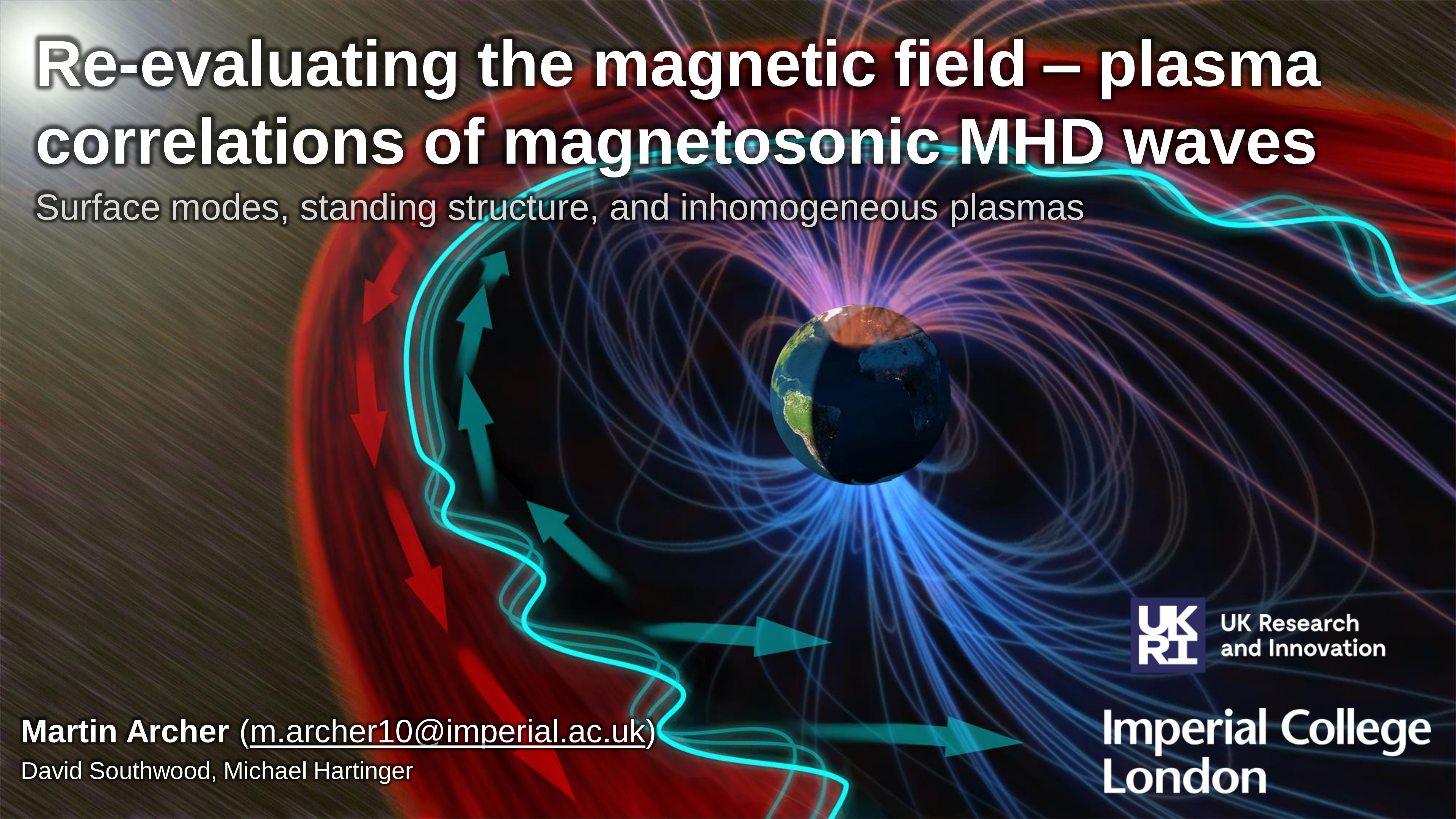
Re-evaluating the magnetic field – plasma correlations of magnetosonic MHD waves

Surface modes, standing structure, and inhomogeneous plasmas



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UK Research and Innovation

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Correlations in Magnetosonic Waves

