Optimal Irrigation Water Allocation among Different Growth Stages

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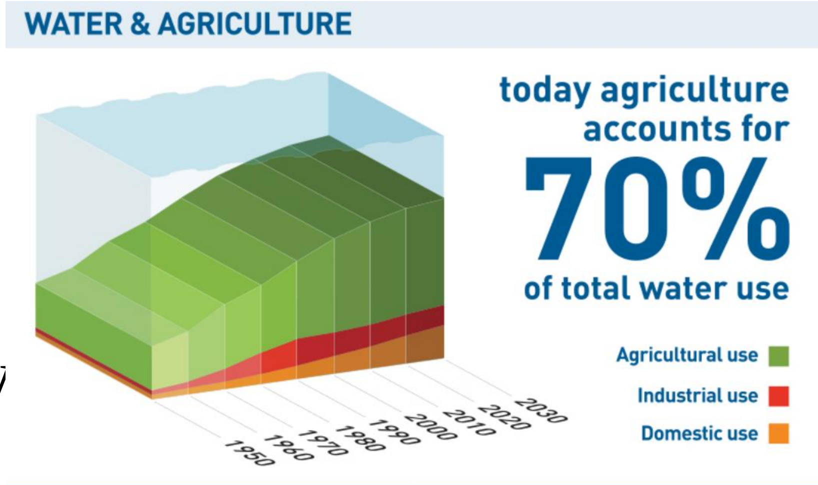
Outline

- Agricultural water use about 70% of global water use

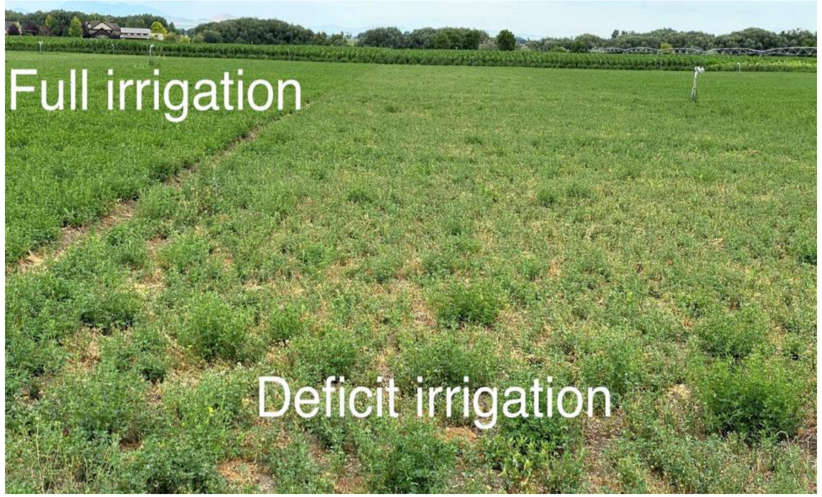
- Agricultural is both the major cause and the casualty of water shortage

- Irrigation is not only a matter of quantity but requires scheduling

- The crops withered away in any time, supplying more water later is never able to save them



FAOWATER www.fao.org/nr/water



https://extension.usu.edu/crops/research/strategies-for-deficit-irrigation-of-forage-cr

- Many studies often simply convert the total water volume directly into the yield as the total benefit of irrigation
- To solve this problem, we uses the concept of agricultural water use and two-stage dynamic operation
- This study aims to explore the dynamic decision-making of irrigation schedules with the consideration of uncertainty under water shortage.

Yield calculation

The relationship between crop yield and water use was been addressed by FAO-33

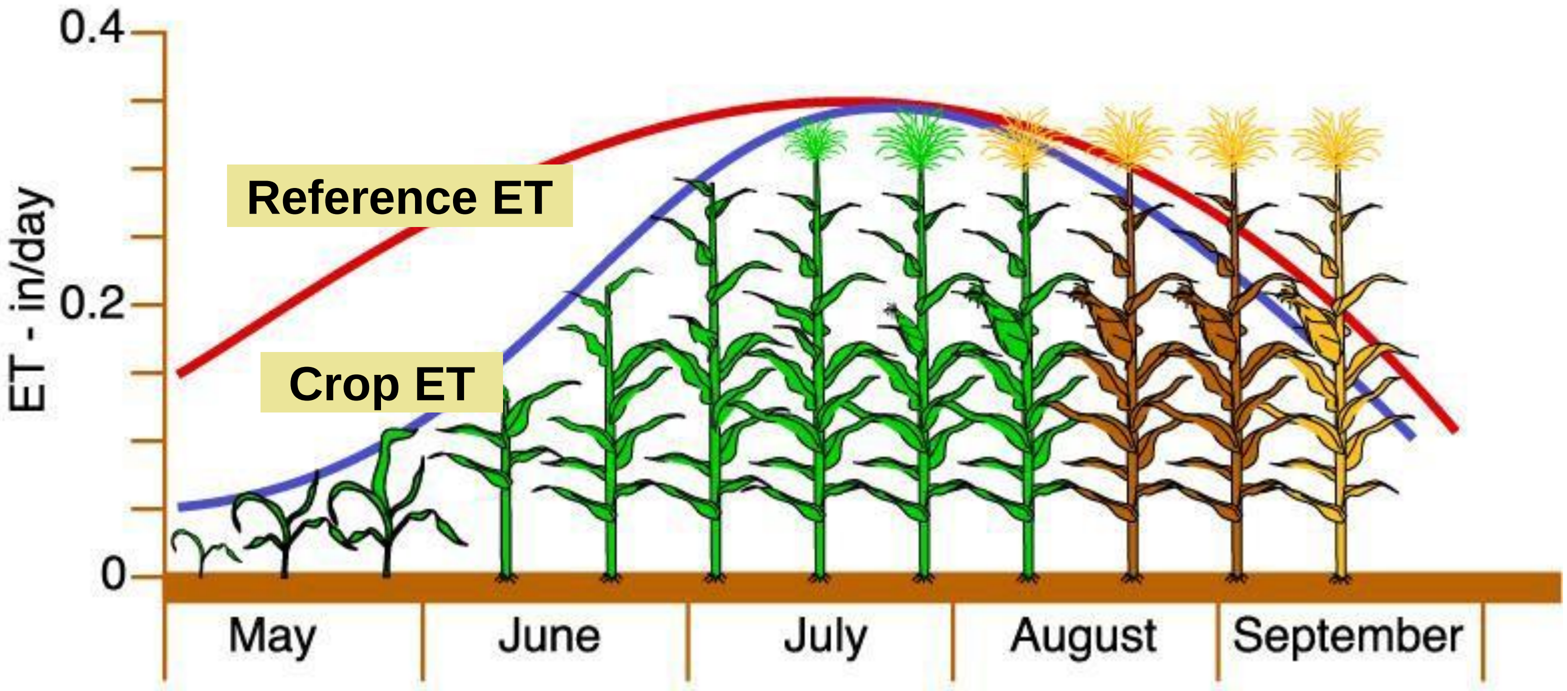
$$\left(1-\frac{\mathrm{Y}}{Y_{max}}\right)=K_y\left(1-\frac{ET_a}{ET_p}\right)=K_y\left(1-\frac{R+S}{ET_p}\right)$$

