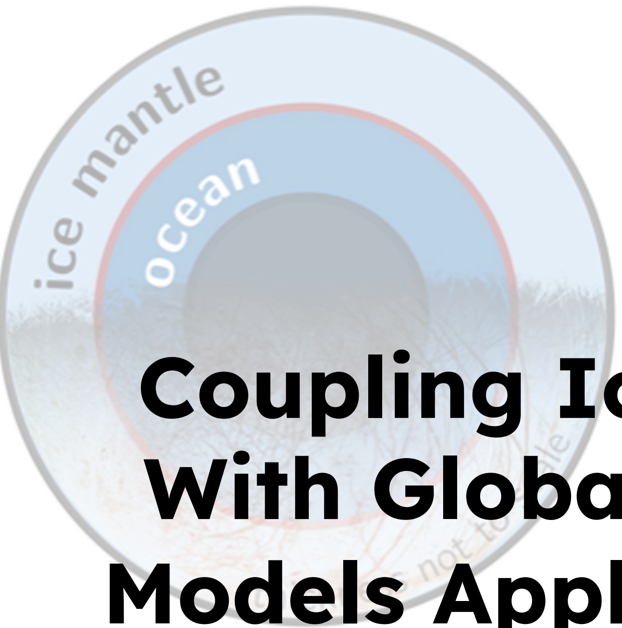

Coupling Ice-Ocean Interface Models With Global-Scale Ice Shell Evolution Models Applied to Jovian Moon Europa

EGU 2023, April 24, Vienna, Austria

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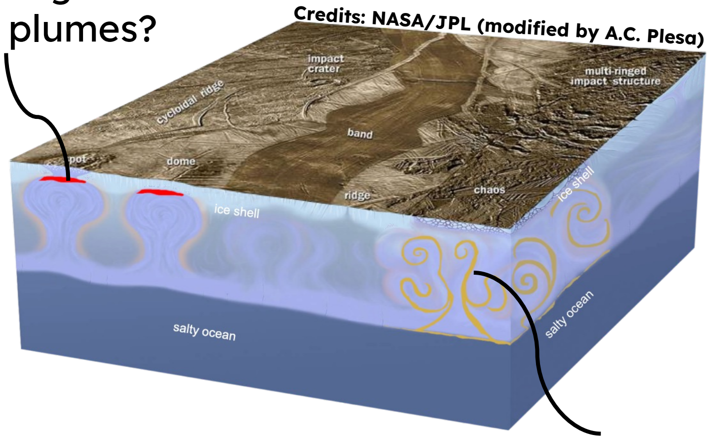
The Importance of Salts in the Outer Solar System

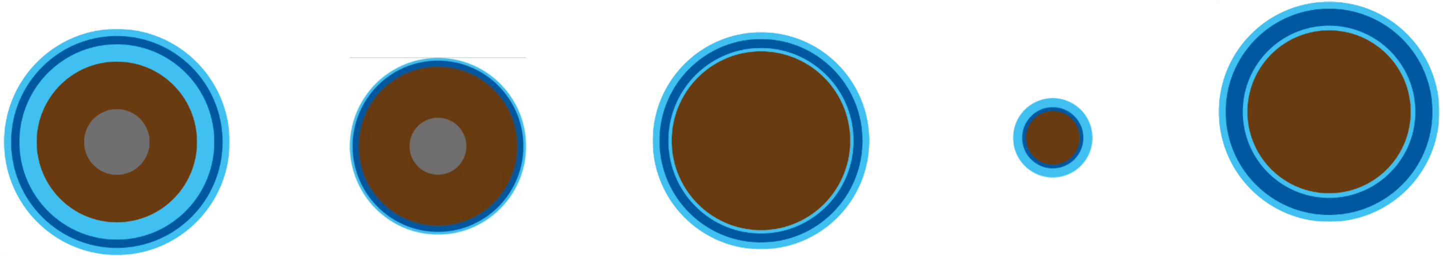
- Liquid reservoirs in our solar system moons (oceans, brine inclusions) **potential habitats** for extraterrestrial life

- How do these liquid reservoirs form and can they stay liquid over long period of times?