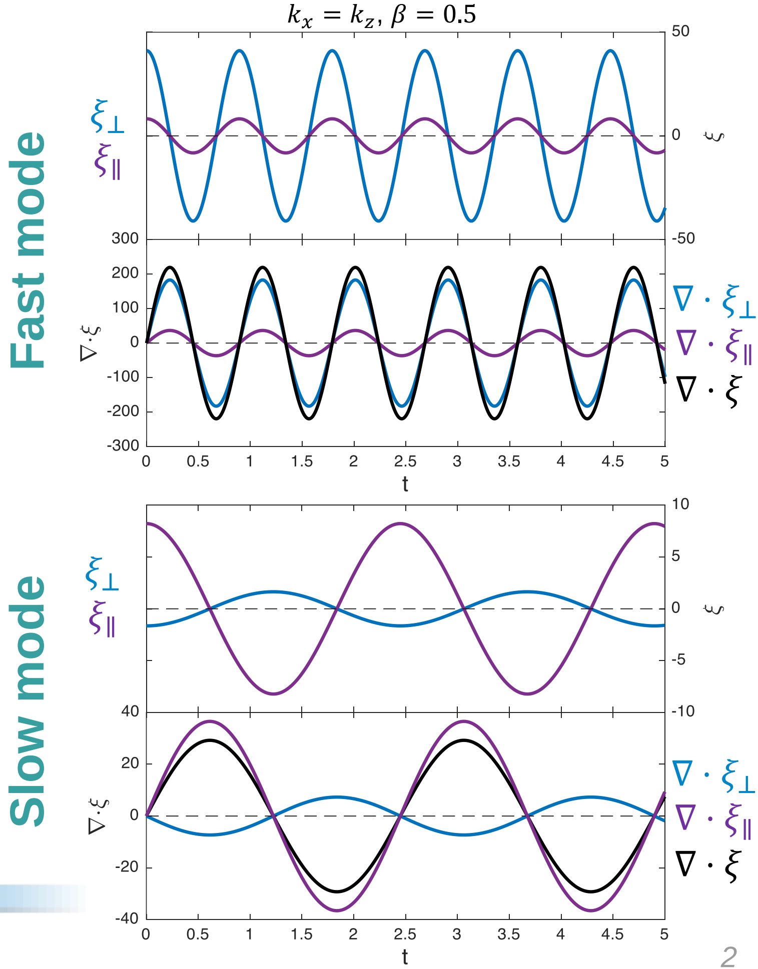
The three familiar MHD wave modes are derived from linearised Ideal MHD equations

Compressional wave perturbations are given in terms of displacement:

$$\delta n = -n_0 \nabla \cdot \boldsymbol{\xi}$$

$$\delta p = -\gamma p_0 \nabla \cdot \boldsymbol{\xi}$$

$$\delta B_{\parallel} = -B_0 \nabla \cdot \boldsymbol{\xi}_{\perp}$$



Results in familiar property of magnetosonic waves

- **Fast modes** correlated magnetic field & density/pressure
- **Slow modes** anticorrelated magnetic field & density/pressure

Often used as a test to classify observed waves

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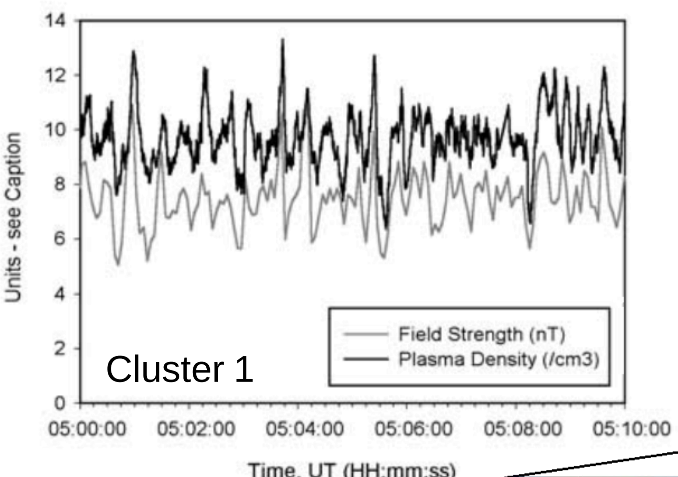
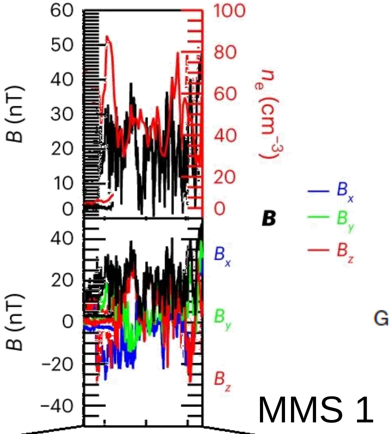
Observational Examples: Fast modes

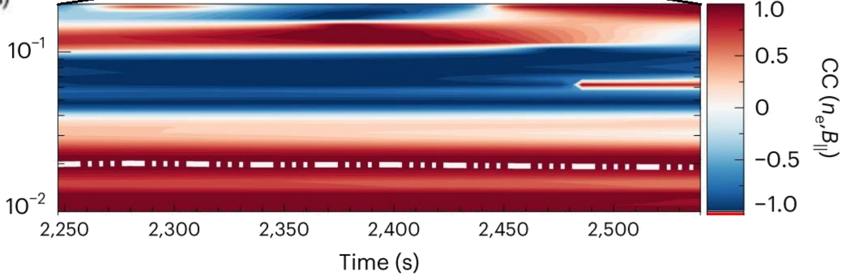
Foreshock



Eastwood+ (2002, GRL)

