Regional modeling of water storage variations in a Kalman filter framework

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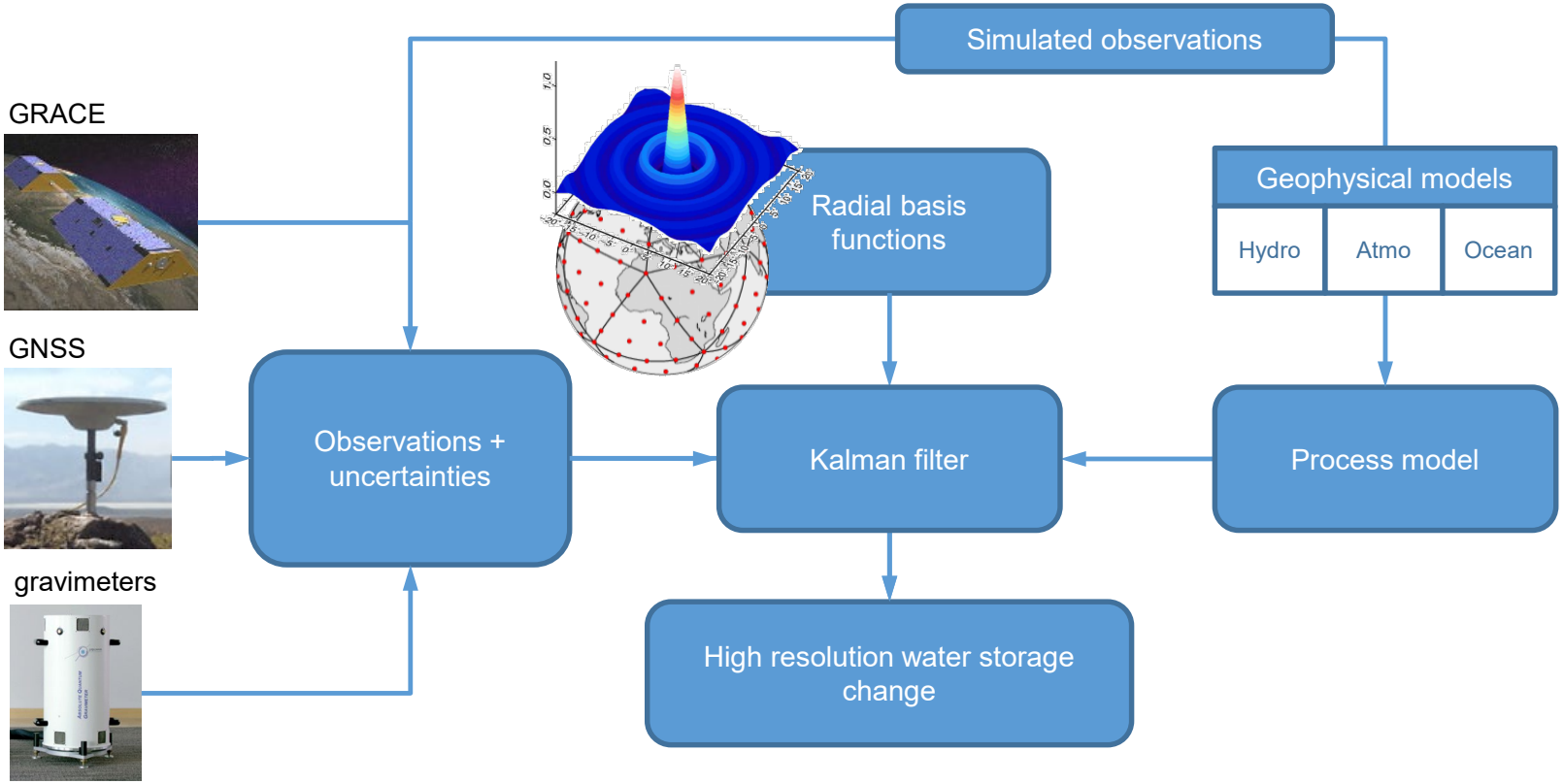
27 April 2023

