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ITS2.1/NP0.4: Tipping points, resilience,
and stochasticity in the Earth's climate and ecosystems

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The deterministic excitation paradigm, with application to the glacial-interglacial transitions

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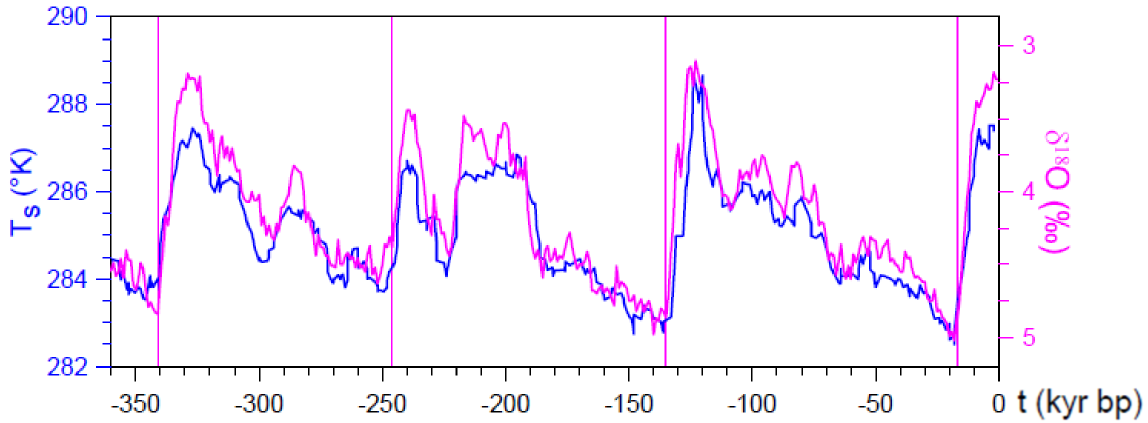
Introduction

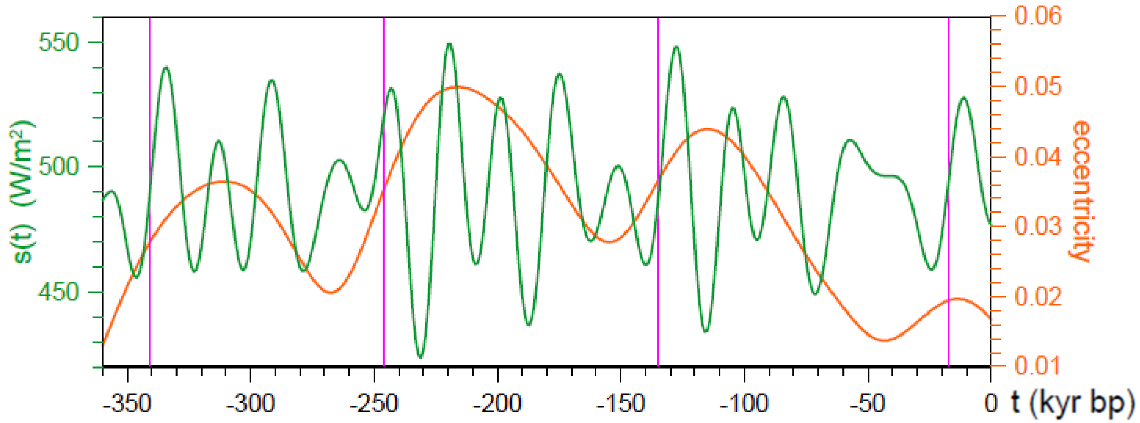
Oscillations of the climate system lasting about 100 kyr, occurred after the Mid-Pleistocene Transition, have been revealed by proxy data. They evidence strong changes in various parameters and are all composed of a **long glacial state**, an **abrupt shift to a brief interglacial state** and a **slow return to a new glacial state**

Milutin Milankovitch hypothesized a century ago that the glacial-interglacial transitions were **paced by an increased solar radiation received in the summer in the northern hemisphere** due to the orbital forcing.



Magenta line: **LR04 benthic $\delta^{18}O$ (1) stack** constructed by the graphic correlation of 57 globally distributed benthic $\delta^{18}O$ records (from Lisiecki and Raymo, 2005; the magenta vertical lines mark the onset of the abrupt glacial-interglacial transitions). Blue line: **global surface temperature** estimate (from Hansen et al., 2013).

(1) benthic $\delta^{18}O$ of ocean sediments is a proxy of global ice volume and deep ocean temperature change



Mean daily insolation at 65°N on summer solstice (green line) and **Earth's orbital eccentricity** (orange line; both from Laskar et al., 2011). For the definition of the magenta vertical lines see the figure on the left.

