Size Distributions Reveal Regime Transition of Dominant Driving Force in Lake Systems

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1. Research Background

Significance: The power law size distribution discovered in natural lakes indicates that lakes are complex systems with characteristics of scale-invariance, critical tipping and self-organization.

Knowledge gaps:

Q1： Power law distribution is not sufficient to describe the whole size range of lakes, with deviations at both ends. What is the full size distribution of lakes?

Q2： What is the driving mechanism underlying the power law distribution observed in lake systems?

2. Methodology

2.1 Statistical analysis

- Power law distribution

$$p(X \geq x) = \left(\frac{x}{x_{min}}\right)^{-b}$$ (Clauset et al., 2009)

$\geq x_{min}$ power law

$< x_{min}$ exponential $p(x) = C_1 e^{-c_1 x}$

stretched-exponential $p(x) = C_2 x^{\beta-1} e^{-c_2 x^{\beta}}$

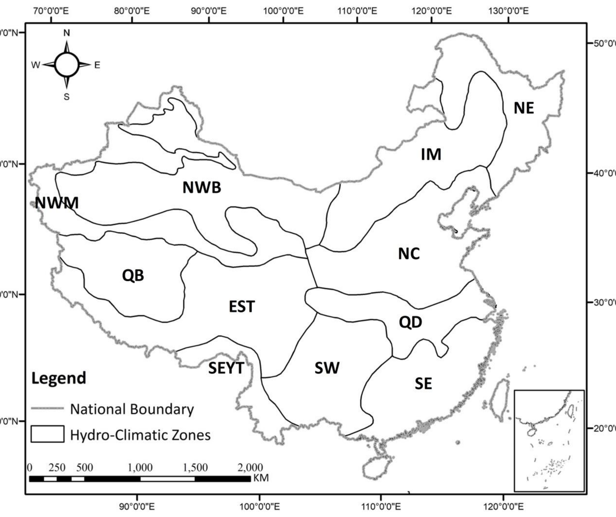


Fig. 1 Study sites: 11 hydro-climatic zones in China.

2.2 Phase properties quantification

- Coefficient of variation: $CV = {}^{\sigma}/_{\mu}$ (Loreau & Mazancourt, 2013)
- Entropy: $AE = \frac{1}{N}[-\sum_{i=1} n_i p_i(x) \log_2 p_i(x)]$ (Lesne, 2014)

2.3 Power spectrum analysis

- Power spectrum: $S(\varphi) = \int dx \frac{xp(x)}{1+(\varphi x)^2} \propto \varphi^{\gamma} \quad (\gamma = -3 + b)$ (Bak et al., 1987)
- Hurst exponent: $H = \frac{\gamma-1}{2} + 1$ (Womell, 1993)
- Autocorrelation: $\gamma > 0, \quad R(\tau) \approx \frac{1}{\pi\tau}\varphi^{\gamma} \sin(\varphi\tau)|_{\varphi \to +\infty}(diverge)$ (Kubo&Hashitsume, 2012)

2.4 Isolation of driving factors and processes

- Potential lake simulation: $TWI = \log({}^{AS}/_{tan\beta})$ (Hu et al., 2017)
- Comparison between observed and simulated lakes & satellite images
- Driving forces:

  Endogenic force: topography and topographic processes

  Exogenic force: other factors and non-topographic processes

3. Results

3.1 Full size distribution of lakes —— [Fig. 2]

- Three components and three phases

  Shoulder component: stretched-exponential or exponential distribution

  Body and tail components: power law distributions

3.2 Phase transition and dominant driving force change —— [Fig. 3]

- Power law : heterogeneous and ordered; endogenic force dominated
- Exponential: random; exogenic force dominated
- Stretched-exponential: homogeneous and disordered; joint control