Project Structure



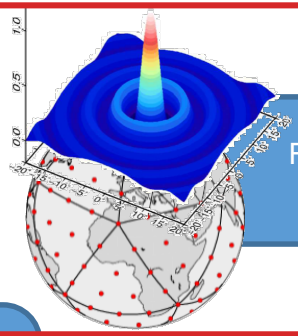
Project Structure

Simulated observations

GRACE

GNSS

gravimeters



Radial basis functions

Geophysical models

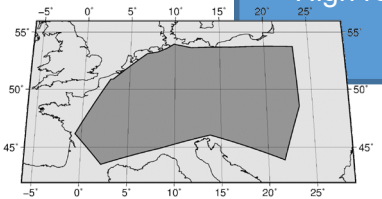
Hydro | Atmo | Ocean

Observations + uncertainties

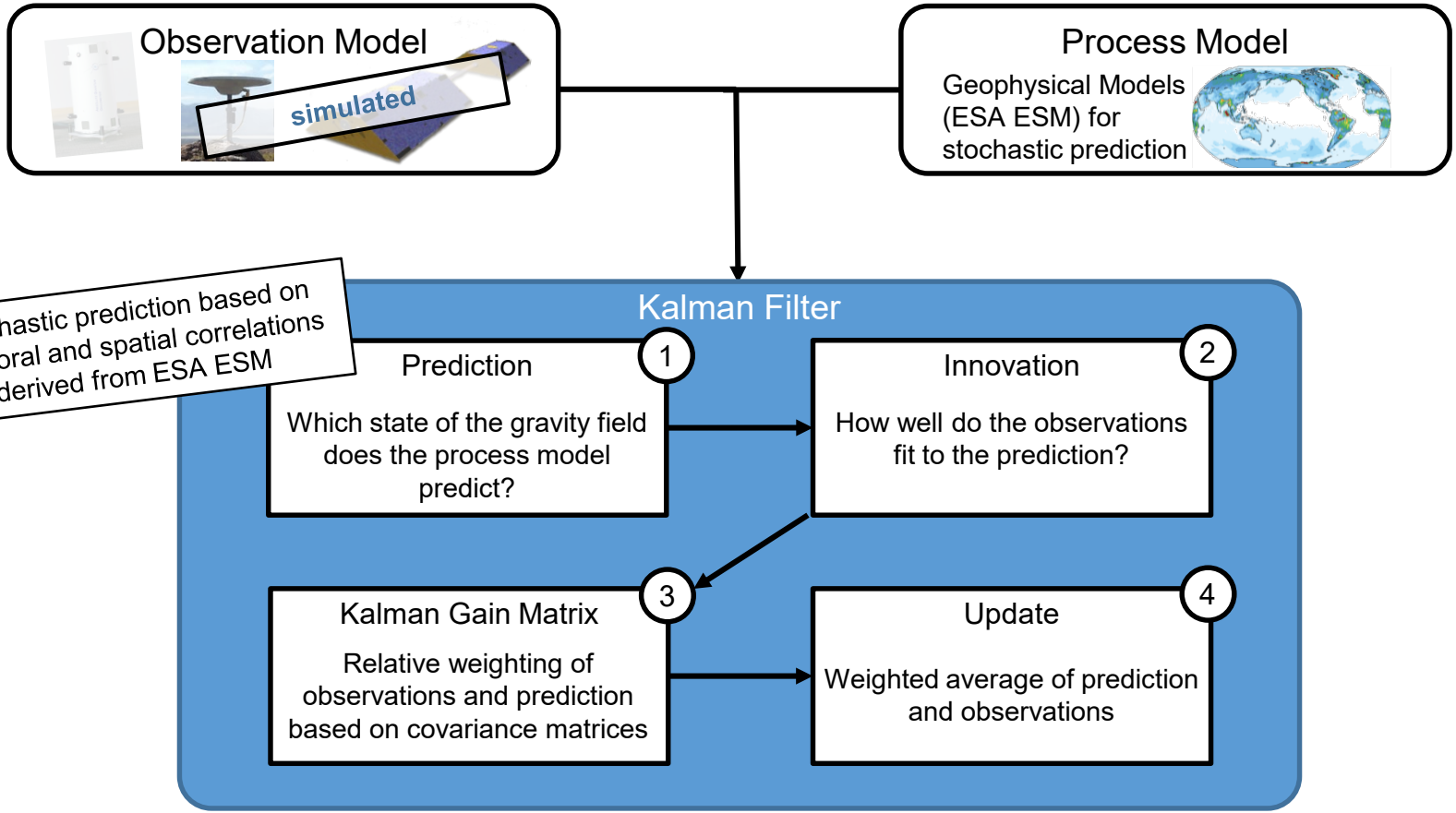
Kalman filter

Process model

High resolution water storage change



Kalman Filter



Kalman Filter – Observation Model

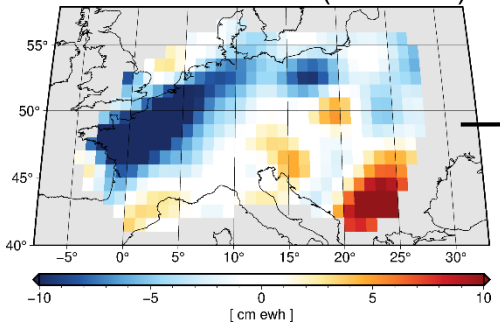
Observation Model

$$\underline{l_t} = A_t x_t + v_t \qquad v_t \sim \mathcal{N}(0, R_t)$$

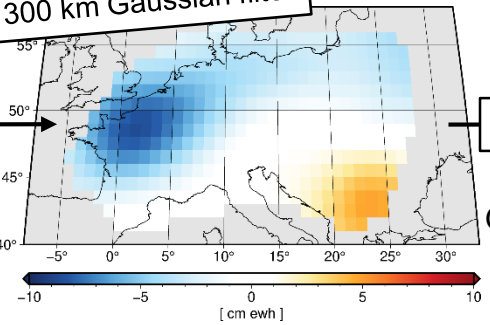
Process Model

$$x_t = B x_{t-1} + w \qquad w \sim \mathcal{N}(0, Q)$$

Gridded GRACE data (simulated)



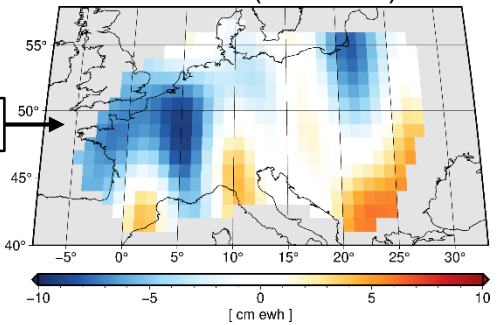
300 km Gaussian filter



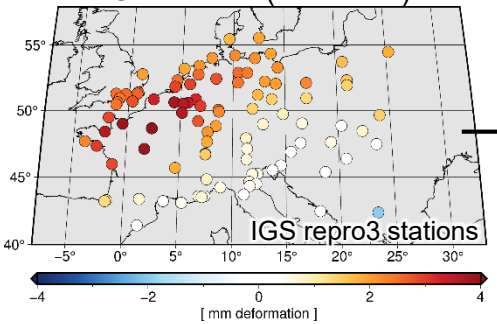
+ correlated noise

From realistic GRACE covariance matrix

GRACE (simulated)



GNSS data (simulated)

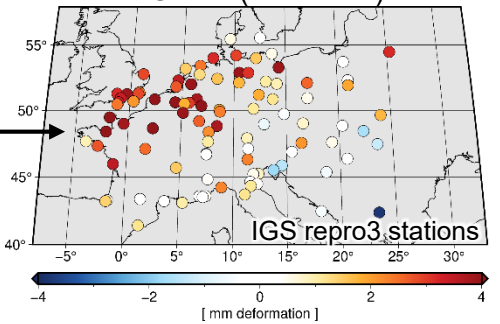


Assuming elastic loading theory

+ uncorrelated noise

To be discussed

GNSS (simulated)