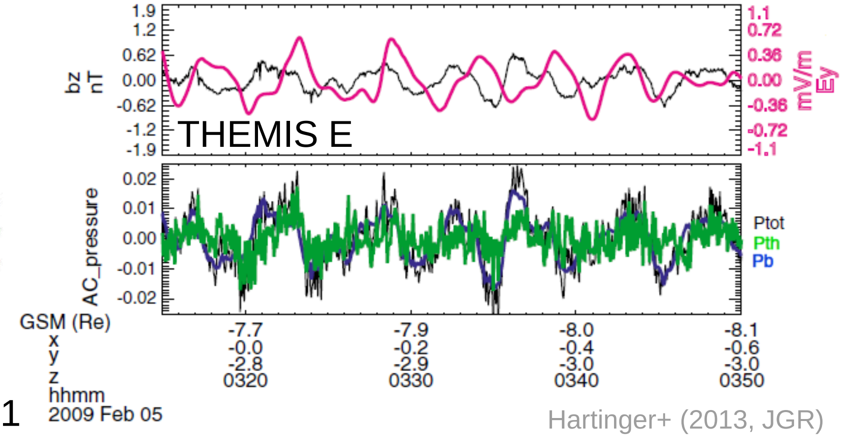
Magnetosheath





Turc+ (2022, Nature Phys.)

Magnetosphere



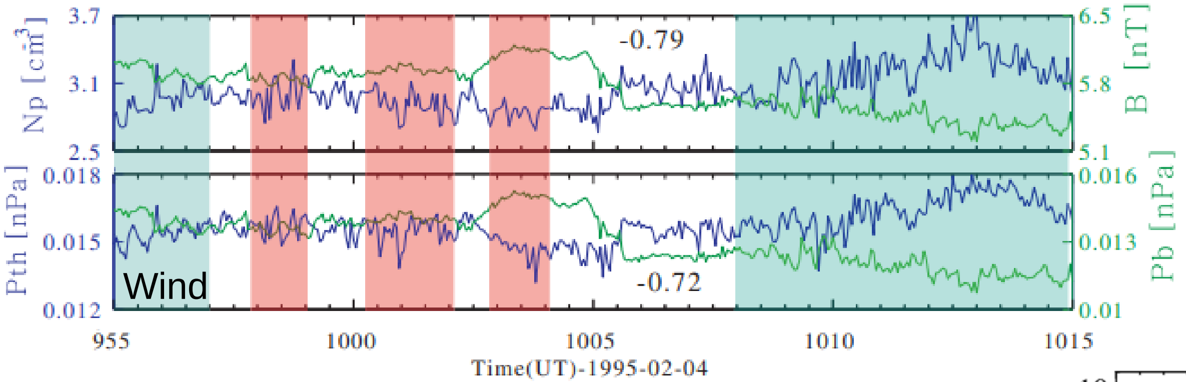
Hartinger+ (2013, JGR)

Fast modes in the solar wind are rarely reported

Observational Examples: Slow modes

Solar Wind

Yao+ (2013, ApJ)



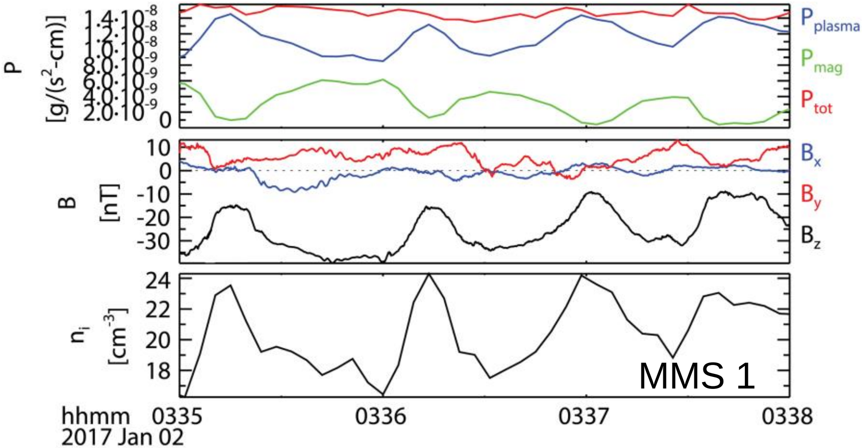
Slow modes in the foreshock are rarely reported