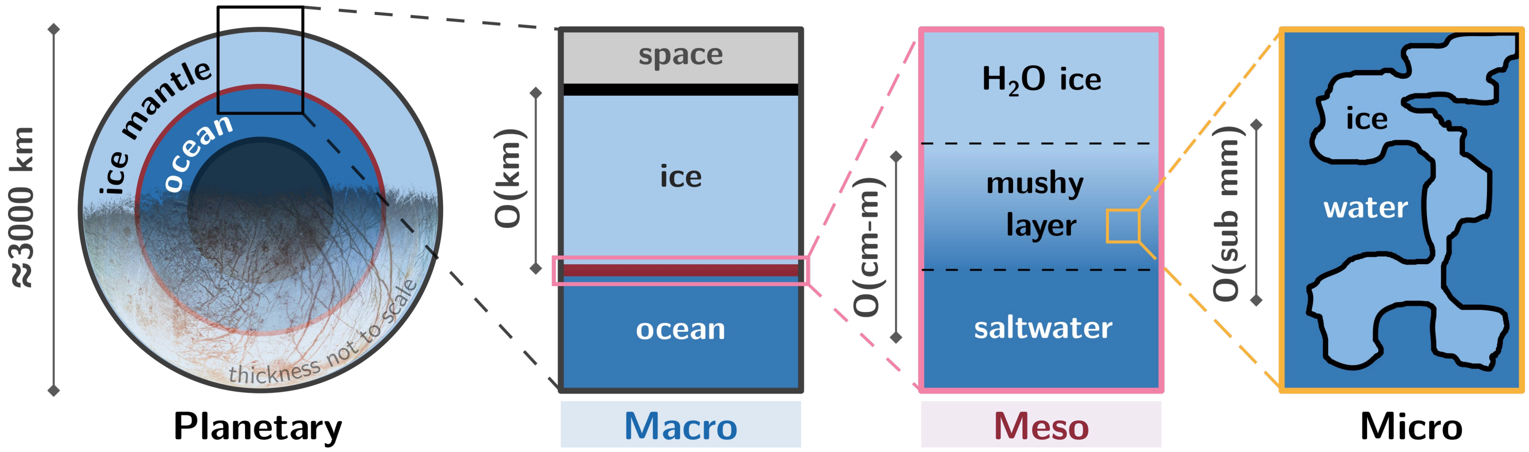
Melting due to hot plumes?



Transport of brines due to convection?