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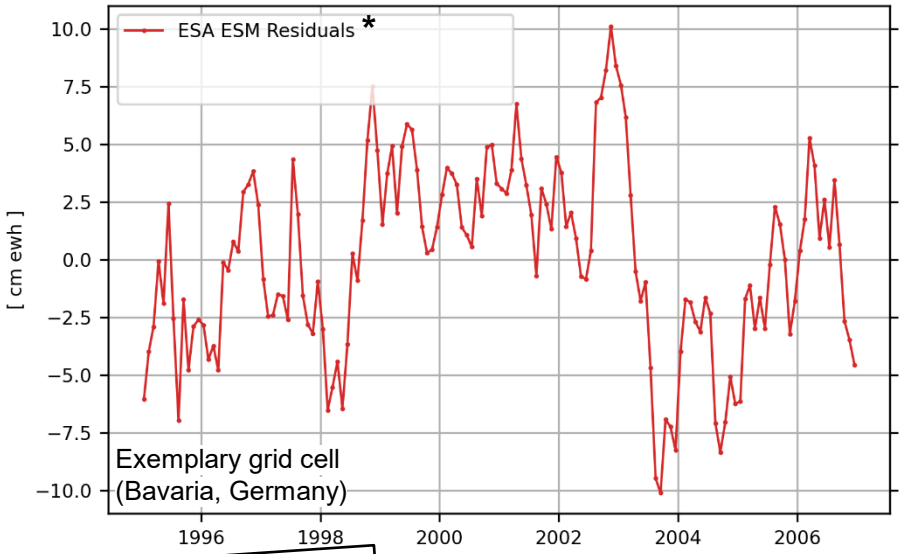
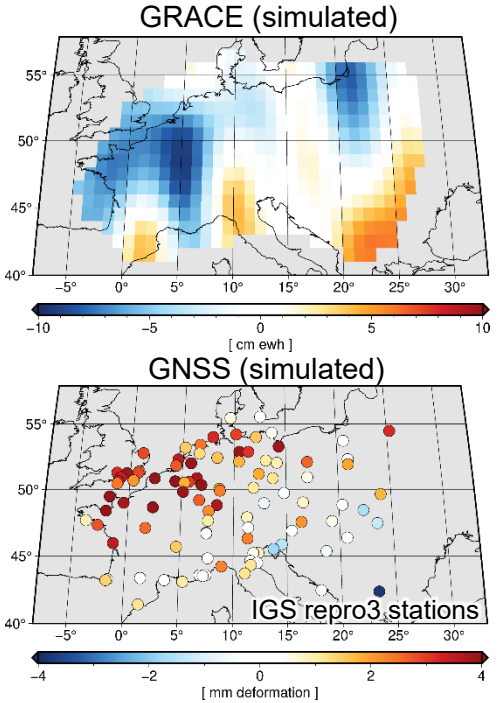


* mean, linear trend and annual signal removed



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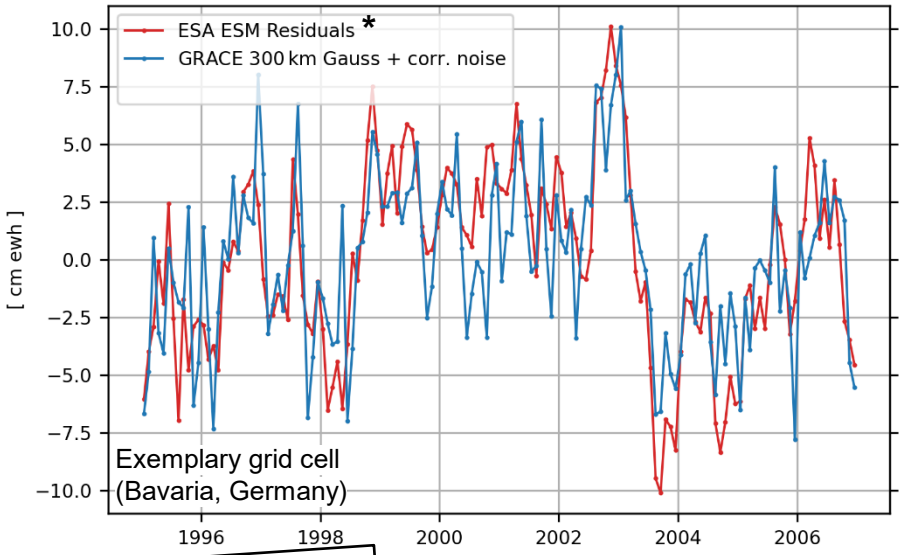

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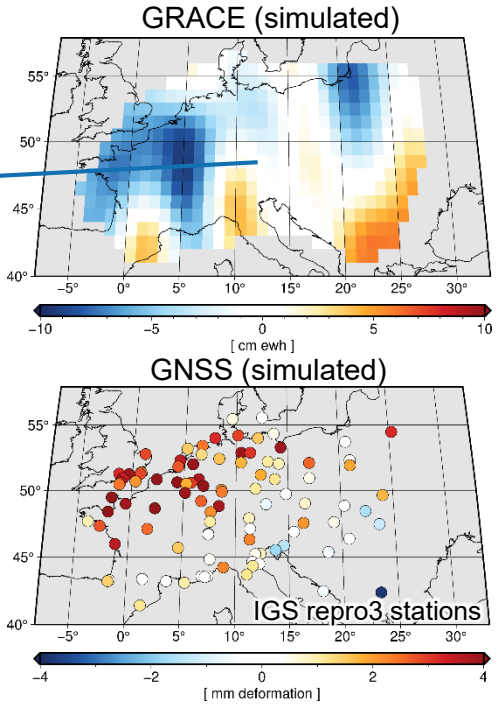