where Y_{max} and Y are the maximum and actual yields,
ET_p and ET_a are the maximum and actual evapotranspiration,
R is the rainfall and S is the irrigation water supply
K_y is a yield response factor

Yield calculation

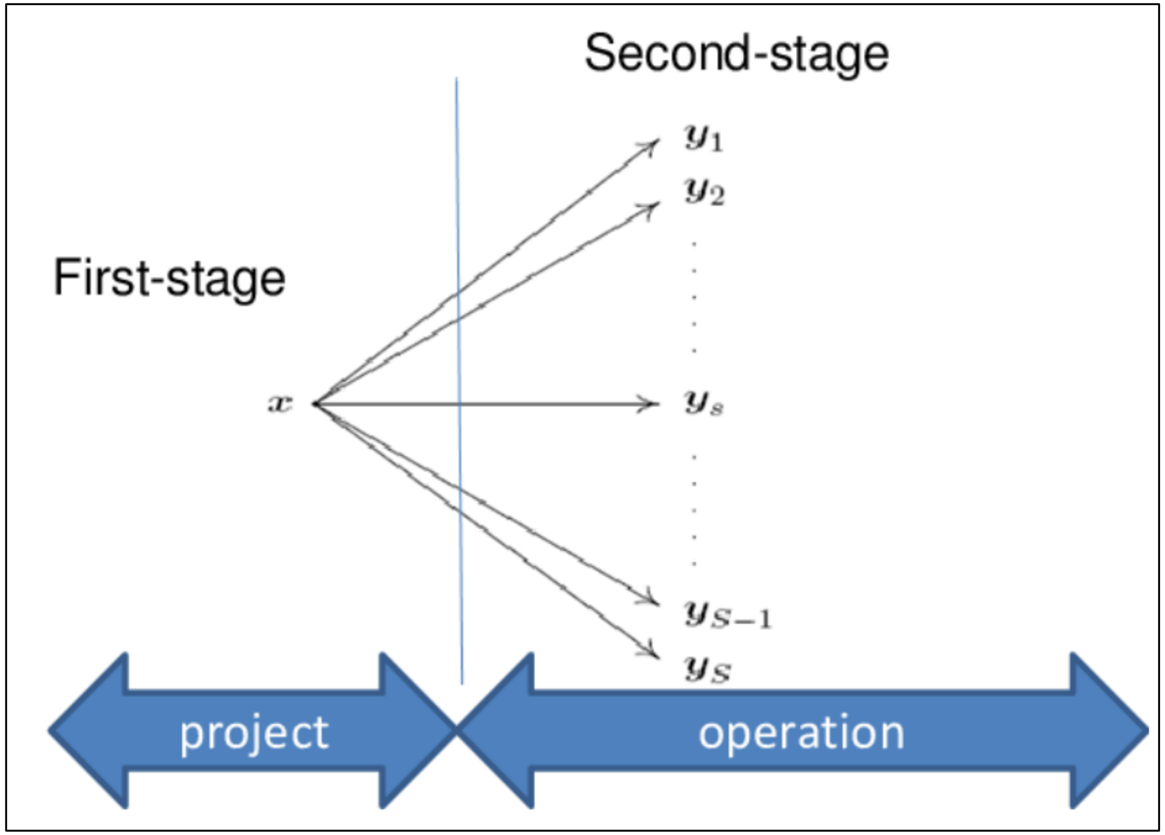
- Quantity, timing and scheduling are also important, as crops require different amounts of water at different growth stages

- In the crop growth cycle, the surviving number will only be maintained or decreased over time, and it cannot be reviviscence