However, **such changes in insolation are by far too weak** to account for the high interglacial mean temperatures, etc.. They must therefore **trigger nonlinear positive feedbacks** that speed up deglaciation

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Introduction

“… ***the ice ages problem*** *is effectively divided into* ***two separate sub problems****:*

- *the first is* ***explaining the phase or timing of the cycles****, and*
- *the second is* ***finding the physical mechanism that gives rise to these cycles*** *…”*

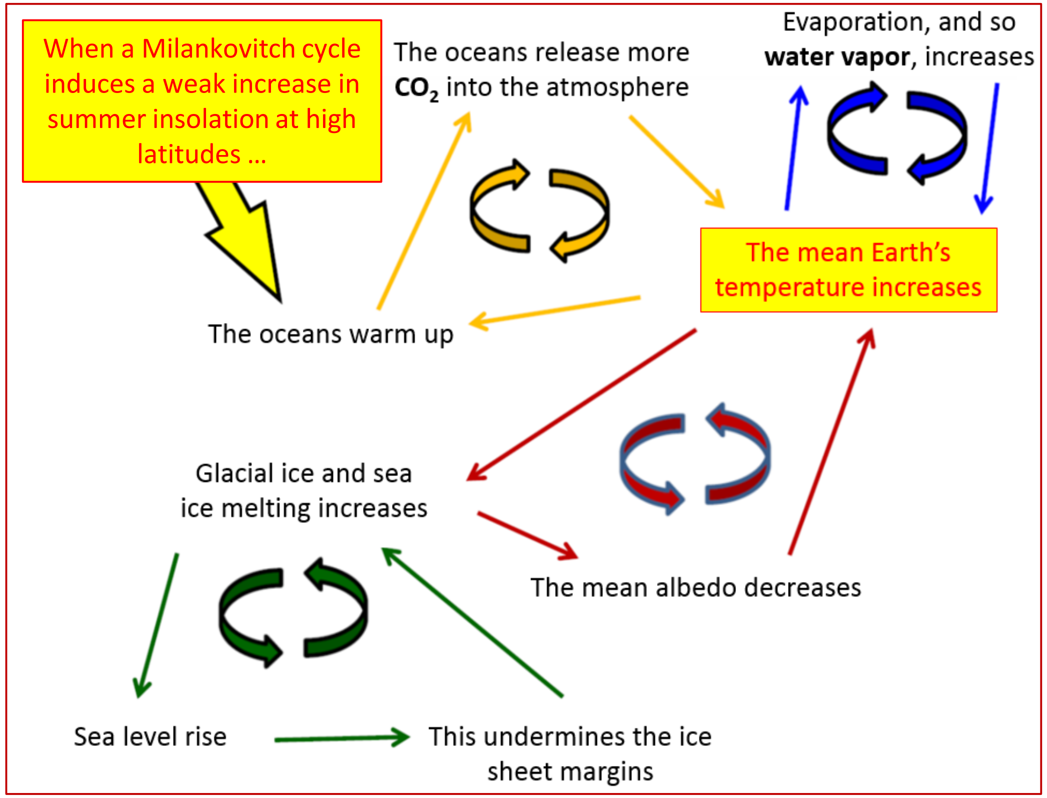
(Tziperman et al., 2006, *Paleoceanogr*. 21, PA4206, https://doi.org/10.1029/2005PA001241)

This presentation deals with the **first sub problem**

(Pierini, 2023, *Chaos* 33, 033108, https://aip.scitation.org/doi/10.1063/5.0127715,
press release: https://publishing.aip.org/publications/latest-content/elegantly-modeling-earths-abrupt-glacial-transitions/)

Feedbacks

Thus, in any **glacial-cycle model** such feedbacks must be
- **simulated**
- **parameterized**



When a Milankovitch cycle induces a weak increase in summer insolation at high latitudes …

The oceans release more CO_2 into the atmosphere

Evaporation, and so **water vapor**, increases

The mean Earth's temperature increases

The oceans warm up

Glacial ice and sea ice melting increases

The mean albedo decreases

Sea level rise

This undermines the ice sheet margins

Threshold crossing rules

Several **deterministic conceptual models*** subjected to the orbital forcing, in which specific **threshold crossing rules** are prescribed, provide a substantially correct timing of the glacial-interglacial transitions

However, **those rules are often not appropriately characterized in terms of dynamical systems theory** and **are overshadowed by the technical details** of the model study