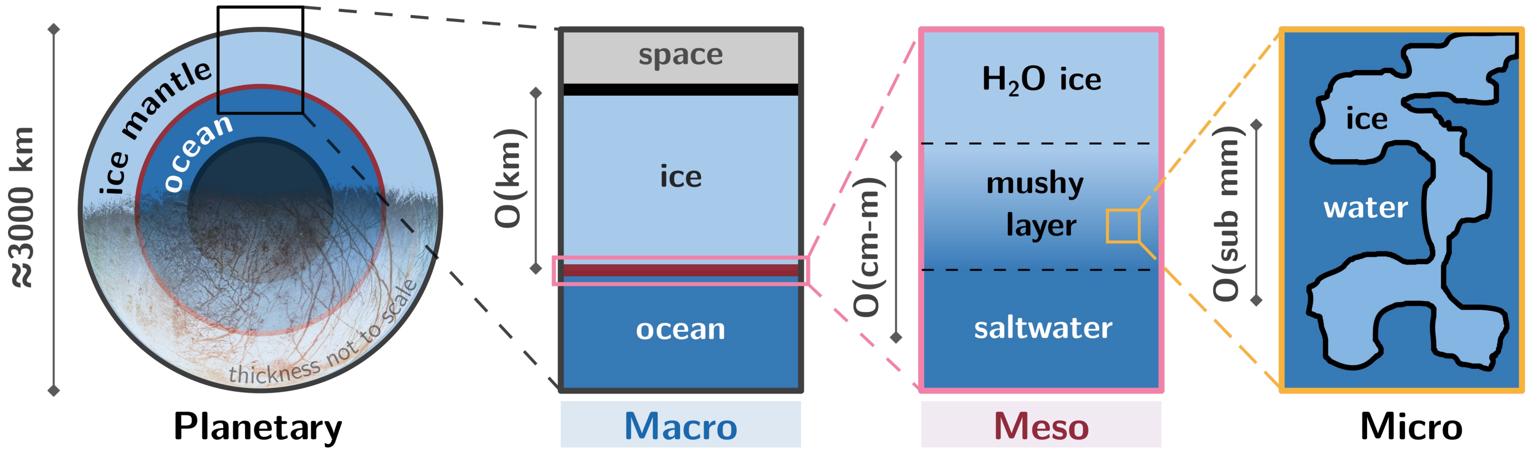


Salt Uptake - The Ice-Ocean System



- Mushy layer connection between ice and ocean
- ⇛ **Macro scale:** solid state convection and/or two-phase flow in the ice shell
- ⇛ **Meso scale:** two-phase flow in highly porous mushy layer

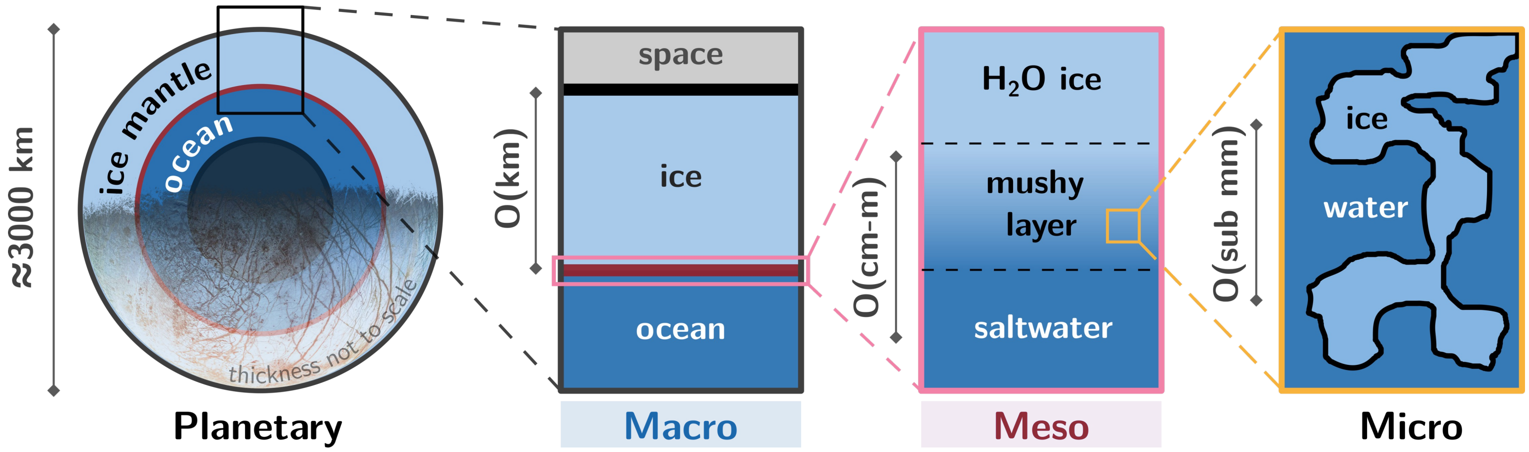
Salt Uptake - The Ice-Ocean System



- Barr et al. , 2004, 2005, 2007
- Han and Showman 2005
- Howell and Pappalardo 2018
- Harel et al. 2020
- ...