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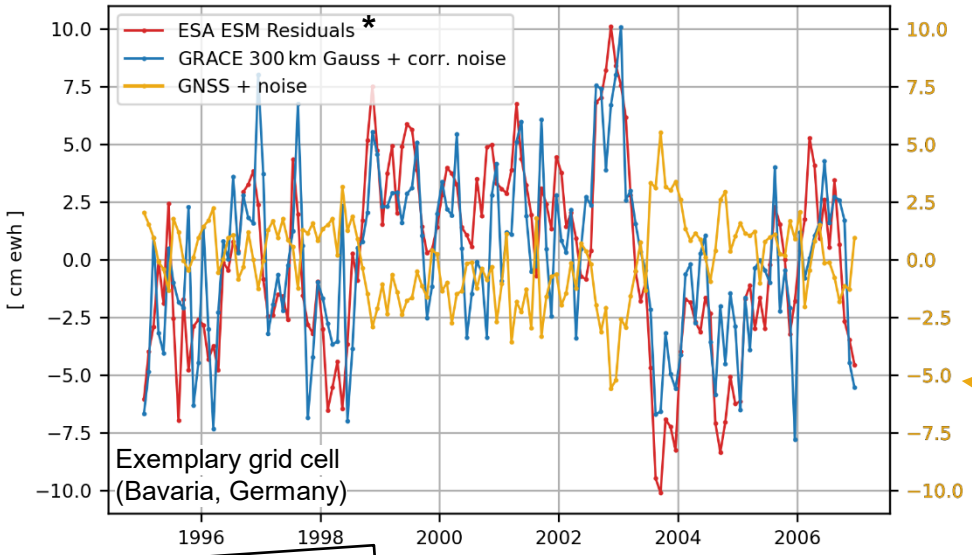

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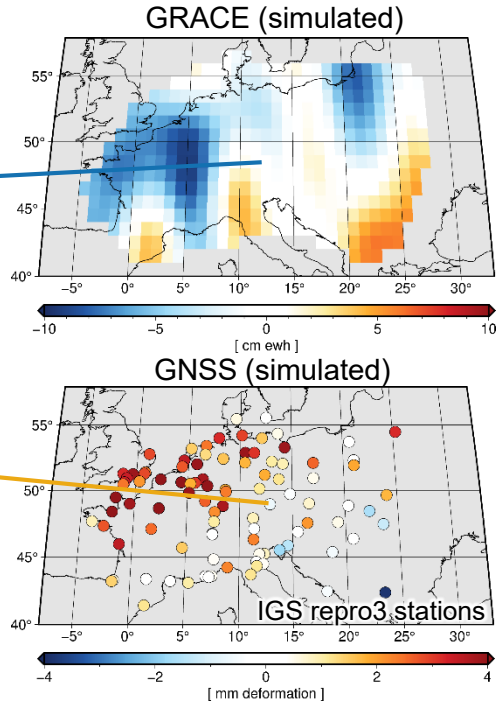

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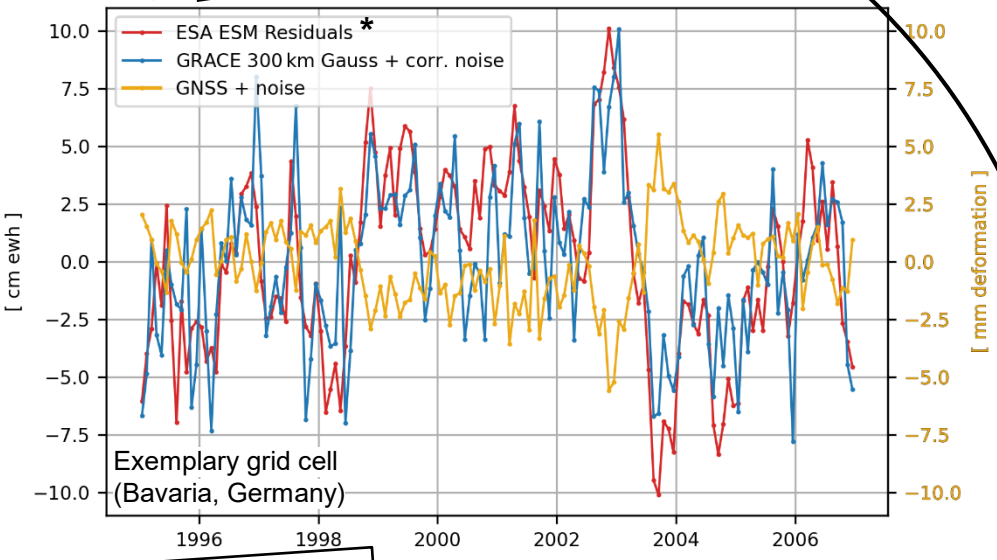

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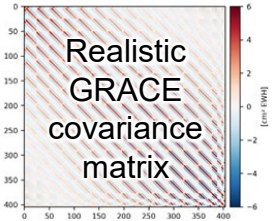
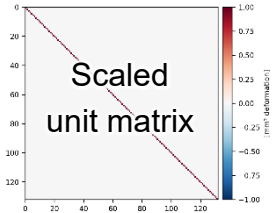
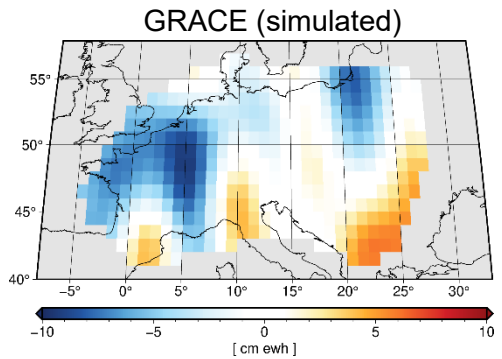
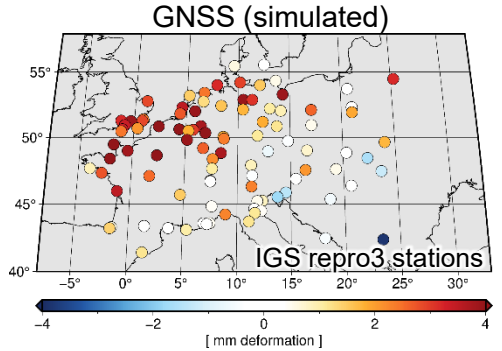