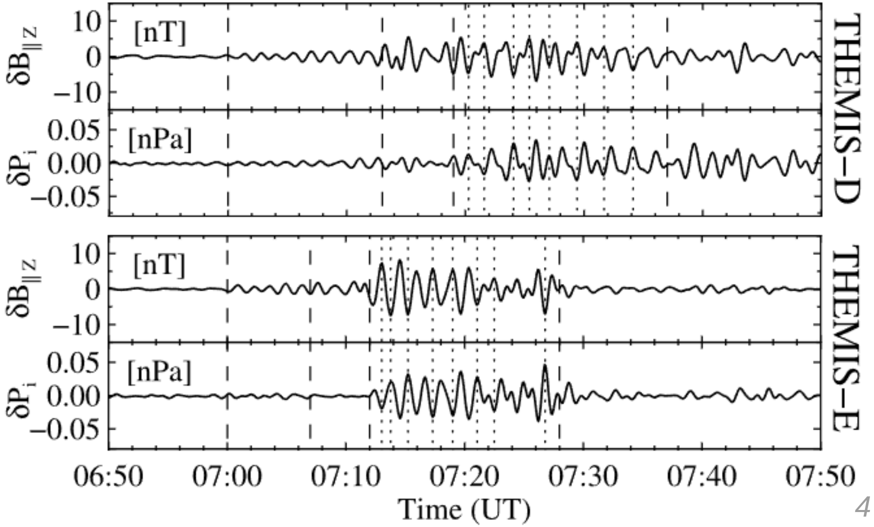
Magnetosphere

Du+ (2011, ApJ)

Magnetosheath

Zhao+ (2019, JGR)

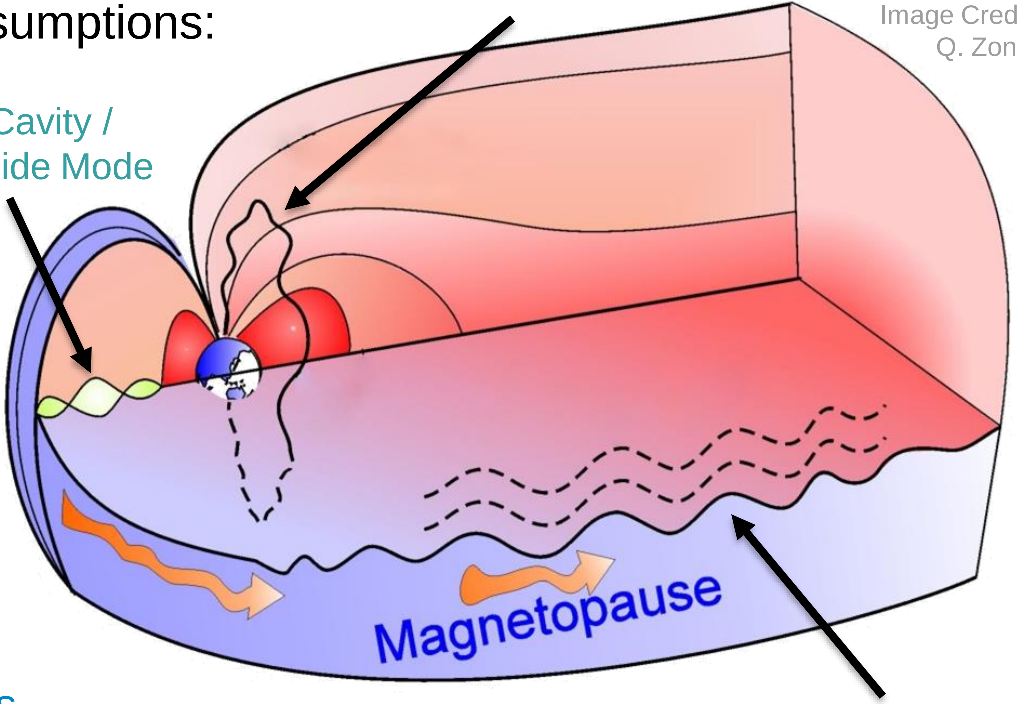




BUT…

This property was derived under the assumptions:

- Propagating plane wave
- Real wavenumbers
- Infinite uniform plasma



Alfvén Field Line Resonance

Fast Cavity / Waveguide Mode

Quasi-Fast Surface Mode

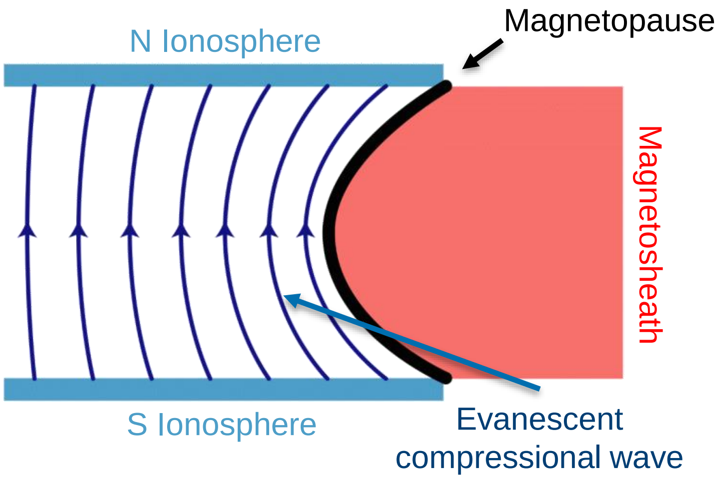
Image Credit Q. Zong

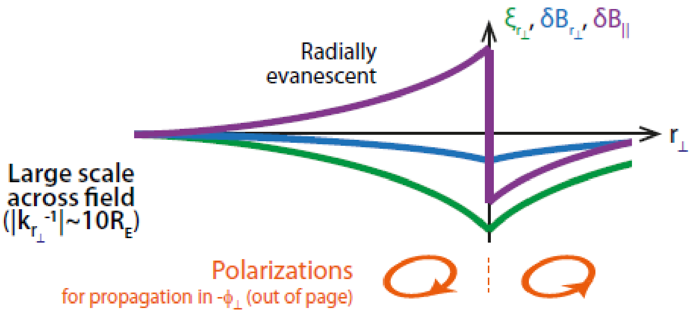
Often not valid in a magnetosphere where MHD waves are system-scale & fall in ULF band (~0.1-100mHz)