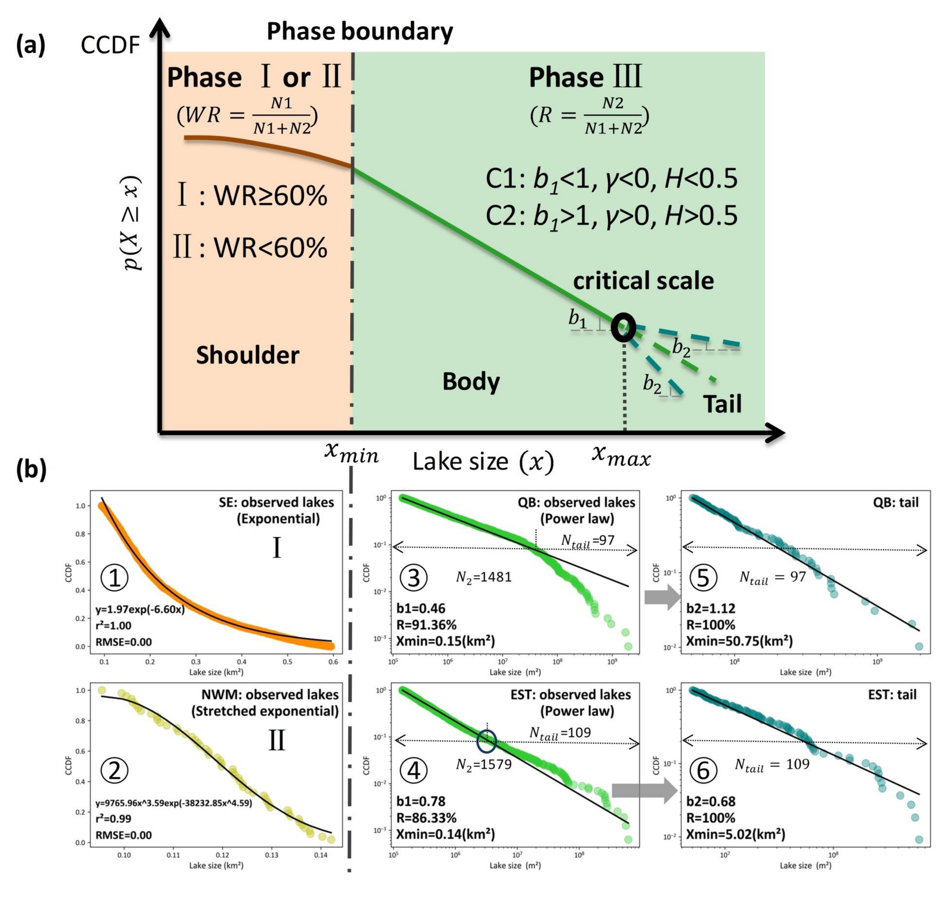
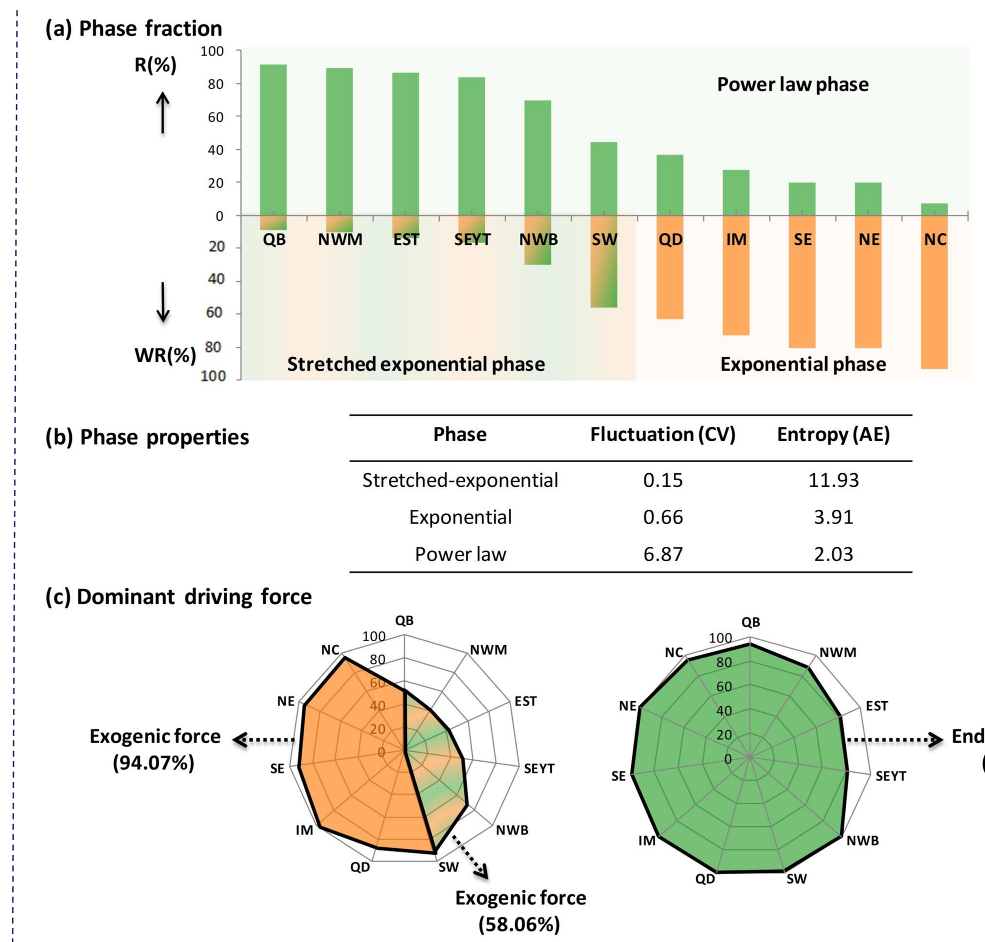


Fig. 2 Lake size distribution consists of three phases featured by distinct distribution functions.