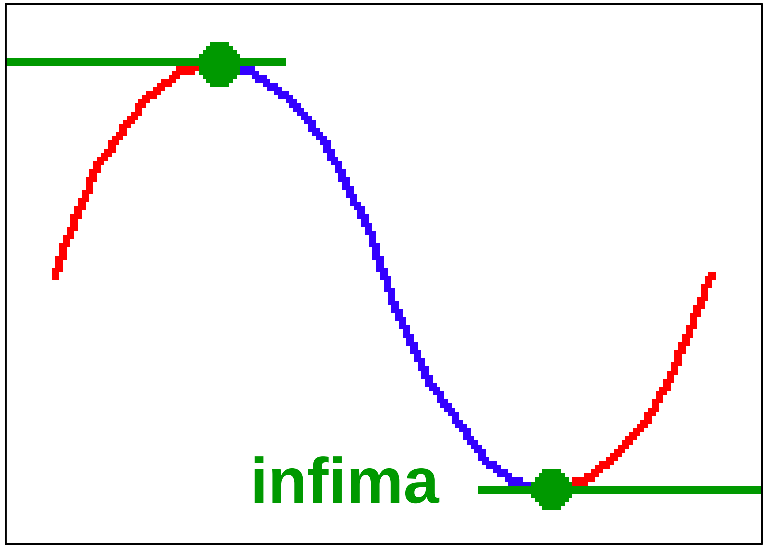
Two-stage dynamic programming

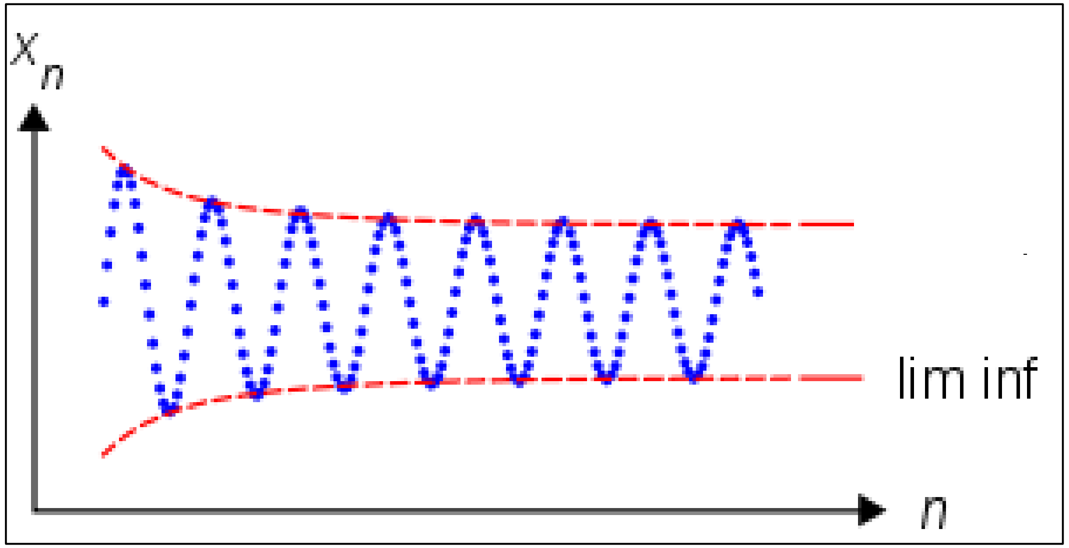
- More accurately analyze the allocation of agricultural water use across time
- Make decisions based on the data available at the current time
- Focuses on two-stage problems, and will extend to more stages in the future



The concept of max-min

- Since the structure of FAO33, we will use the concept of max-inf
- Infima is the lower limit of the function, and can also be expressed as the lower bound of the function limit
- Find the minima yield in all stages and maximize it





Yield expectation calculation

- Combining the concepts of two-stage and max-inf, if only consider the total yield of two stages can be expressed as:

$$Y_a = max(inf(Y_1, Y_2))$$

- Taking into account the uncertainty of rainfall in the second stage, we maximize yield expectation:

$$E(Y_a) = \left[1 - K_{y1}\left(1 - \frac{R_1 + S_1}{ET_{p1}}\right)\right] * Y_{max} * P_\alpha + \int_0^{R_o} \left[1 - K_{y2}\left(1 - \frac{R_2 + S_2}{ET_{p2}}\right)\right] * Y_{max} * f(R_2)d(R_2)$$

$Y_1 < Y_2, P_\alpha$

$Y_1 > Y_2, 1 - P_\alpha$

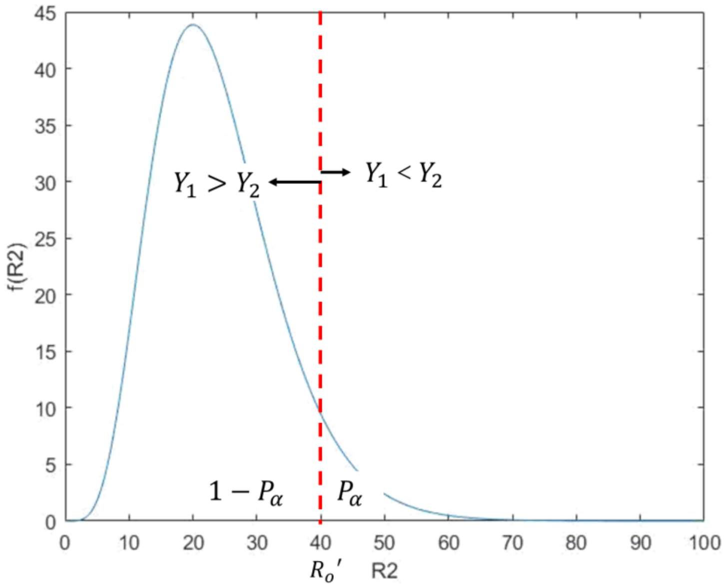
Decision point of which stage dominates

The decision point is under the condition of $Y_1 = Y_2$, we assume that $R_2 = R_o$, and R_o can be expressed as the following:

$$R_o = ET_{p2} * \left[-\frac{K_{y1}}{K_{y2}} * \left(1 - \frac{R_1 + S_1}{ET_{p1}} \right) + 1 \right] - (S_a - S_1)$$

at the same time

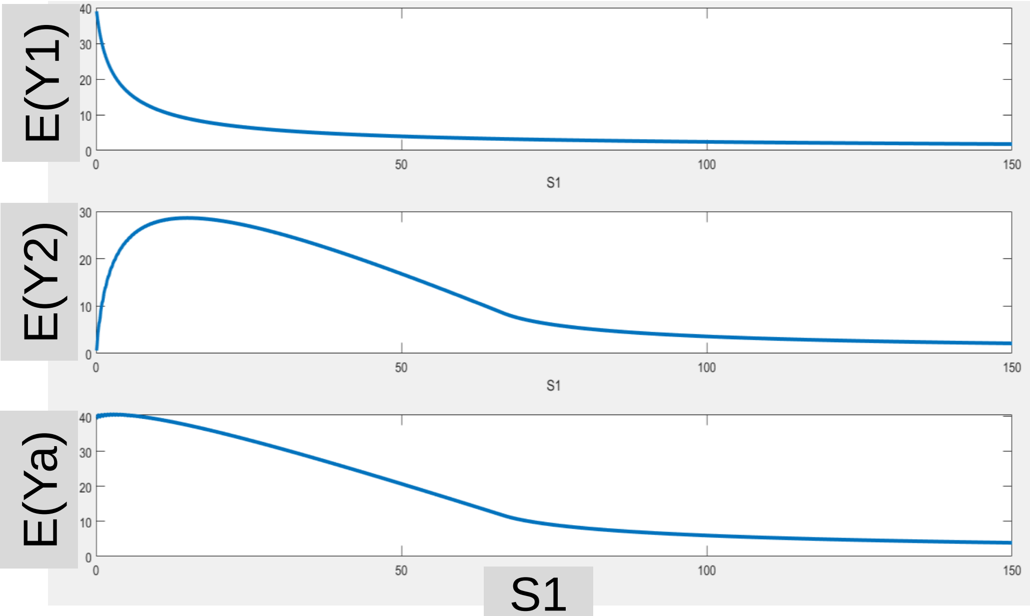
$$P_\alpha = 1 - \int_0^{R_0} f(R_2) d(R_2)$$

