Realistic uncertainties for Surface Wave dispersion curves and their influences on 1D S-wave profiles

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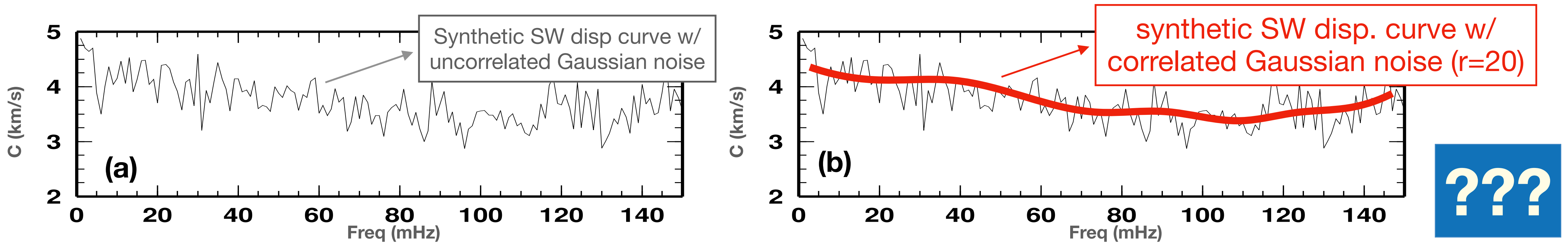
ABSTRACT: Surface wave (SW) dispersion curves are widely used to retrieve 1D S-wave profiles of the Earth at different depth-scale, from local to global models. However, such models are generally constructed with a number of assumptions which could bias the final results. One of the most critical issue is the assumption of a diagonal error covariance matrix as representative of the data uncertainties. Such first-order approximation is obviously wrong for any SW practitioner, given the smoothness of dispersion curves, and could lead to overestimate the information content of the dispersion curves themselves.

In this study, we compute realistic errors (i.e. represented by a non-diagonal error covariance matrix) for Surface Wave dispersion curves, computed from earthquakes data. Given the huge amount of data available worldwide, realistic errors can be easily estimated using empirical formulations (i.e. repeated measurements of the same quantity). Such approach leads to the computation of a full-rank empirical covariance matrix which can be used as input in standard Likelihood computation (e.g. to drive a Markov chain Monte Carlo, McMC, sampling of a Posterior Probability Distribution, PPD, in case of a Bayesian workflow).

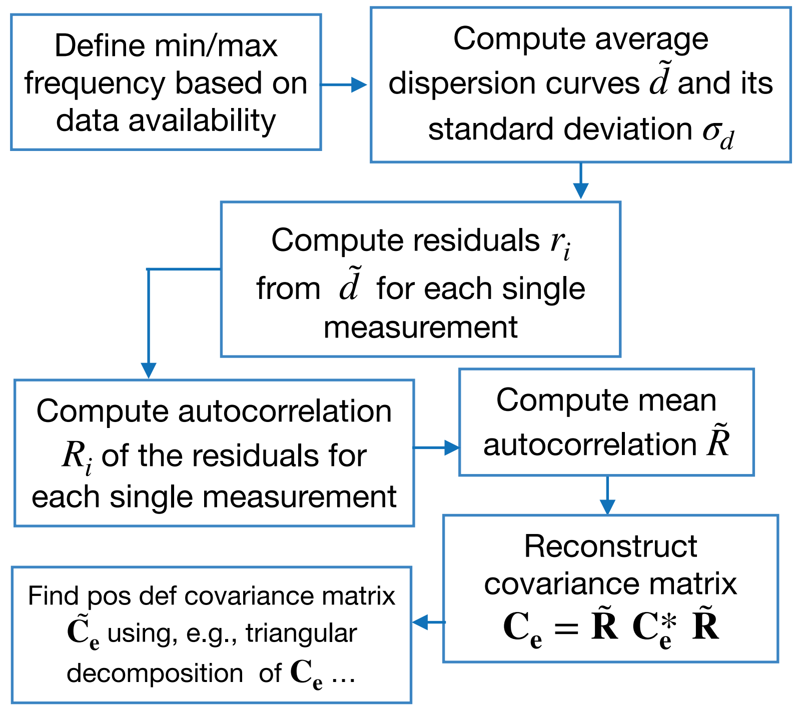
We apply our approach to field measurements recorded along one decade in the British Islands. We first compute the empirical error covariance matrices for 12 two-stations dispersion curves, under different assumptions, and, then, we invert the curves using a standard trans-dimensional McMC algorithm, to find relevant 1D S-wave profiles for each curve. We perform both an inversion considering the full-rank error covariance matrix, and one inversion using a diagonal version of the same matrix. We compare the retrieved profiles with published results. Our main finding is that 1D profiles obtained using a full-rank error covariance matrix are often similar to profiles obtained with a diagonal matrix and published profiles obtained with different approaches. However, relevant differences occur in a number of cases, which leads to potentially question some details in 1D models. Given the extreme easiness of computing the full-rank error covariance matrix, we strongly suggest to include realistic error computation in SW studies.