- Feltham et al. 2006
- Huppert and Worster 1985
- Griewank and Notz 2013
- **Buffo et al. 2018, 2020, 2021**
- ...

Salt Uptake - The Ice-Ocean System



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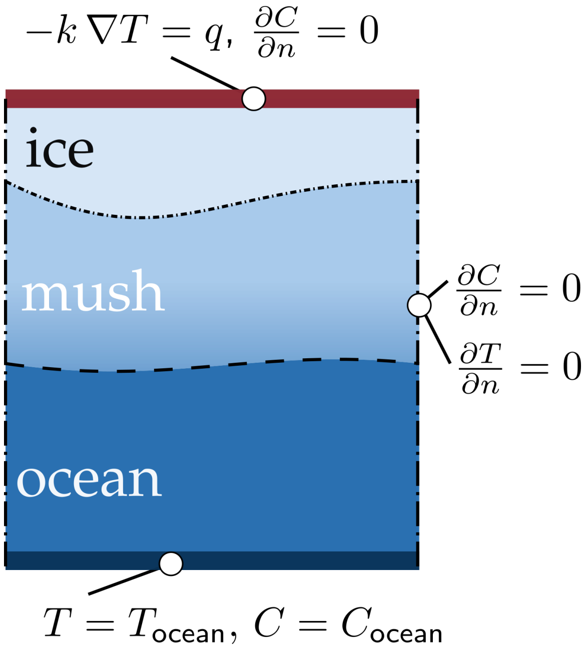
- ...

Meso-Scale Model: BRYNE



Compute:

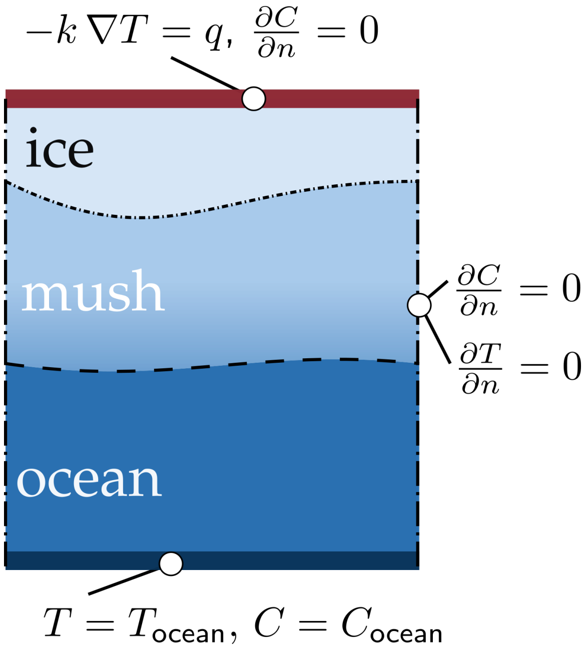
salt concentration C

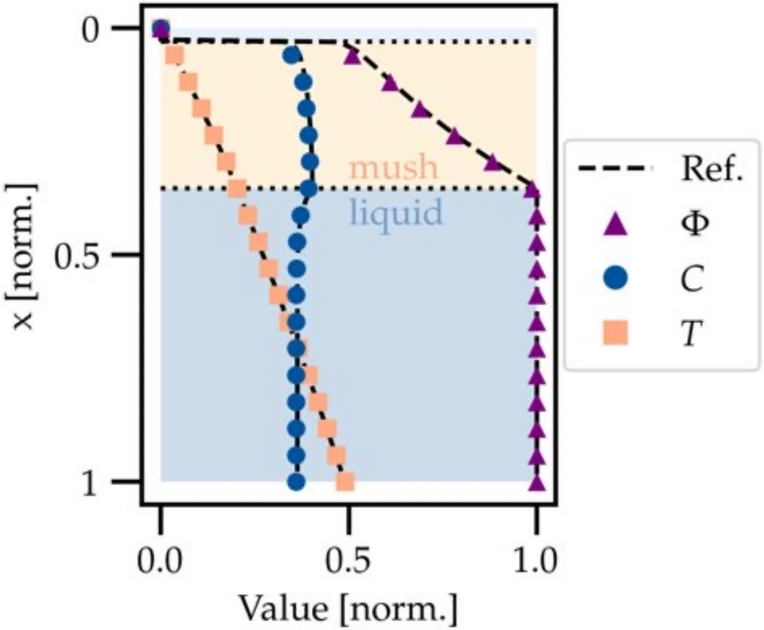
temperature T

liquid fraction $\Phi(C,\,T)$

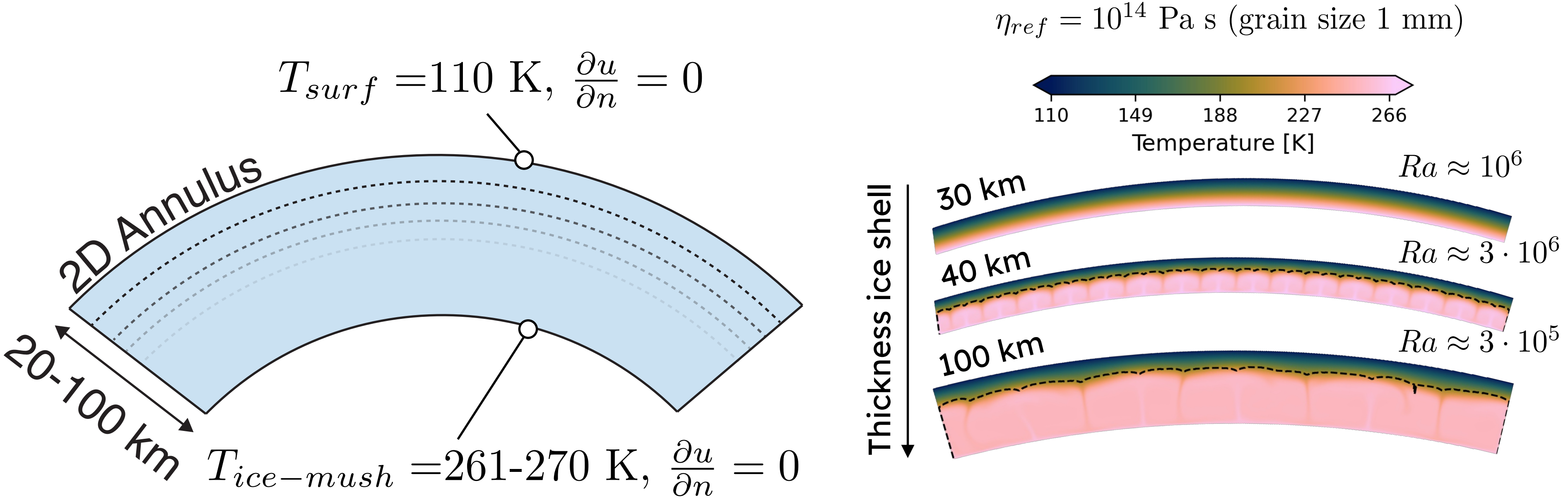
across the meso-scale mixture soldification interface.