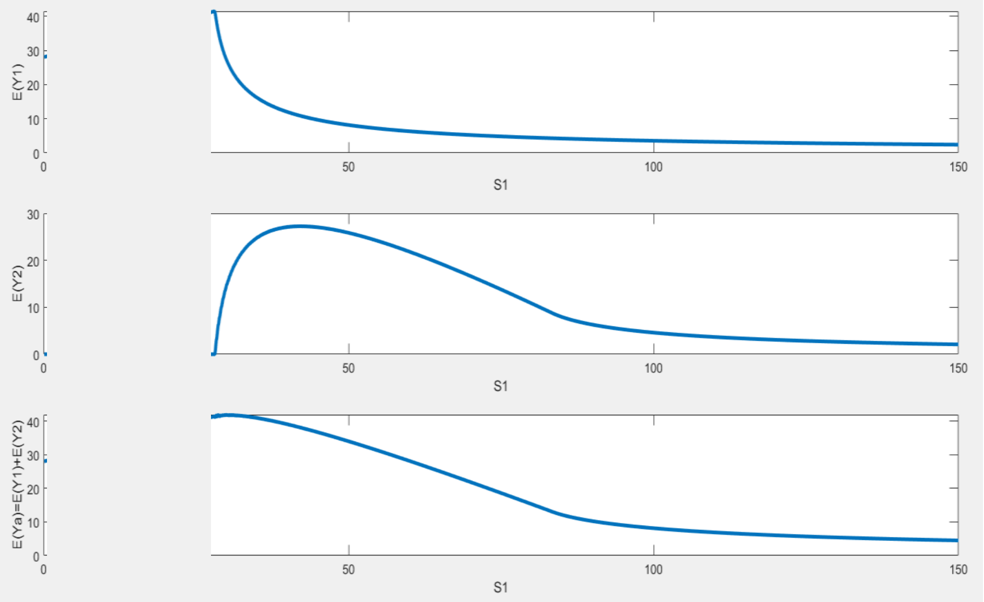


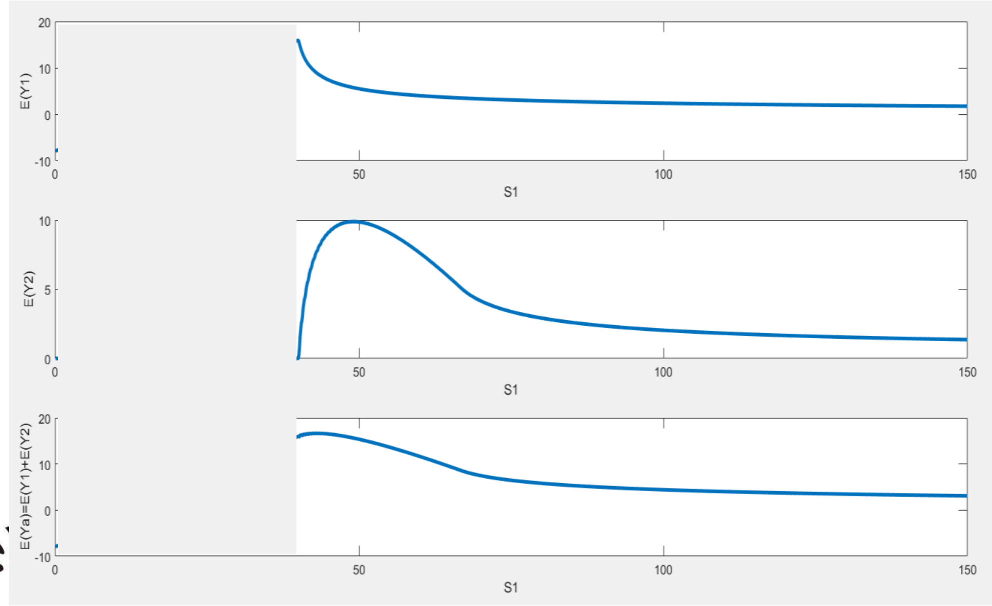
case simulation

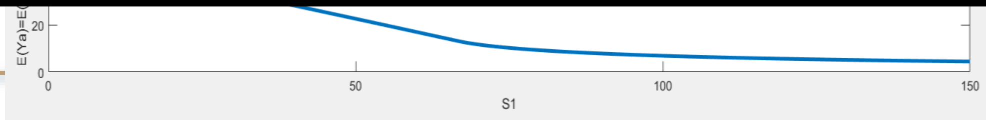
The probability distributions of R_2 are all lognormal
Mean: 1.8; Standard Deviation: 1.77

Case 1	Case 2	Case 3	Case 4
ETP1=200	**ETP1=250**	ETP1=200	ETP1=200
ETP2=250	**ETP2=200**	ETP2=250	ETP2=250
Ky1=1.2	Ky1=1.2	Ky1=1.2	Ky1=1.2
Ky2=1.5	Ky2=1.5	Ky2=1.5	Ky2=1.5
Sa=150	Sa=150	Sa=150	Sa=150
R1=100	R1=100	**R1=20**	**R1=180**
Ymax=100	Ymax=100	Ymax=100	Ymax=100