Getting started: how do your SW dispersion curves look like: (a) or (b)?



Getting started: how to compute realistic $\mathbf{C_e}$ from repeated measurements

Workflow


Define min/max frequency based on data availability

Compute average dispersion curves $\tilde{d}$ and its standard deviation σ_d

Compute residuals r_i from $\tilde{d}$ for each single measurement

Compute autocorrelation R_i of the residuals for each single measurement

Compute mean autocorrelation $\tilde{R}$

Reconstruct covariance matrix $\mathbf{C_e} = \tilde{\mathbf{R}}\ \mathbf{C_e^*}\ \tilde{\mathbf{R}}$

Find pos def covariance matrix $\tilde{\mathbf{C}}_\mathbf{e}$ using, e.g., triangular decomposition of $\mathbf{C_e}$ ...

Equations

$$\tilde{d}(f_j) = \frac{1}{N_d}\sum_{i=1}^{N_d} d_i(f_j) \qquad \sigma^2(f_j) = \frac{1}{N_d}\sum_{i=1}^{N_d}(d_i(f_j) - \tilde{d}(f_j))^2 \qquad (1)$$

$$r_i(f_j) = d_i(f_j) - \tilde{d}(f_j) \qquad (2)$$

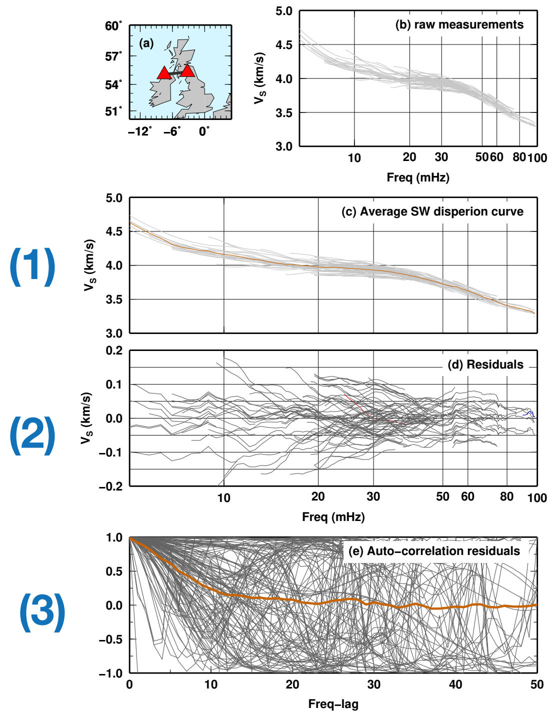
$$R_i(\Delta f) = \frac{\sum r_i(f_j) r_i(f_j + \Delta f)}{\sum r_i^2} \qquad \tilde{R}(\Delta f) = \frac{1}{N_d}\sum_{i=1}^{N_d} R_i(\Delta f) \qquad (3)$$

$$\mathbf{C_e^*} = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{N_f}^2 \end{bmatrix} \qquad \tilde{\mathbf{R}} = \begin{bmatrix} \tilde{R}_0 & \tilde{R}_1 & \tilde{R}_2 & \dots & \tilde{R}_{N_f} \\ \tilde{R}_1 & \tilde{R}_0 & \tilde{R}_1 & \dots & \tilde{R}_{N_f-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{R}_{N_f} & \dots & \tilde{R}_2 & \tilde{R}_1 & \tilde{R}_0 \end{bmatrix}$$

Variables

$j = 1,...,N_f$: set of frequencies ; $i = 1,...,N_d$: set of dispersion curves; f_{min} : minimum frequency ; f_{max} : maximum frequency ; $d_i(f_j)$: i-th dispersion curve; $\tilde{d}(f_j)$: average dispersion curve; $\sigma_d(f_i)$: standard deviation of the average dispersion curve; r_i : residual of the I-th dispersion curve; R_i : autocorrelation of the i-th residual; $\tilde{R}$: mean autocorrelation