To be discussed





Radial Basis Functions

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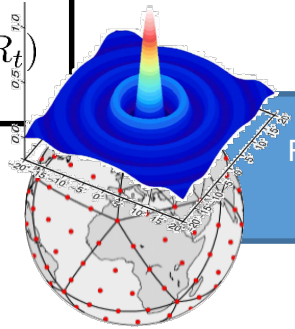
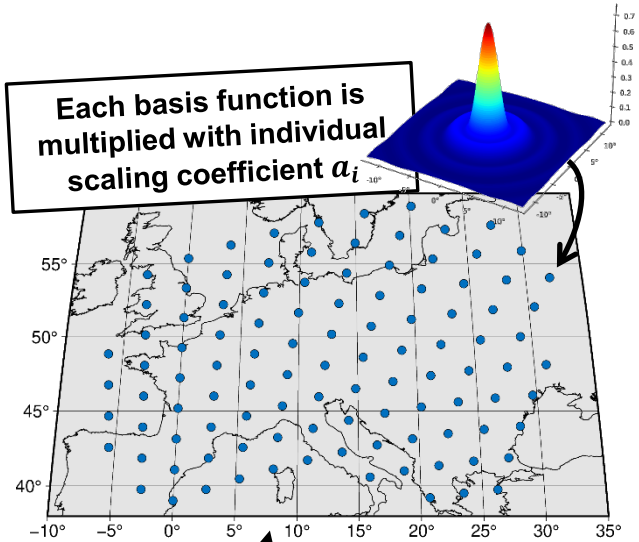


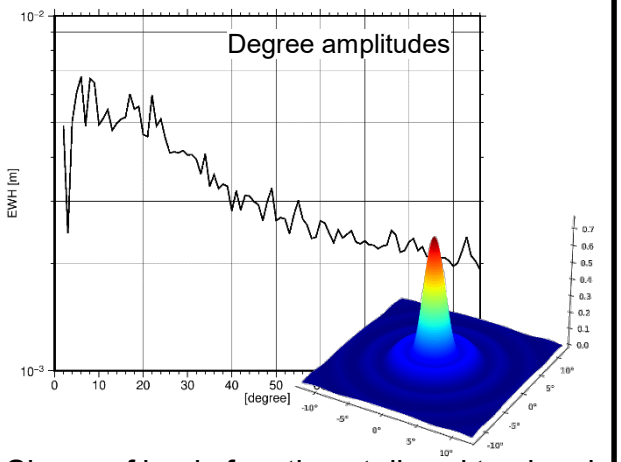
Radial basis functions

Kalman filter

Parametrization with space localizing (radial) basis functions

Each basis function is multiplied with individual scaling coefficient a_i





Shape of basis functions tailored to signal content of expected gravityfield signal

Scaling coefficients

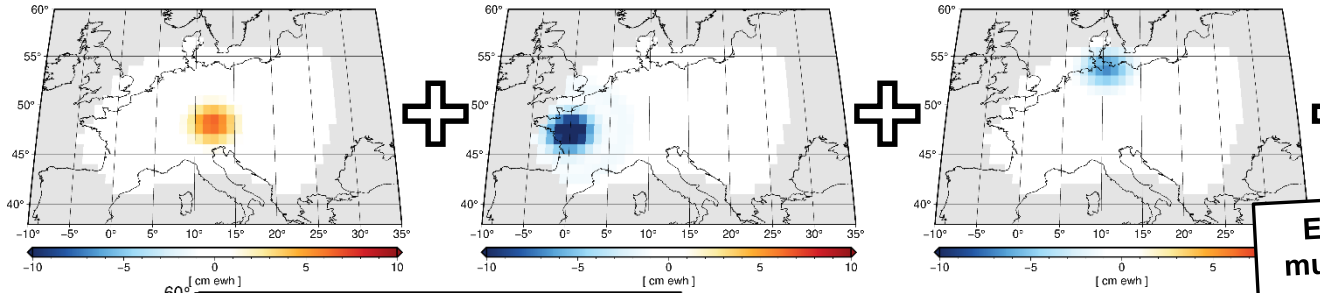
Gravity field functional at point x

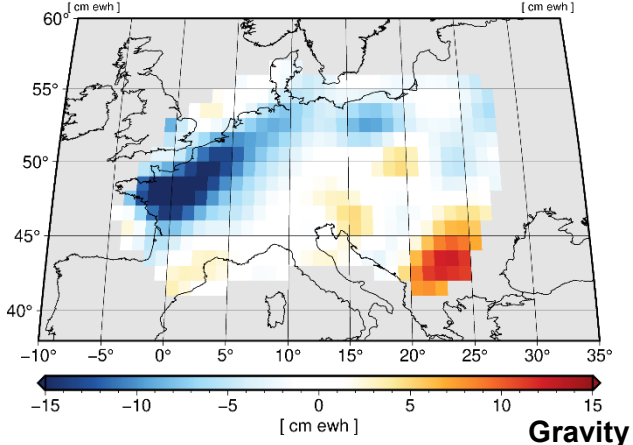
$$s(\boldsymbol{x}) = \sum_{i=1}^{I} a_i \Phi_i(\boldsymbol{x}, \boldsymbol{x}_i)$$

Radial basis functions (spherical splines) located at points x_i

$$\Phi_i(\boldsymbol{x}, \boldsymbol{x}_i) = \sum_{n=0}^{\infty} \sqrt{2n+1} \cdot k_n P_n(\boldsymbol{x}, \boldsymbol{x}_i)$$