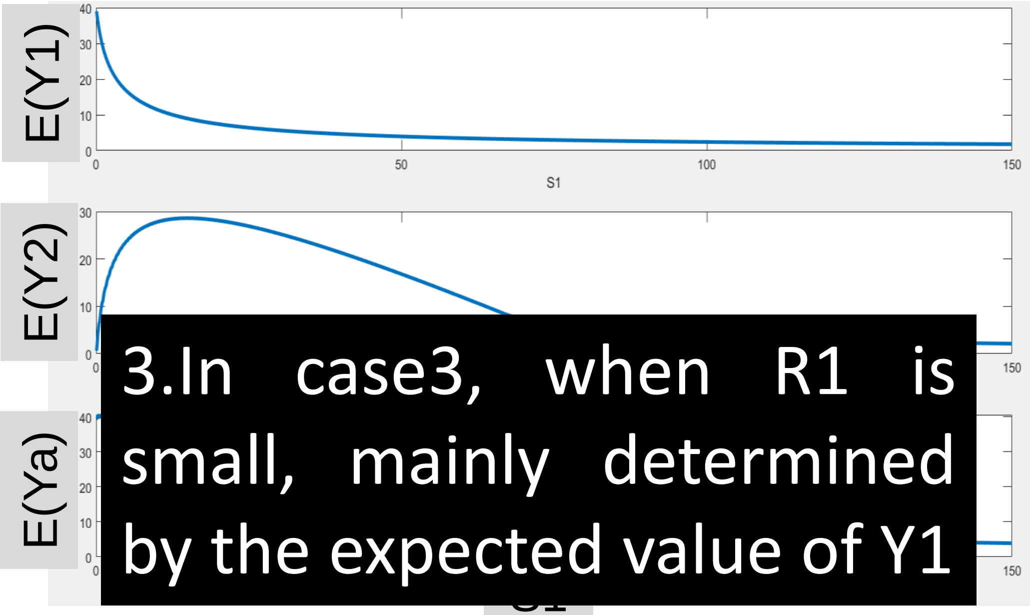




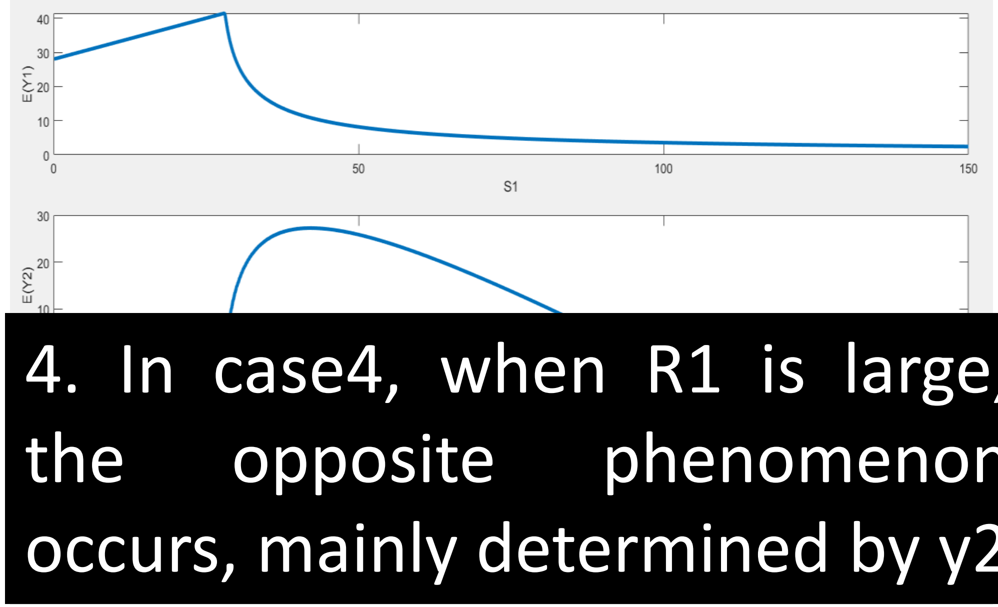


1.The water supply in the first stage

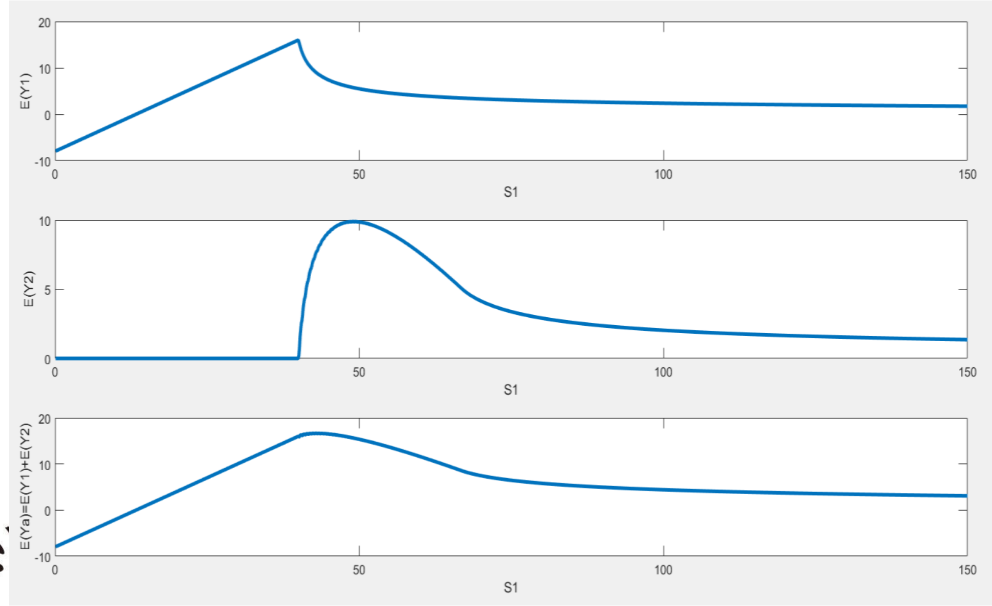
2.In case2 and 3, the expected value of y1 to the maximum , the trend of change between s1 and the expected value is the same as case1.