Phase	Fluctuation (CV)	Entropy (AE)
Stretched-exponential	0.15	11.93
Exponential	0.66	3.91
Power law	6.87	2.03

Fig. 3 Lake size distribution phases (a), its properties (b) and dominant driving forces (c).

3.3 Power law phase and system resilience

- "Blue" shift in size power spectrum
- Autocorrelation diverges
- Reduced system resilience

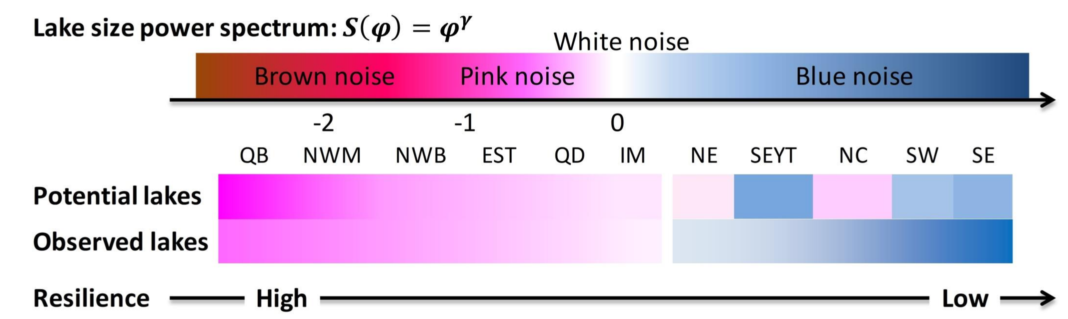


Fig. 4 Comparison of size power spectrum characteristics between observed and potential lakes.

3.4 Conceptual model

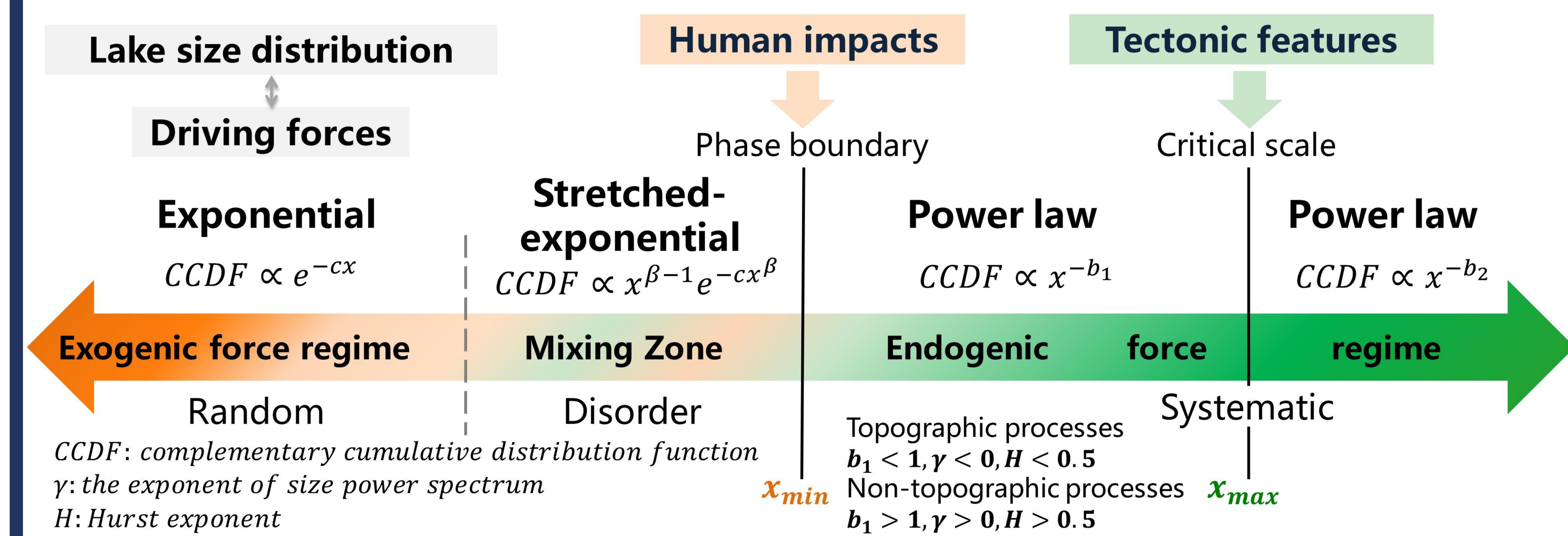


Fig. 5 A conceptual model of the driving mechanism underlying lake size distribution.

4. Conclusions

- Full size distribution of lakes contains exponential, stretched-exponential and power law phases with successively increasing degrees of heterogeneity and orderliness.
- Dominant driving force changed from endogenic to exogenic shifts the size distribution from power law to stretched-exponential and further to exponential.
- Exogenic force tends to shrink the power law phase and increase its scaling exponent, leading to the loss of resilience in lake systems.