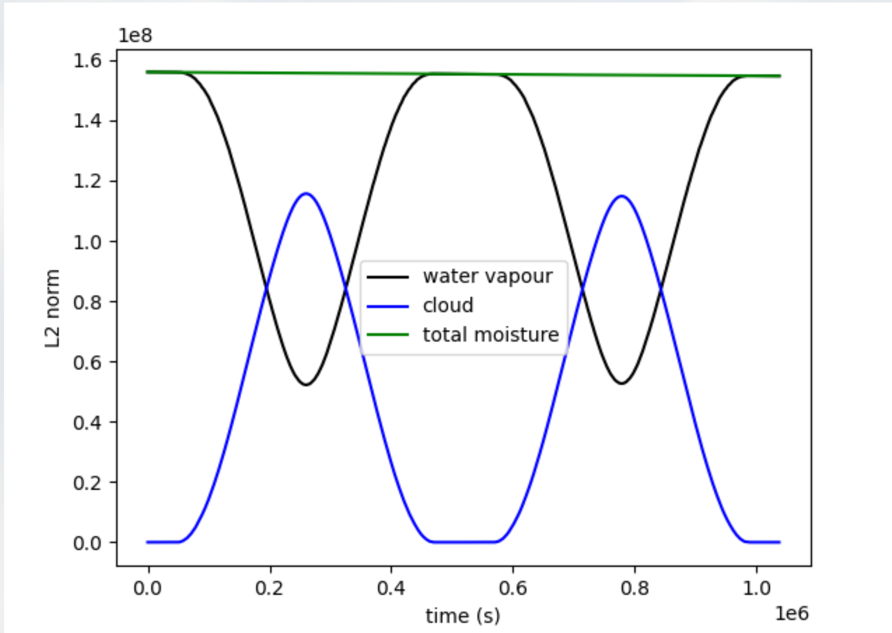


Figure 2: Results of the reversible moist advection test. An initial vapour cosine bell is converted between vapour and cloud as it is advected around the sphere, through a prescribed saturation curve that varies with latitude. The test aims to measure how well the total moisture is conserved in time.

Test 3: Solid Body Rotation

- Tests the ability of the full model to maintain a steady state.
- Zonally balanced flow on a sphere from Zerroukat and Allen.
- Modifies test 2 of Williamson *et al.* to reflect the extra terms in the system, by adding balanced initial conditions for the buoyancy field *b* and the moisture field *q*.

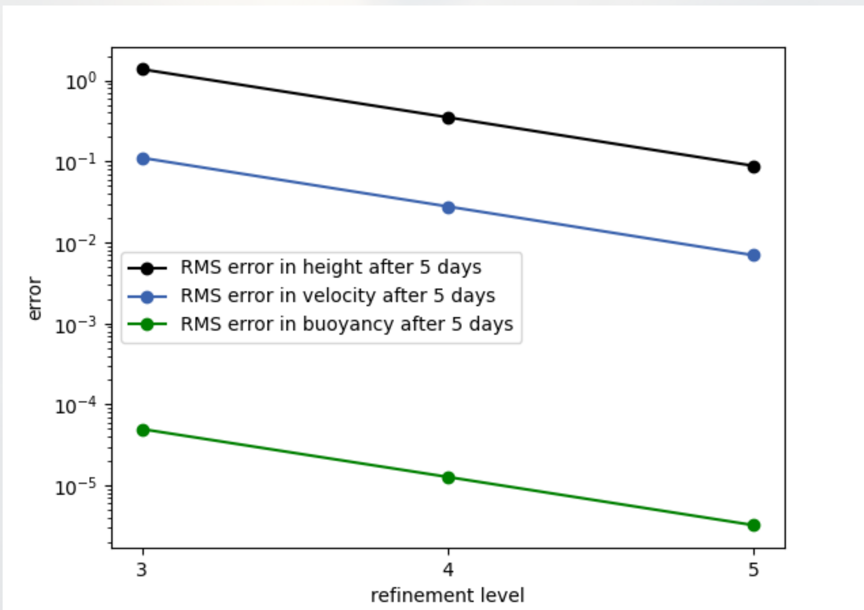


Figure 3: Root mean squared error for the thermal solid body rotation test after 5 days for the height, velocity and buoyancy fields, at three different resolutions. The test aims to measure how well the initial conditions are maintained.