Radial Basis Functions





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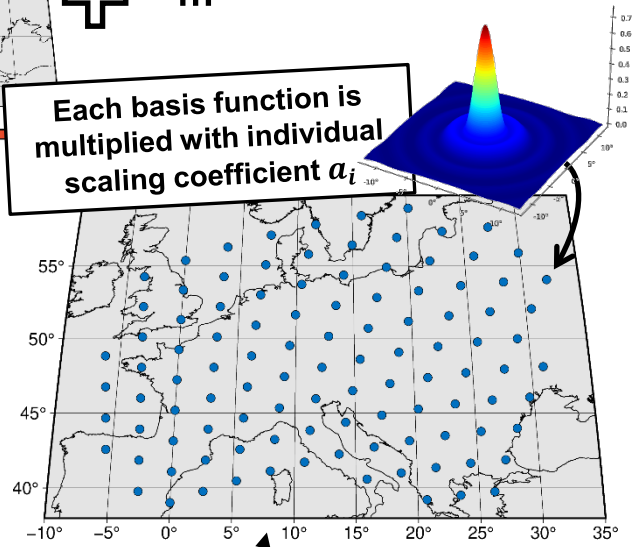

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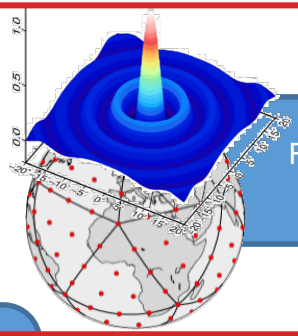

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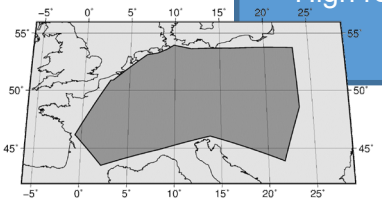
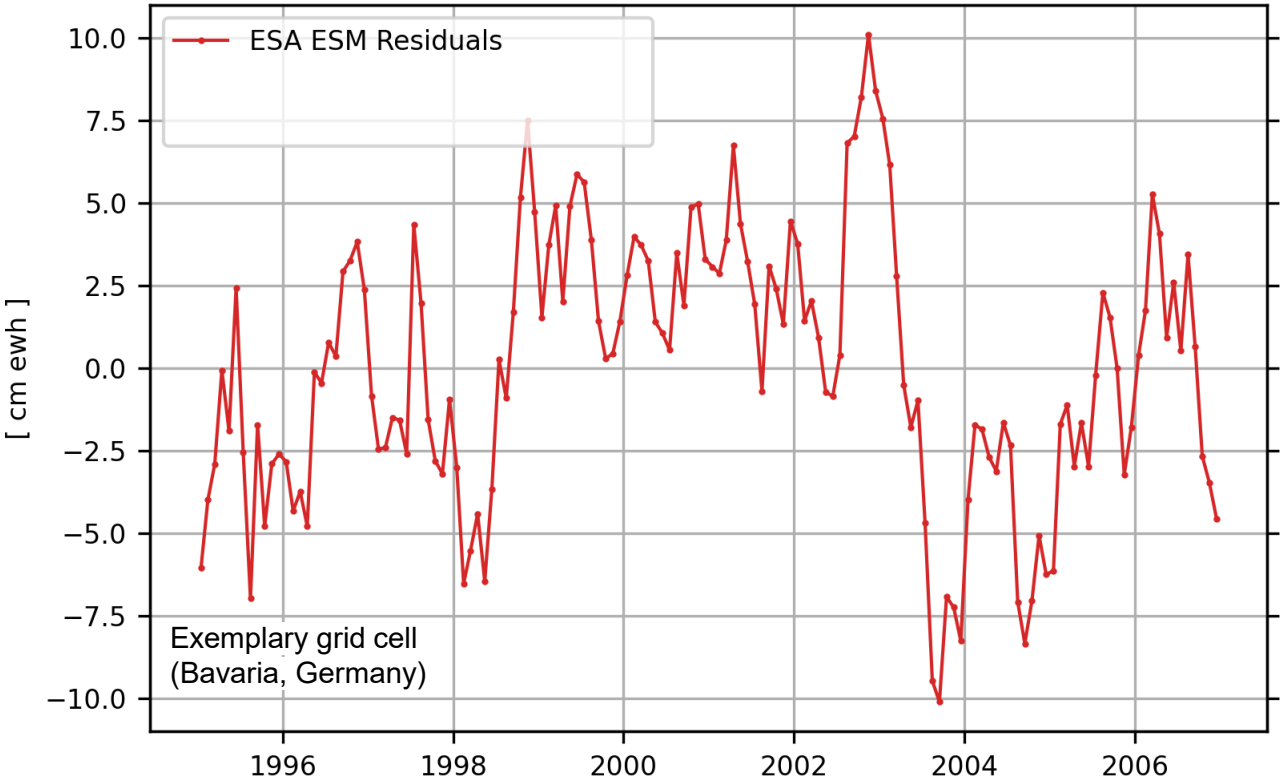