- Inhomogeneous plasmas
- Curvilinear magnetic field geometry
- Standing waves between boundaries
- Evanescent surface modes on boundaries
- Non-Ideal plasma (e.g. temperature anisotropy)

Do these alter the correlation between the magnetic field & plasma?

Magnetopause Surface Waves





Archer+ (2022, JGR)

- First considered in simple box models
  (Sen, 1963, Phys. Fluids; Chen & Hasegawa, 1974, JGR; Pu & Kivelson, 1983, JGR; Plaschke & Glassmeier, 2011, AG)
- Collective mode consisting of evanescent magnetosonic wave within magnetosphere
- Quasi-fast mode has correlated $\delta B_{\parallel}$ & δn, δp
  (propagating plane wave in infinite compressible plasma)
- Reflection by ionosphere results in standing wave **magnetopause surface eigenmode (MSE)**
- Lowest frequency normal mode of system

$$\omega \approx \mathrm{k}_{\parallel} \frac{B_{0,sph}}{\sqrt{\mu_0 \rho_{0,msh}}} \ll \omega_A$$

- Discovered in multipoint spacecraft observations
  (Archer+, 2019, Nature Comm.)

Investigate field & plasma correlations associated with surface waves

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Parallel Standing Structure

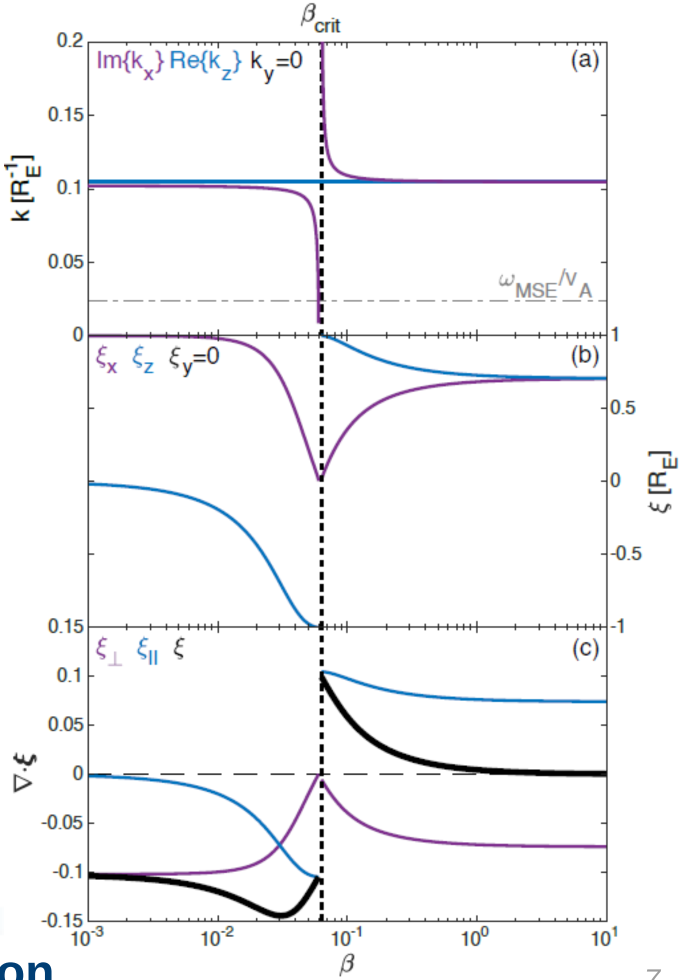
In compressible box model magnetosphere for a wide range of plasma β, we calculate for MSE:

- Wavenumber k_x via magnetosonic dispersion relation
- Displacement amplitudes for standing wave solutions (numerical solution)
- Surface wave perturbations from displacement:

$$\frac{\delta n}{n_0} = \frac{\delta p}{p_0} = -\nabla \cdot \xi$$
$$\delta B_{\|}/B_0 = -\nabla \cdot \xi_{\perp}$$

Transition occurs at $\beta_{\text{crit}} = 2\left\{\gamma\left[\left(\frac{\omega_A}{\omega}\right)^2 - 1\right]\right\}^{-1}$

- $\beta < \beta_{\text{crit}}$ plasma & field remain **correlated**
- $\beta > \beta_{\text{crit}}$ plasma & field become **anticorrelated**
- $\beta \gg 1$ plasma becomes **incompressible**