Example



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Data

The use of the two-station method in surface-wave analysis: introduced by Sato (1955). We use the implementation of the two-station method as in Meier et al. (2004), Soomro et al. (2016), Bonadio et al. (2021). Allows us to compute phase-velocity dispersion of the surface waves that travel approximately along the GCP between stations of a pair. To minimize the effect of the errors in the curves on the final, average measurements, we only accept smooth portions of phase-velocity curves. We also exclude the outlier measurements and, also, accept only the curves not unrealistically far from a pre-calculated reference dispersion curve, as computed for the region in Bonadio et al. (2021), which provides a data-based initial reference curve for the area. In **Figure 2** we show the final data-sets obtained for 12 station pairs

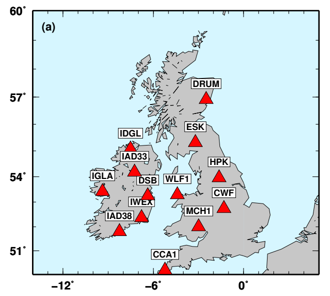
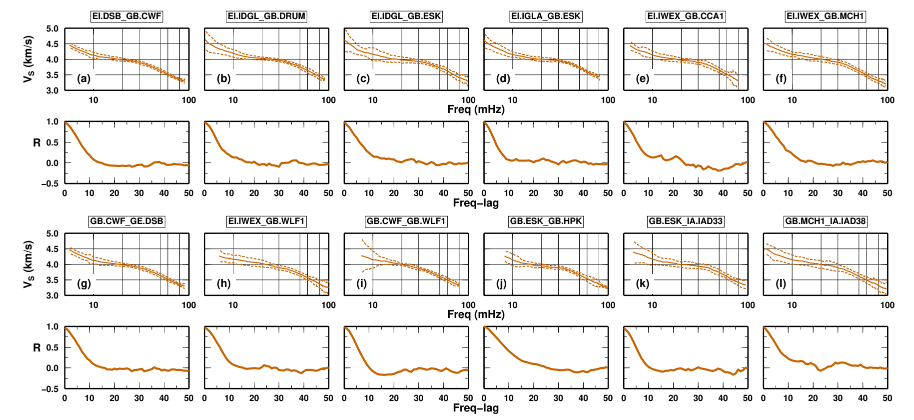
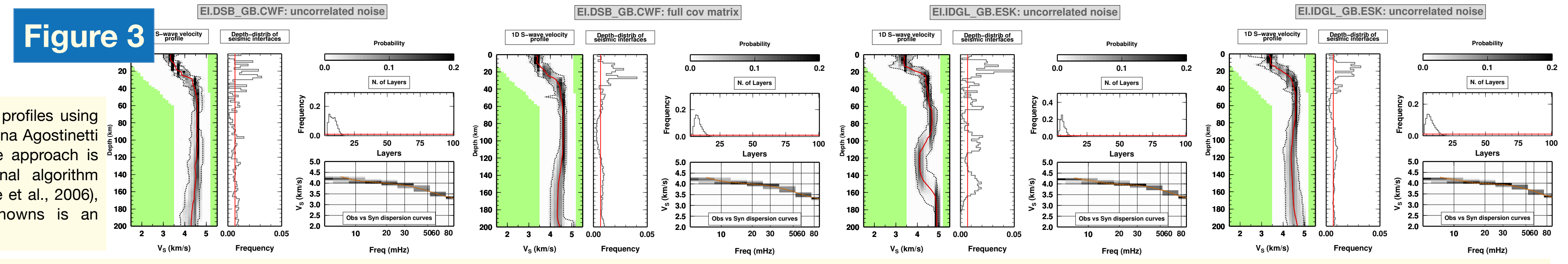


Figure 2



Method

Figure 3



We infer 1D S-wave velocity profiles using an approach derived from Piana Agostinetti and Malinverno (2018). The approach is based on a trans-dimensional algorithm (Malinverno, 2002; Sambridge et al., 2006), where the number of unknowns is an unknown itself

We apply the new methodology to all stations pairs, considering both correlated and un-correlated noise. In **Figure 3** we report the results of the analysis for two stations pairs, considering both noise statistics. We report both S-wave velocity profiles and interface depths. The red lines indicates the mean posterior 1D S-wave velocity profiles, used to compare different noise statistics (see below).