Meso-Scale Model: BRYNE





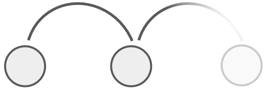
Macro-Scale Model: GAIA



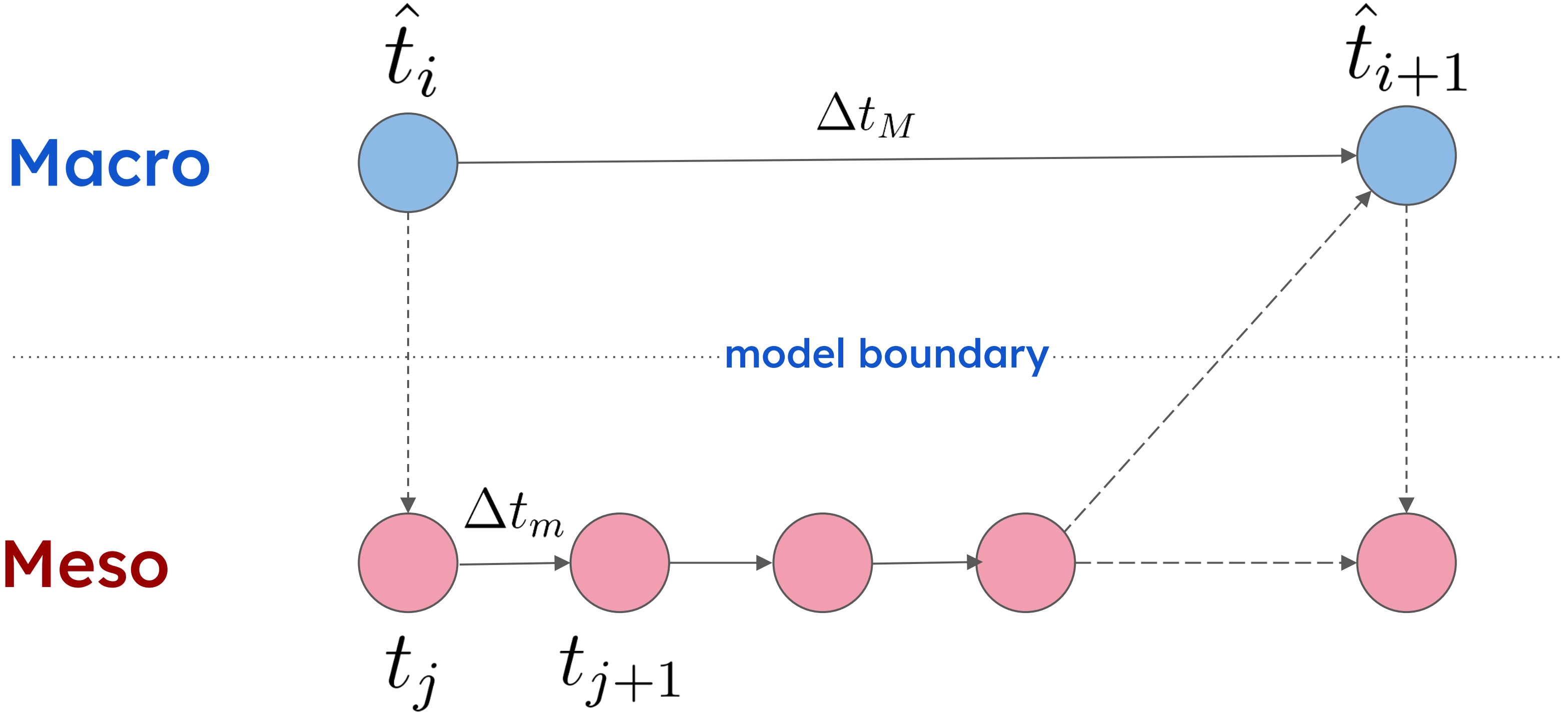
- Conservation of mass, momentum, and energy (non-dimensional) for incompressible, low Re fluid (Stokes flow) solved with fluid flow solver GAIA **(Hüttig and Stemmer, 2008)**