Reversal due to interference pattern in parallel direction

Inhomogeneous Plasma

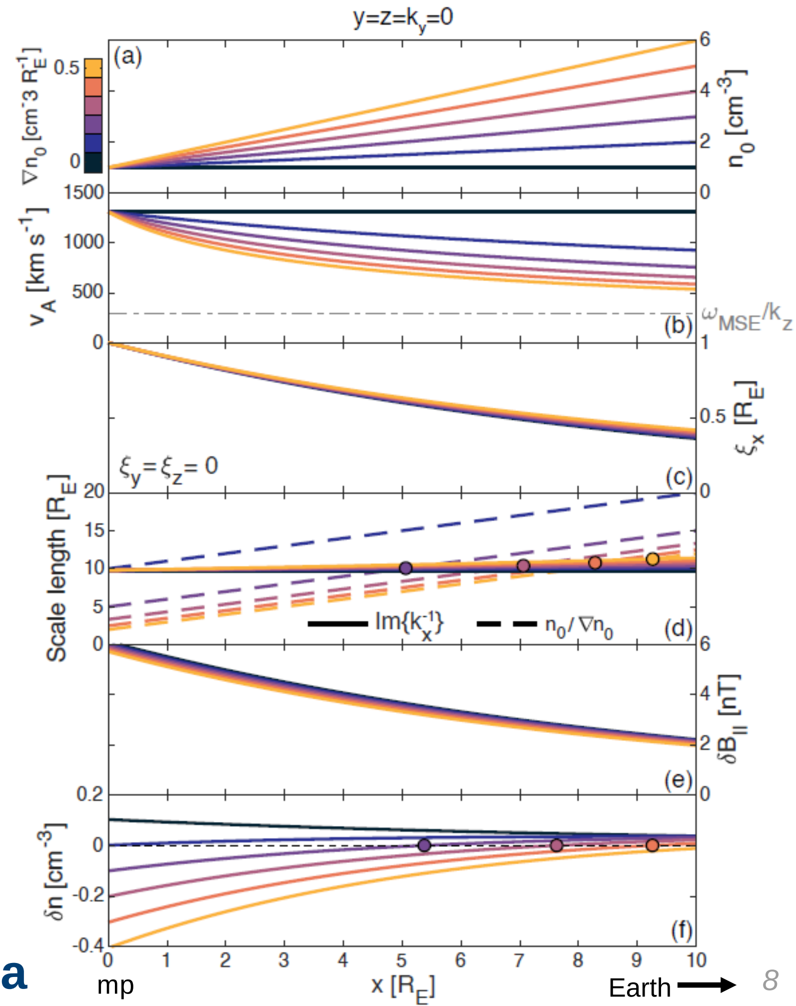
In cold ($\beta = 0$) plasma box model magnetosphere with background density gradient, we calculate:

- MSE displacement amplitudes using WKB
- Surface wave perturbations, which now include both **compression** and **advection** effects:

$$\delta n = -n_0 \nabla \cdot \xi - \xi \cdot \nabla n_0$$
$$\delta B_{\parallel} = -B_0 \nabla \cdot \xi_{\perp} - [(\xi_{\perp} \cdot \nabla)\boldsymbol{B}_0]_{\parallel}$$