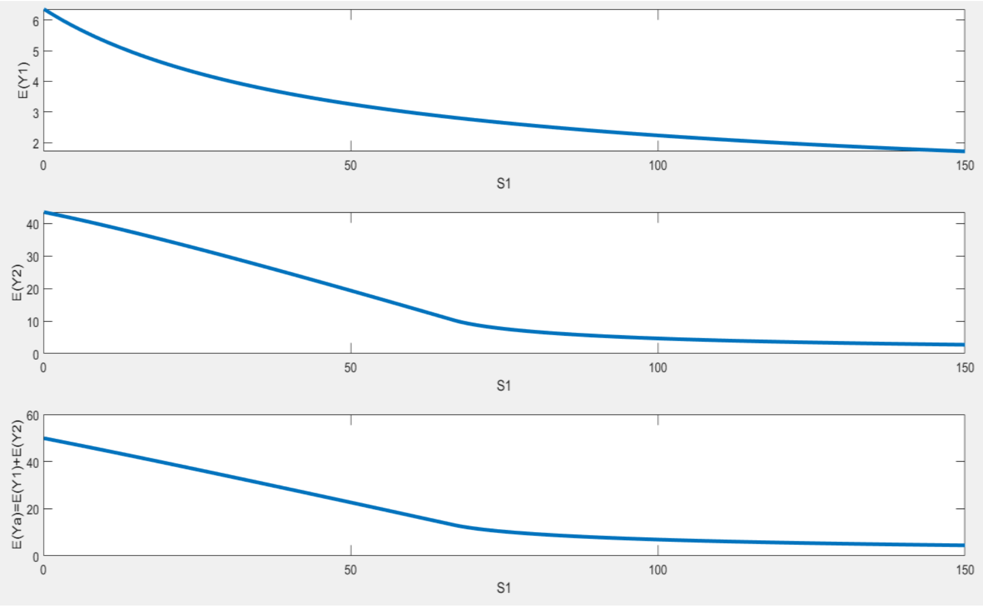


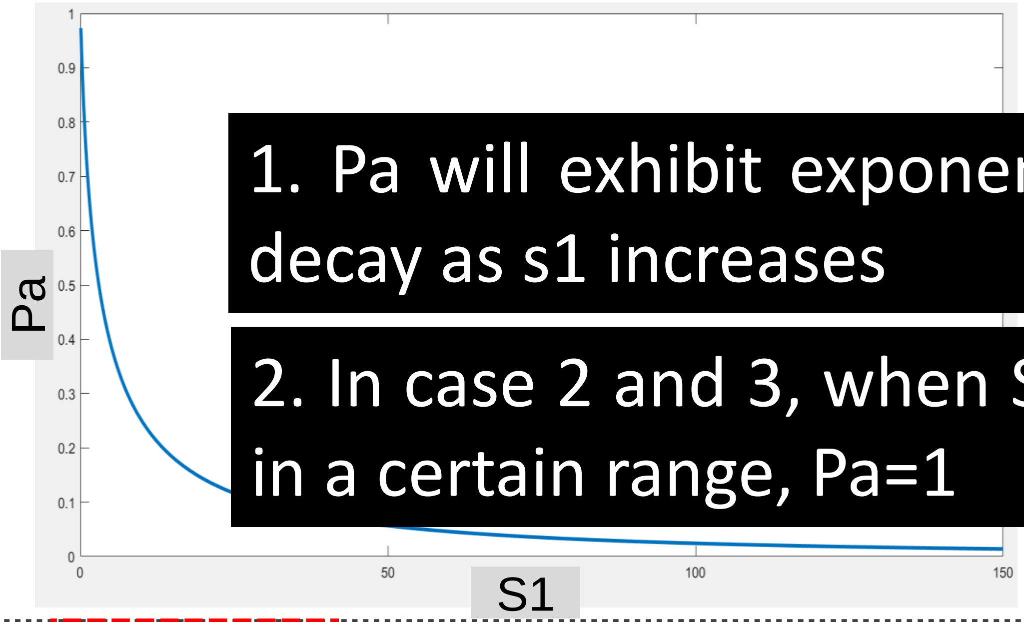
3.In case3, when R1 is small, mainly determined by the expected value of Y1