Results

We report the results for all the 12 stations pairs, using both correlated and uncorrelated noise in the 1D inversion. In **Figure 4**, we compare the mean posterior models obtained using both noise statistics (red- full covariance matrix; blue- uncorrelated noise). For reference, we also report the 1D S-wave velocity profile obtained using a linearised inversion (grey line)

1D S-wave profiles using not-correlated and correlated noise display:

🟥 significantly different features (marked with ➡)

🟧 some different features (marked with ➡)

🟩 similar patterns (almost overlapping values)

1D S-wave profiles using same station pair as above, in different years (double check raw-data analysis)

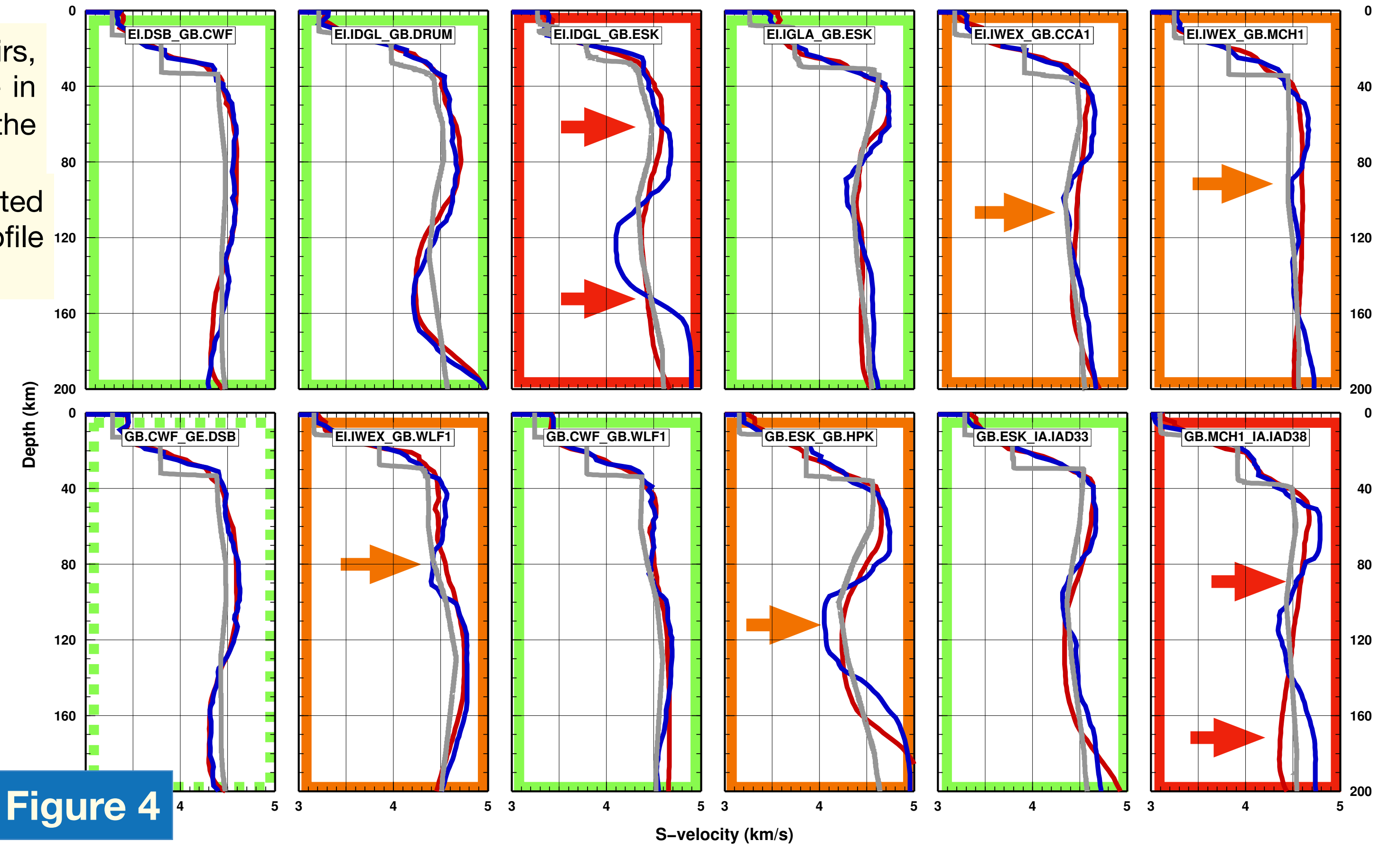


Figure 4

CONCLUSIONS:

1. **Error statistics for SW dispersion curves can be easily computed from repeated measurements of a 2-stations baseline**
2. **SW dispersion curves at least display a sample correlation as large as 10 lag-times**
3. **Using a realistic error statistics for retrieving a 1D S-wave profile (eg. a full Covariance matrix in the computation of the likelihood) can modify the overall result of the analysis.**

Given the easiness of the computation of the full Covariance matrix, we suggest to avoid using un-correlated noise statistics in the inversion of SW dispersion curves

Sharing is encouraged