Here, a dynamical paradigm denoted ***deterministic excitation*** (*DE*) is formulated on the basis of some **general dynamical systems properties**

Its application to a conceptual model forced by realistic orbital forcing **characterizes in the simplest possible way the pacing of the glacial terminations by the orbital forcing** implied by the Milankovitch hypothesis

* *e.g., Calder, 1974; Imbrie et al., 1993; Paillard, 1998, 2001; Tziperman and Gildor, 2003; Huybers, 2007; Dietlevsen, 2009; Imbrie et al., 2011; 2012; Tzedakis et al., 2017; Berends et al., 2021*

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Excitable relaxation oscillations

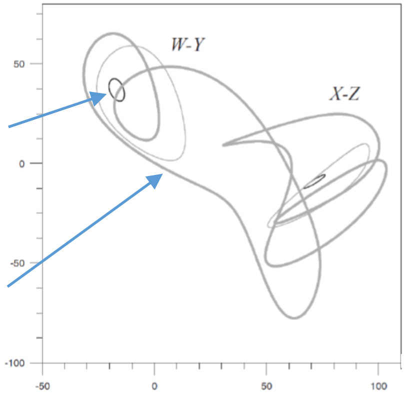
Example adapted from Pierini
(*J. Phys. Oceanogr*. 40, 238-248, 2011;
Phys. Rev. E 85, 027101, 2012)

This system has 4 degrees of freedom: (*W*,*X*,*Y*,*Z*)