4. In case4, when R1 is large, the opposite phenomenon occurs, mainly determined by y2

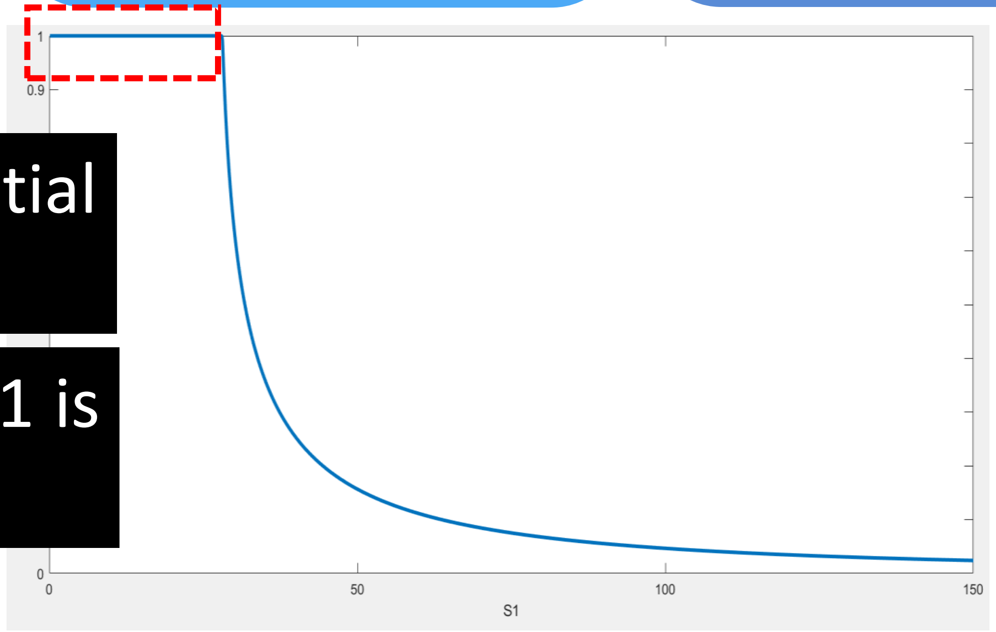


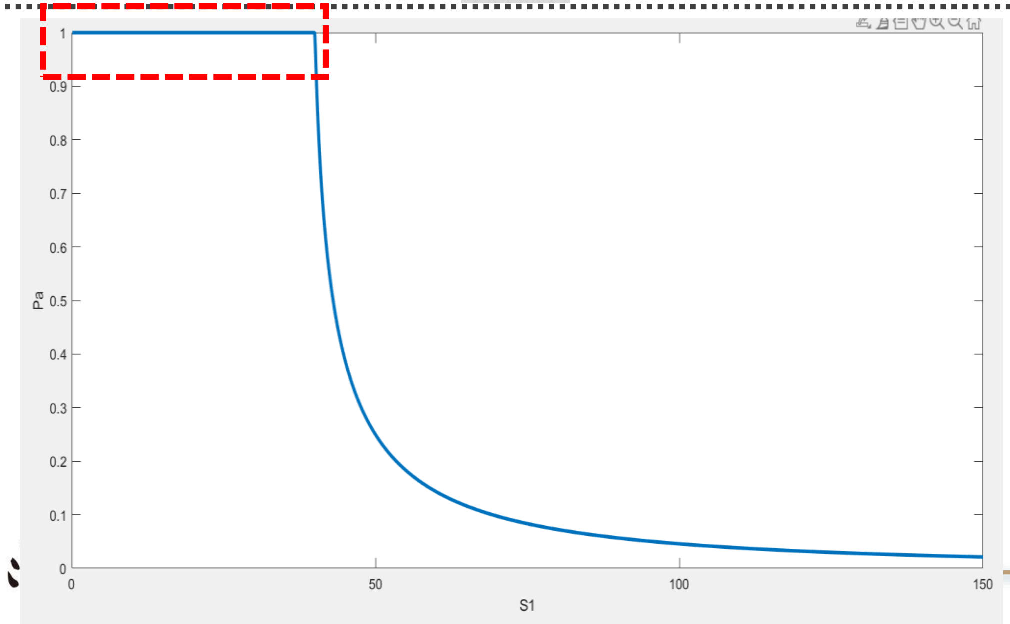


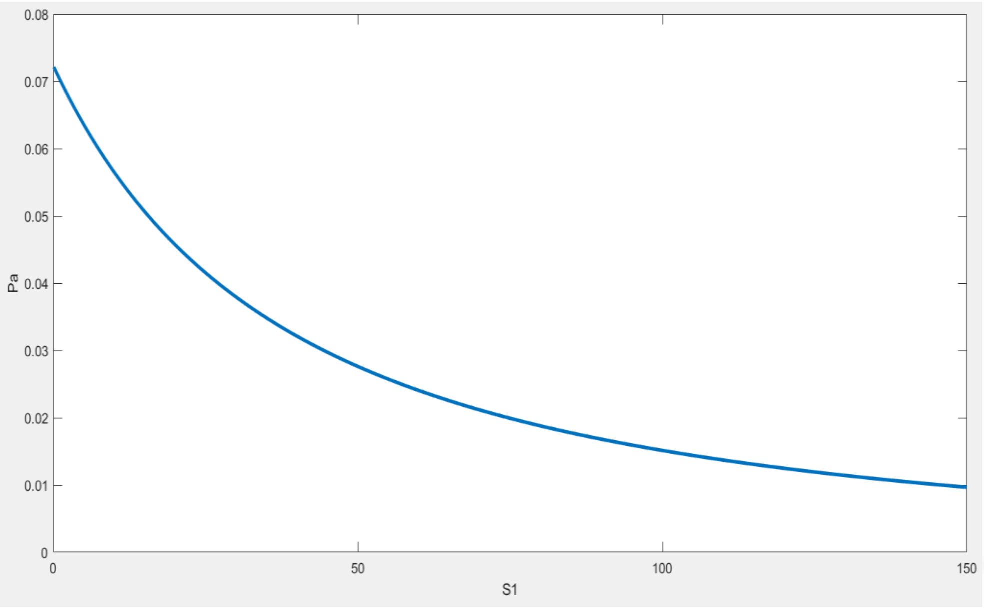


1. Pa will exhibit exponential decay as s1 increases

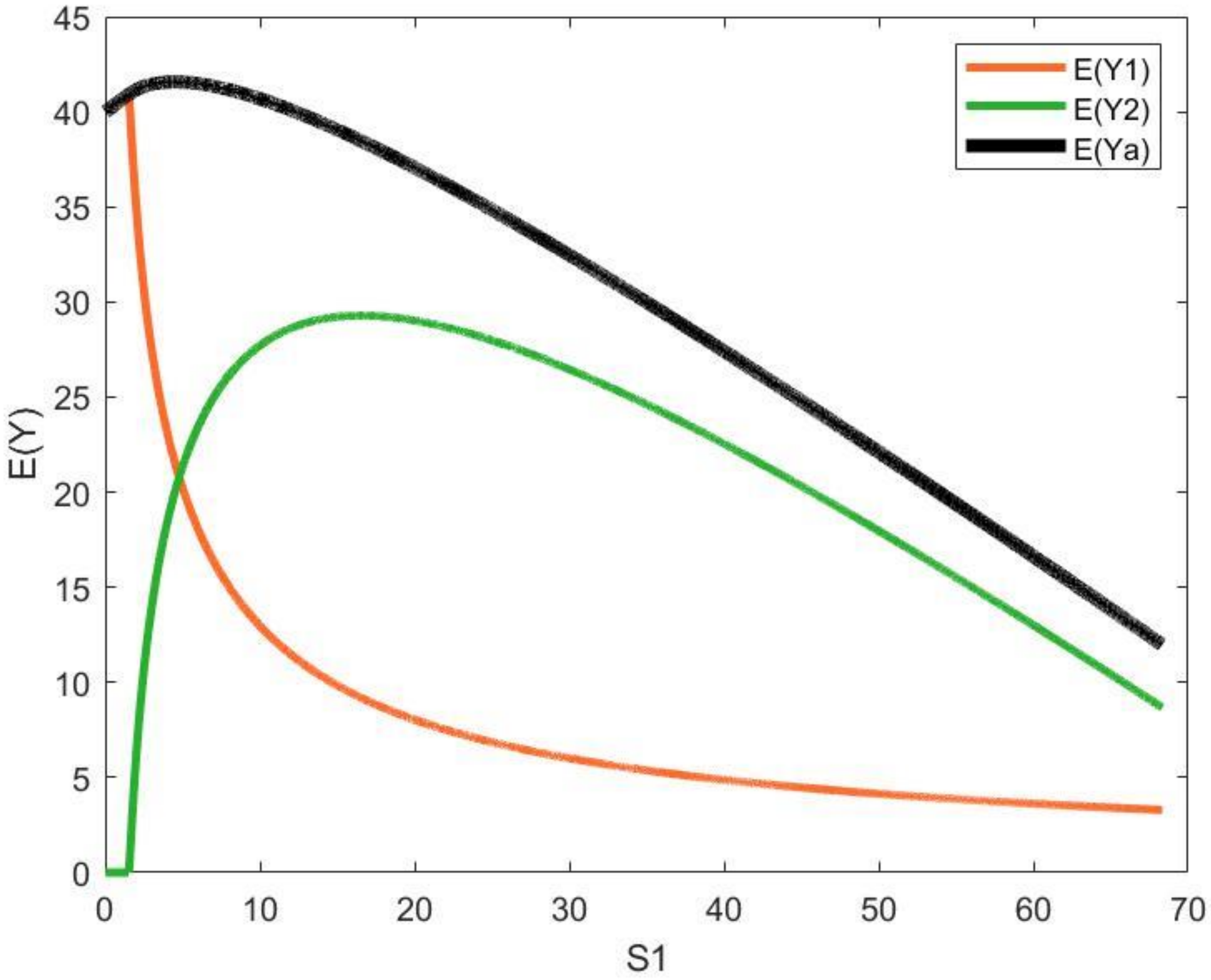
2. In case 2 and 3, when S1 is in a certain range, Pa=1