High resolution water storage change

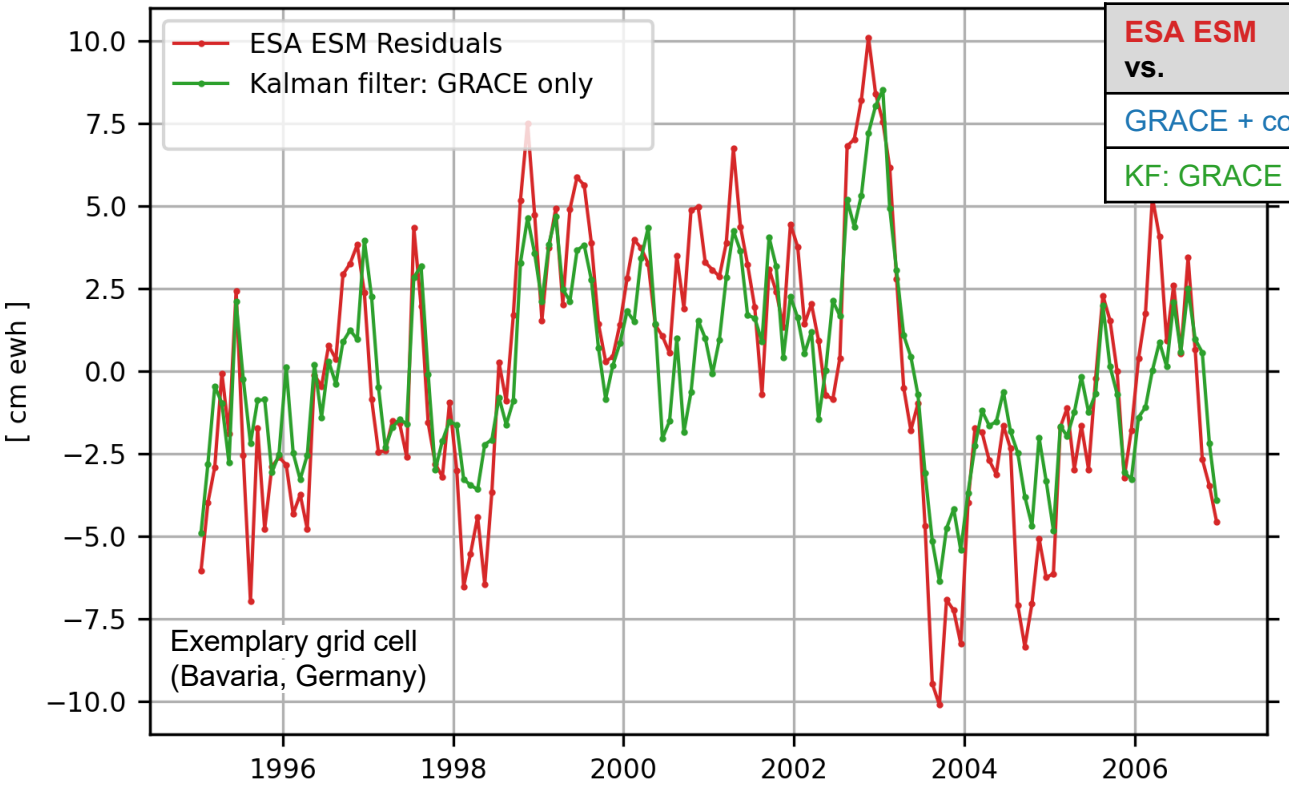
Goal: data combination and noise removal using the Kalman filter

Kalman Filter Time Series



Goal: data combination and noise removal using the Kalman filter
=> High agreement between **reference** and **Kalman filter**

Kalman Filter Time Series



larger is better

smaller is better

ESA ESM vs.	Correlation [-]	RMSD [cm]
GRACE + corr. noise	0.75	2.7
KF: GRACE only	0.89	2.0

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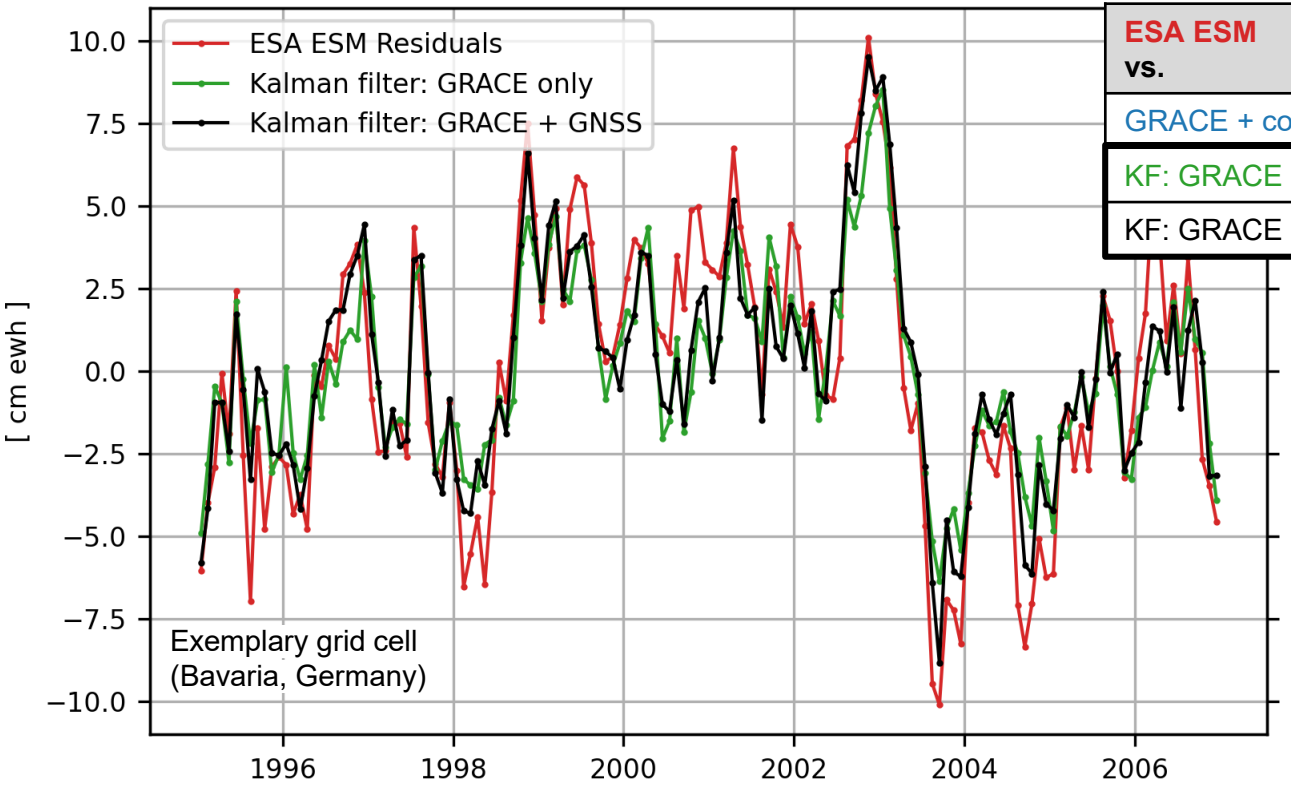
Kalman Filter Time Series