autonomous system

subcritical limit cycle

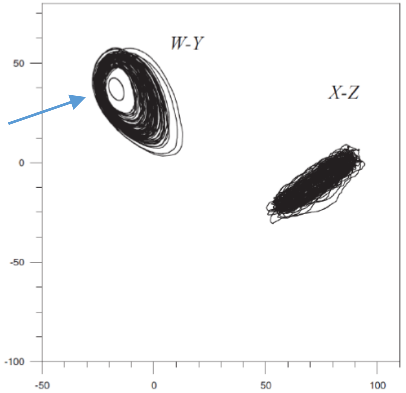
supercritical relaxation oscillation



perturbed **excitable* system,** no relaxation oscillations

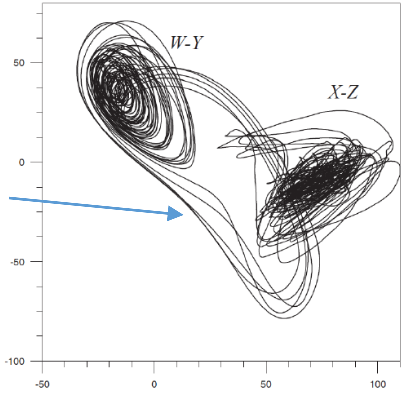
**subcritical*

subcritical limit cycle subjected to an Ornstein-Uhlenbeck process



perturbed excitable system, **excited relaxation oscillations**

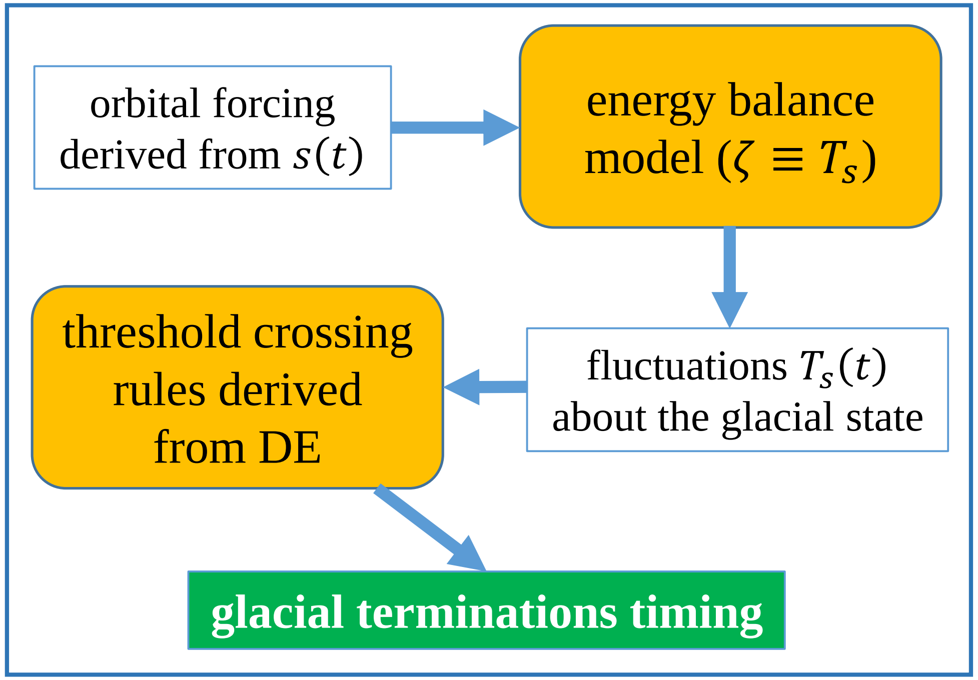
excited relaxation oscillations



The Deterministic Excitation paradigm

- The system is **monostable**, **excitable**, and possesses **relaxation oscillations** (ROs);
- in order to be excited, a RO requires that a given control parameter ζ crosses a given tipping point $\bar{\zeta}$;
- the temporal distance between two successive excitations must be greater than the typical time scale τ_{ro} of the RO.

The conceptual model



orbital forcing derived from $s(t)$

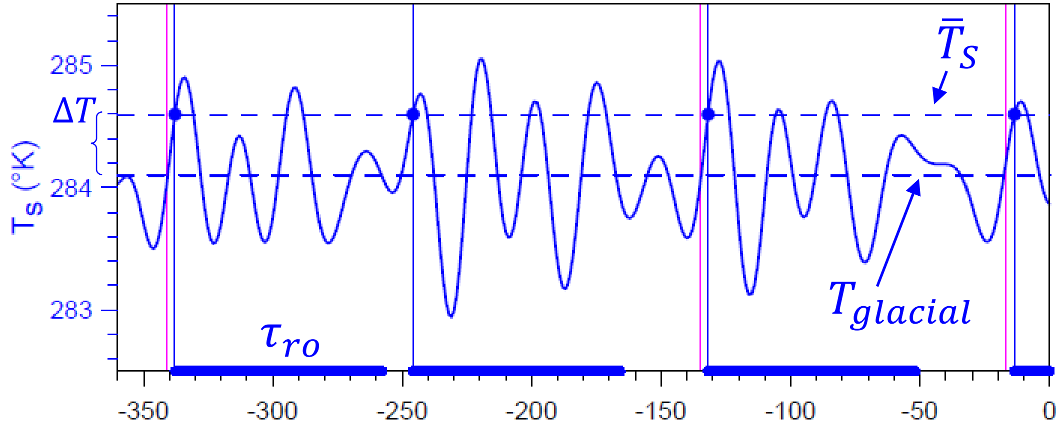
energy balance model ($\zeta \equiv T_s$)

fluctuations $T_s(t)$ about the glacial state

threshold crossing rules derived from DE

glacial terminations timing

Results: correct timing



Timing of the simulated glacial terminations (vertical blue lines) vs. timing derived from proxy data (vertical magenta lines)

Red line: number of simulated glacial termination times as a function of ΔT, or τ_{ro} or the forcing amplitude δ (all the other parameters being unchanged)

Blue line: number of simulated termination times that fit one of the proxy times

The cyan bars/yellow boxes show the ranges in which all (four) the simulated transition times fit all (four) the real transition times, thus yielding a timing that is basically the same as the one shown in the figure above