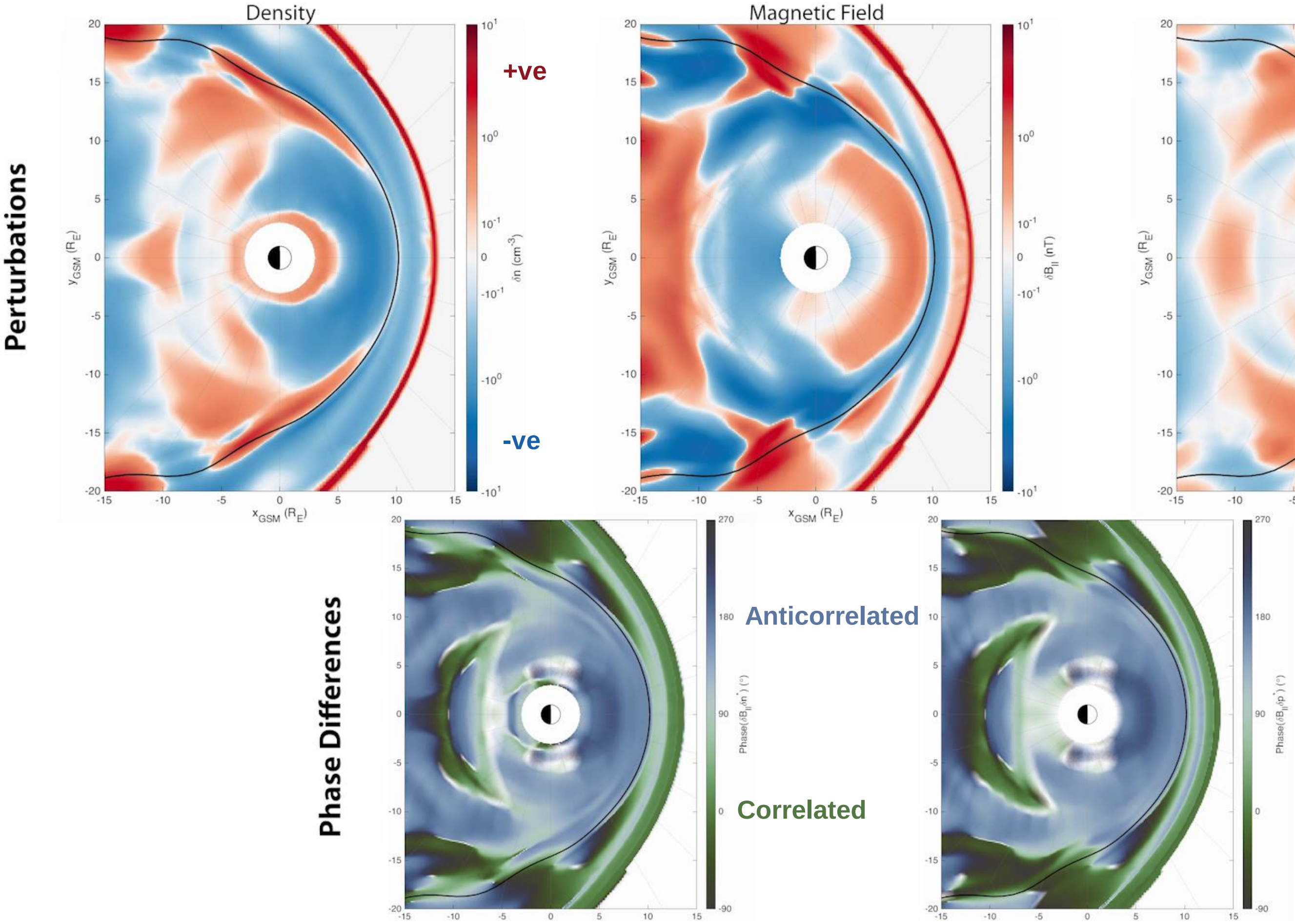
Advection of non-unform plasma by wave becomes more important than inherent wave compression for large density gradients

Leads to anticorrelation of field and density



Reversal due to advection of non-uniform plasma

High-Res Global MHD simulation



Previously published simulation of MSE excited by solar wind density pulse

(Hartinger+, 2015, GRL; Archer+, 2021, Nature Comm.)

Global waves have mostly anticorrelated field & plasma despite MSE being a fast mode

17 min

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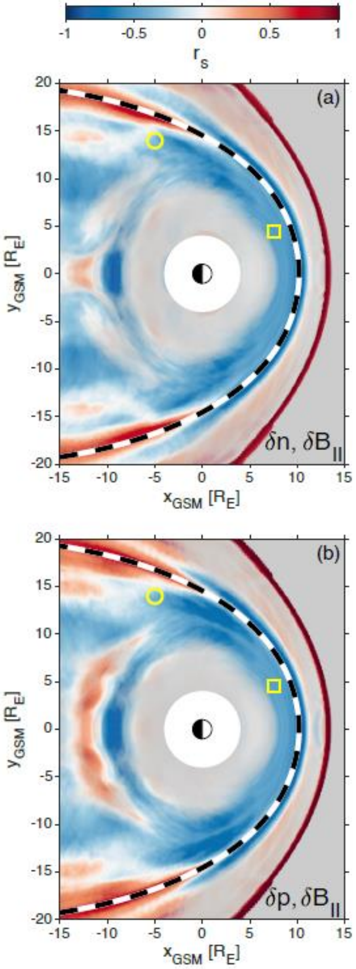
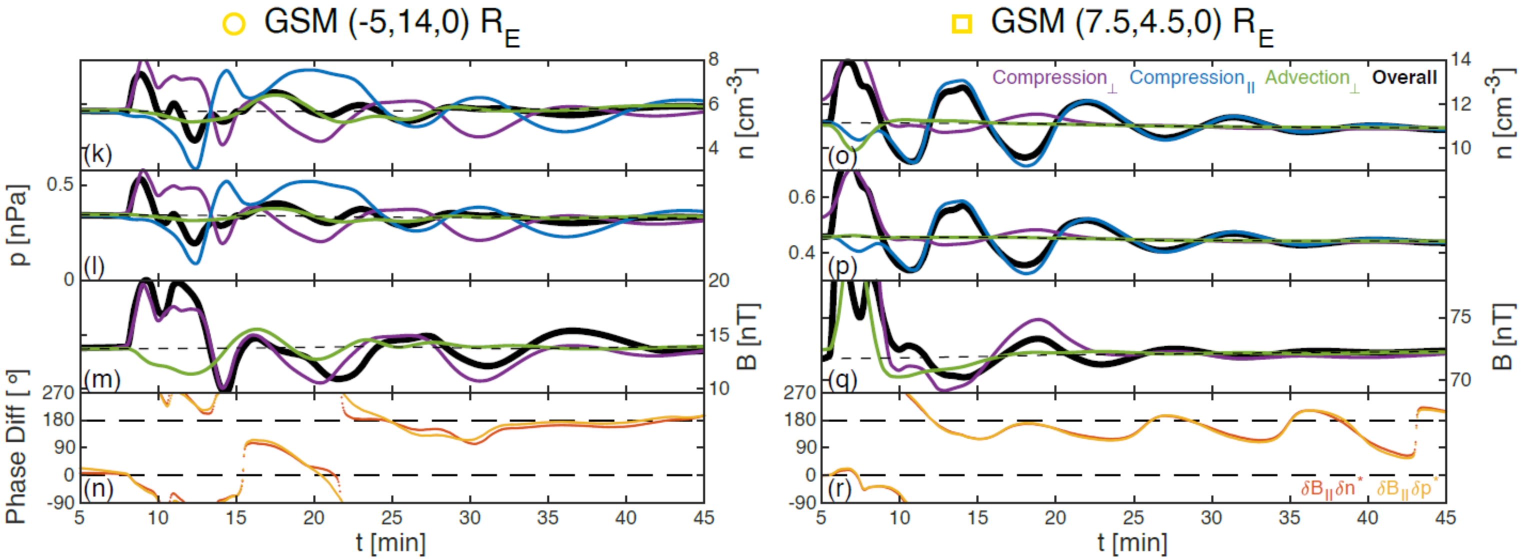
High-Res Global MHD simulation