Test 4: Moist Flow Over Orography

- Tests the ability of the full model to produce cloud.
- Vapour is initialised everywhere close to saturation, in the style of the test from Zerroukat and Allen.
- The height-dependent saturation function causes cloud production near the mountain.

saturation = $q_0 e^{-\alpha\frac{h}{H}}$
where q_0, α are constants and H is the constant background height

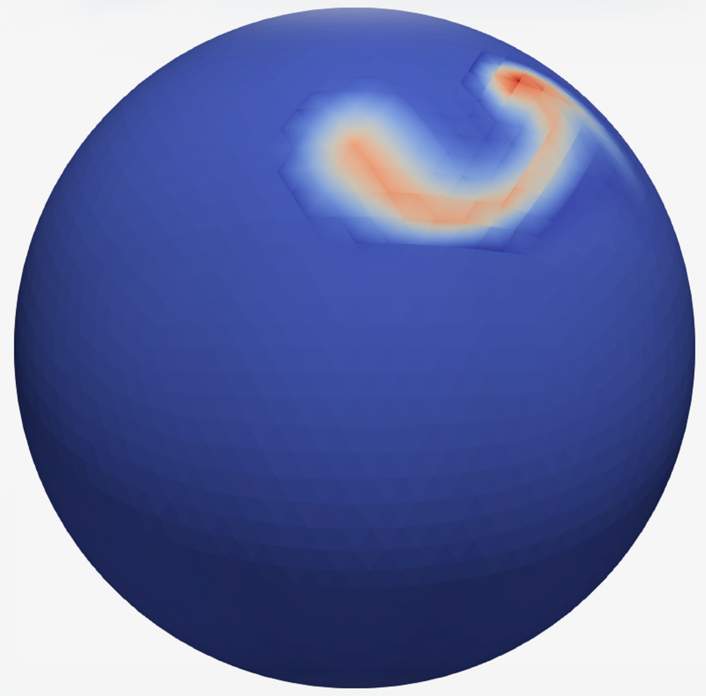


Figure 4: A snapshot of the cloud field during the moist flow over a mountain test. Cloud is generated near the mountain and is then transported around the globe by the flow.

Next Steps

- What can we learn about time-stepping with physics and physics-dynamics coupling from test cases in moist shallow water?
- Test cases that are closer to real-world dynamics.

References

Cotter, C.J. and Shipton, J., 2012. Mixed finite elements for numerical weather prediction. *J. Comput. Phys.*, *231*(21)
Bouchut, F., Lambaerts, J., Lapeyre, G. and Zeitlin, V., 2009. Fronts and nonlinear waves in a simplified shallow-water model of the atmosphere with moisture and convection. *Phys. Fluids*, 21(11)
Kurganov, A., Liu, Y. and Zeitlin, V., 2020. Moist-convective thermal rotating shallow water model. *Phys. Fluids*, 32(6)
Zerroukat, M and Allen, T., 2015. A moist Boussinesq shallow water equations set for testing atmospheric models. *J. Comput. Phys.*, 290(201)
Williamson, D., Drake, J., Hack, J., Rudiger, J. and Swarztrauber, P., 1992. A standard test for numerical approximations to the shallow water equations in spherical geometry. *J. Comput. Phys.*, 102(1)

Sharing is encouraged

This presentation participates in OSPP
Outstanding Student & PhD candidate Presentation contest

Use this QR code to read the abstract for this poster.