This sensitivity analysis insures the robustness of the simulated timing

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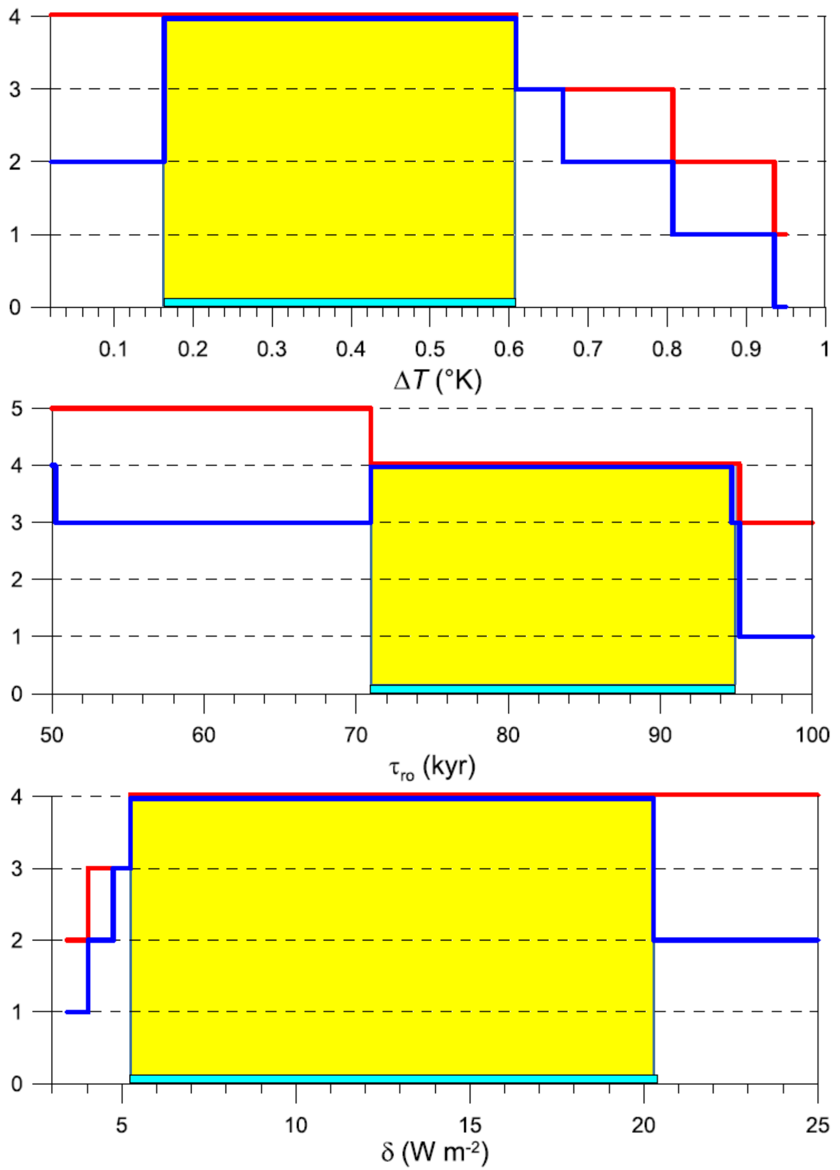
Results: sensitivity experiments



Bistability

A classical paradigm in climate dynamics is **multistability**. In such a context, the **stochastic resonance** mechanism (SR, e.g., Benzi et al. 1982; Nicolis 1982) was proposed in an attempt to explain the glacial-interglacial variability in a **bistable system**.

Noise and a **periodic forcing** cooperate in such a way that glacial↔interglacial transitions occur with the same period of the forcing. The eccentricity $e(t)$ was invoked as the orbital forcing. **However:**

- e has a minor effect on the insolation at high northern latitudes;
- e is not periodic (it yields spectral lines at 95, 100, 123, 131 kyr), but periodicity is required for SR to be applied;
- the noise-switching time matching condition required by SR can hardly be verified;
- in SR the transitions are symmetric, in sharp contrast with the saw-tooth shaped proxy signals.

In conclusion, **SR turns out to be inapplicable** to the glacial-interglacial variability problem.

More in general, **bistability is unlikely to be the correct dynamical paradigm** for this climate phenomenon.

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On the Milankovitch paradox

Deterministic excitation suggests how to possibly deal with the so-called Milankovitch paradox, which points to the presence of a 100 kyr spectral peak in paleorecords despite the absence of an analogous peak in the orbital forcing.

Intrinsic ROs describing a glacial cycle with relaxation time ~70−95 kyr are assumed to exist in a monostable excitable climate system (significant examples are available in the literature).

Moreover, the small temporal scales (20 kyr) of the climatic precession -as modulated by both obliquity and eccentricity- may trigger a RO only few tens of kyr after the end of a previous RO.

This makes the temporal distance between two successive glacial terminations to be around, or just above 100 kyr.

Therefore, according to the DE paradigm, **the highest frequency components of the orbital forcing** included in the climatic precession **would give an essential contribution to the observed 100 kyr cycles** of the late Pleistocene.

Conclusions

- A dynamical systems paradigm denoted **deterministic excitation** is formulated. It applies to a **monostable excitable system** in which relaxation oscillations are triggered by a deterministic external forcing if specific tipping points are crossed.

- Such paradigm has been **successfully applied to the last four glacial-interglacial transitions**. A timing of the glacial terminations in very good agreement with proxy records is obtained; moreover, a sensitivity analysis insures the robustness of the simulated timing. As a result, the Milankovitch hypothesis is now simply and appropriately characterized in terms of dynamical systems theory.

- Deterministic excitation applies to excitable systems, in which **only one stable state** plays a crucial role in the transitions. This is in contrast with other classical theories based on bistable systems. In general, **excitable dynamics appears to be the most appropriate dynamical framework within which several cases of abrupt climate changes could be explained**.

- An extended application to *all* the glacial terminations that occurred after the MPT is in progress.