Contributions to wave due to **compression** (⊥ & ∥) and **advection** (⊥)





Which term dominates the overall wave response?

Magnetic field: Everywhere **compression$_{\perp}$**

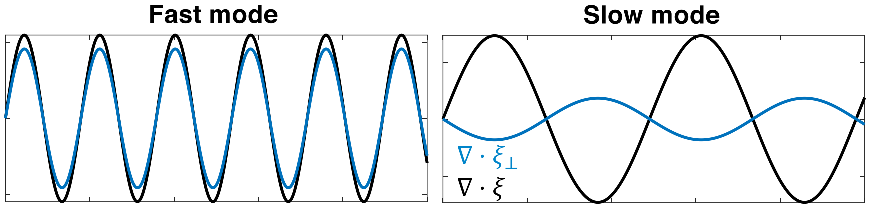
Plasma: Flanks (○) **advection$_{\perp}$** (wave approx. incompressible as $\beta \gg 1$)

Dayside (□) **compression$_{\parallel}$** (finite $\beta \sim 0.1 > \beta_{crit}$)

Both effects discussed important in different regions of system

Conclusions

Re-evaluated the magnetic field – plasma correlation test for magnetosonic waves



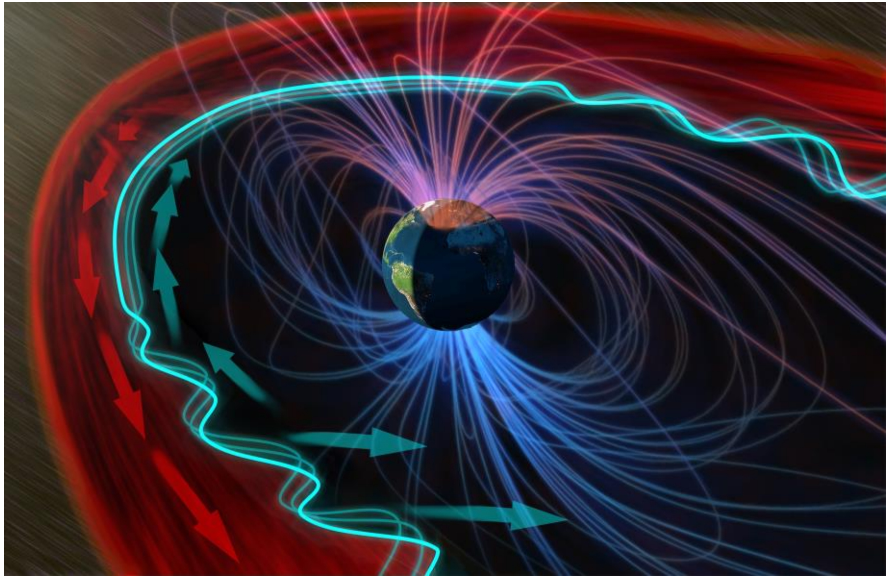
Strictly valid only for:

- Propagating plane wave
- Real wavenumbers
- Infinite uniform plasma

Often not true in the magnetosphere

Examples: fast mode magnetopause surface waves have **anticorrelated** plasma & fields for:

Parallel standing structure	Due to interference pattern of displacement along field above β_{crit}
Non-uniform background plasma	Due to wave's advection of plasma with different background dominating over compression for large gradients



Demonstrated theoretically and in a global simulation

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