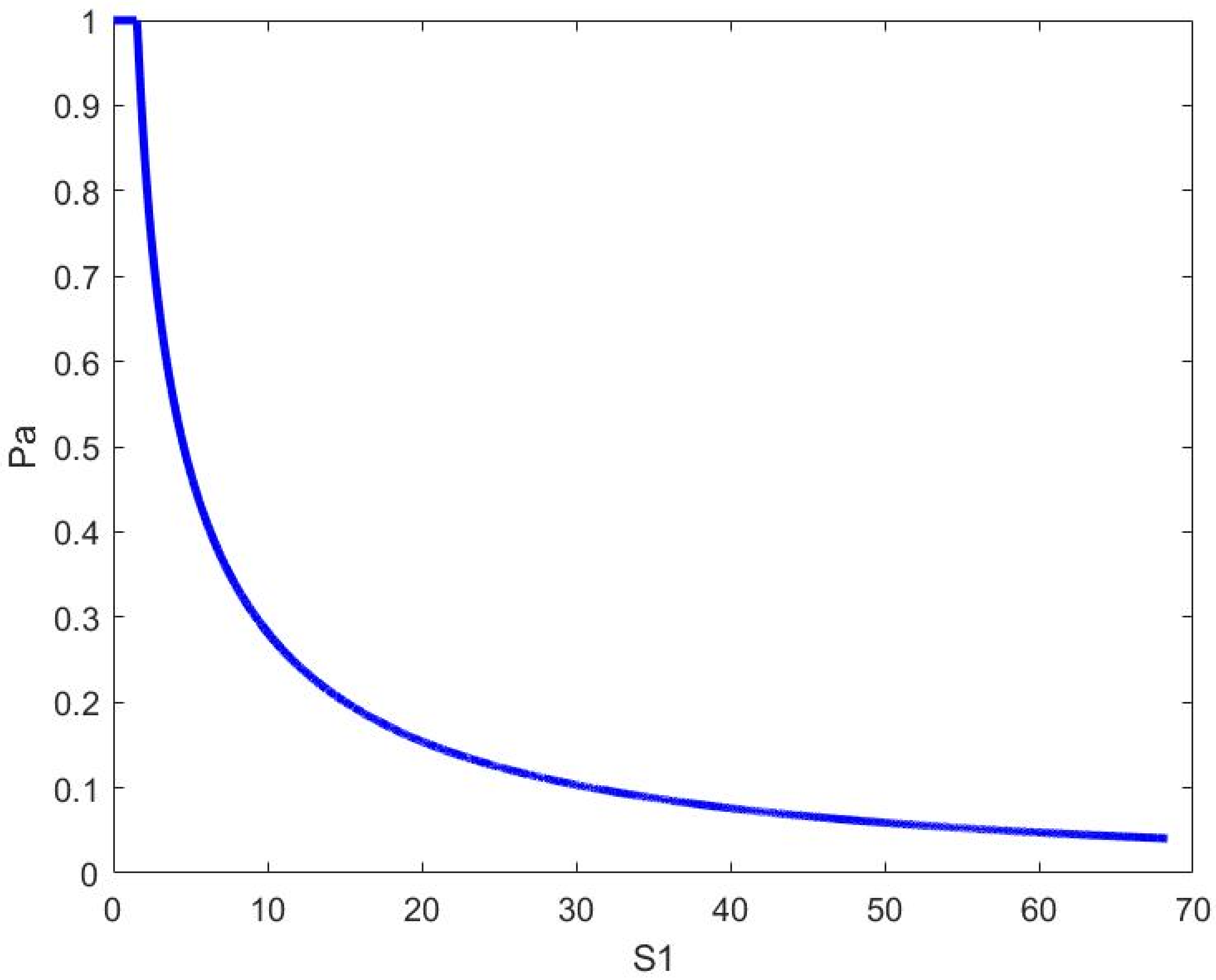




Numerical



Numerical



- Very a few studies really take into account the water consumption at different stages to calculate yield
- Both two stages are representative, but the second stage involves the uncertainty of rainfall and is more complicated
- When the rainfall in the second stage is less than R_o, the second stage is dominant; on the other hand, the first stage is dominant.
- P_α, which could be influenced by S1, will be important in determining yield expectation.
- In the future, we expect to have better understanding of the irrigation decision

THANK YOU