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ESA ESM vs.	Correlation [-]	RMSD [cm]
GRACE + corr. noise	0.75	2.7
KF: GRACE only	0.89	2.0
KF: GRACE + GNSS	0.92	1.7

Next slide: values for all grid cells

Goal: data combination and noise removal using the Kalman filter => High agreement between **reference** and **Kalman filter**

Correlations and RMSD

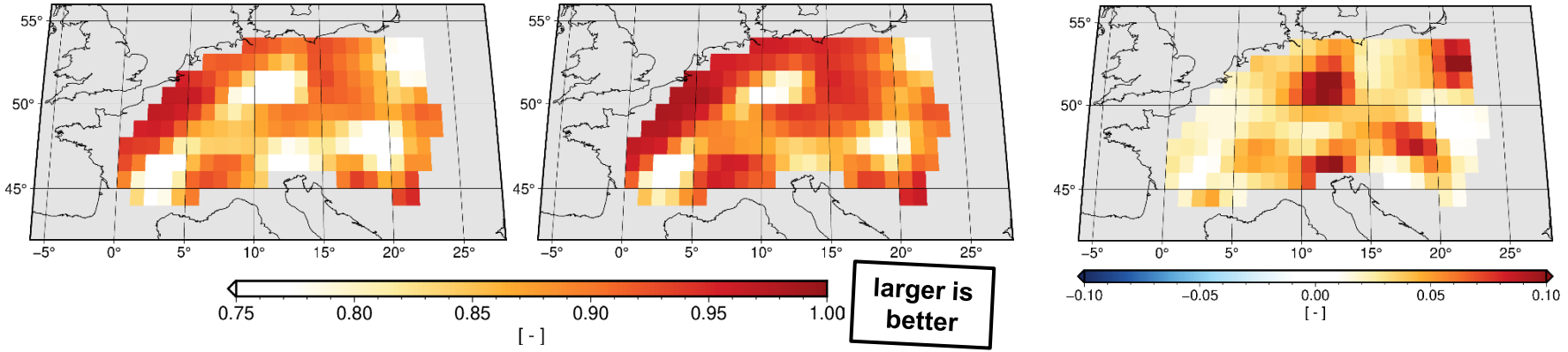
ESA ESM (reference) vs.

Kalman filter: GRACE only

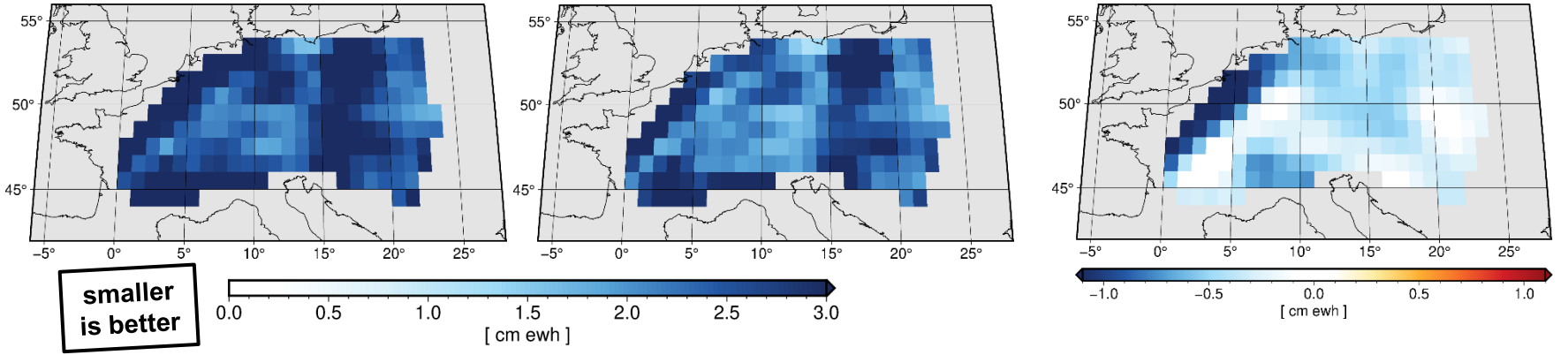
Kalman filter: GRACE + GNSS

Improvement: GRACE only → GRACE + GNSS

Correlation



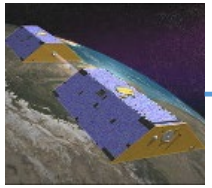
RMSD



Summary and Outlook

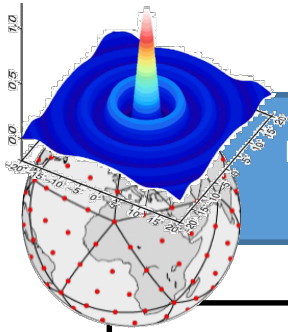
Simulated observations

GRACE



GNSS

gravimeters



Radial basis functions

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Observations + uncertainties

- Regional modeling with space localizing (radial) basis functions implemented in Kalman filter
- Kalman filter implemented for data combination of simulated GRACE and GNSS data

Next steps:

- GNSS error model and relative weighting (GRACE vs. GNSS) to be discussed
- Introduce real GRACE and GNSS data
- Add (simulated) observations from gravimeters

Questions? viviana.woehnke@hcu-hamburg.de

We thank the DFG for funding this project