Multi-Scale Coupling

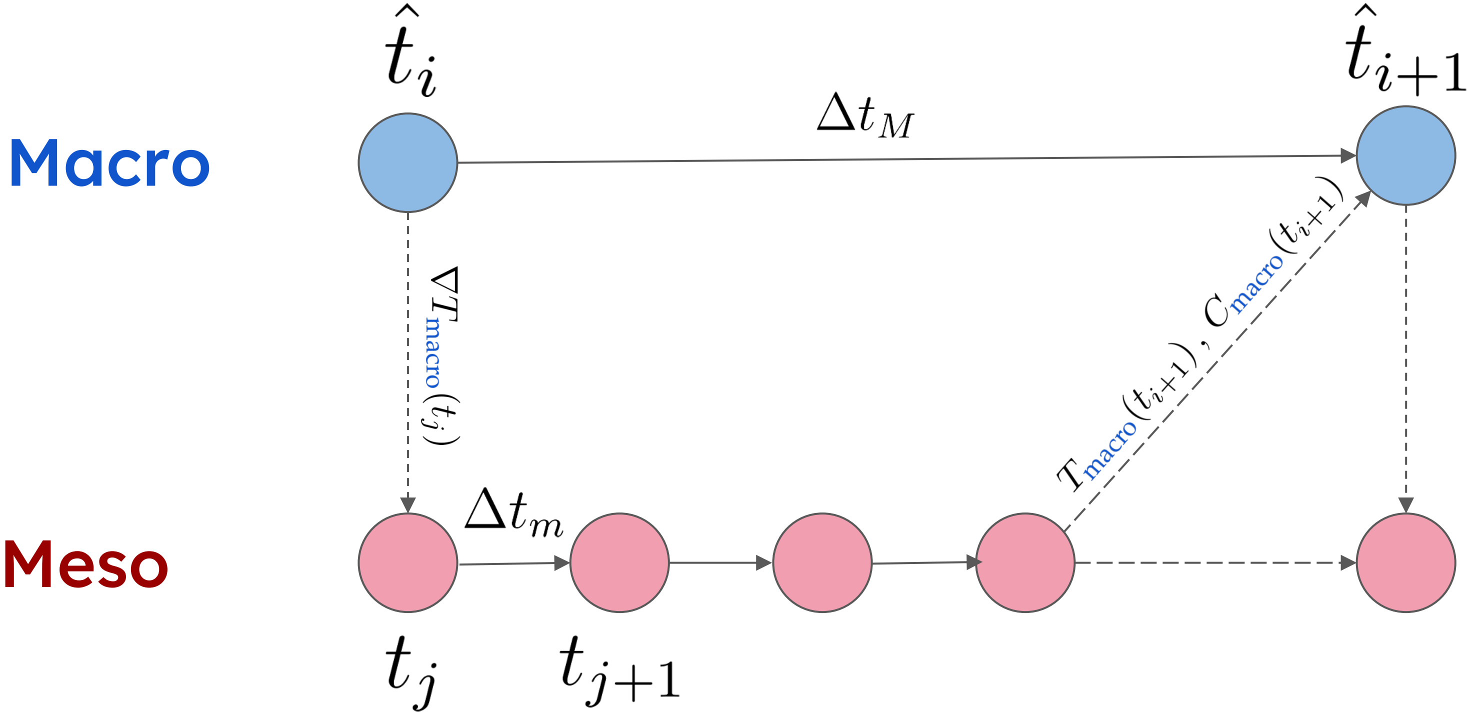
Characteristic Scales

	Macro	Meso
 Spatial Scale	$10^2 - 10^3$ m	$10^{-2} - 10^0$ m
 t_i t_{i+1} t_{i+2} Temporal Scale	$10^0 - 10^2$ [years]	[days] – [years]
$\alpha, \beta, \gamma, \ldots$ Vector of Unknowns	$\mathbf{U}_M = (\mathbf{v}_s, p_s, T, C_{\text{bulk}})$	$\mathbf{U}_m = (\mathbf{v}_l, p_l, T, C_{\text{bulk}})$

Multi-Scale Time Coupling



Multi-Scale Time Coupling

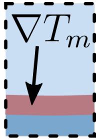


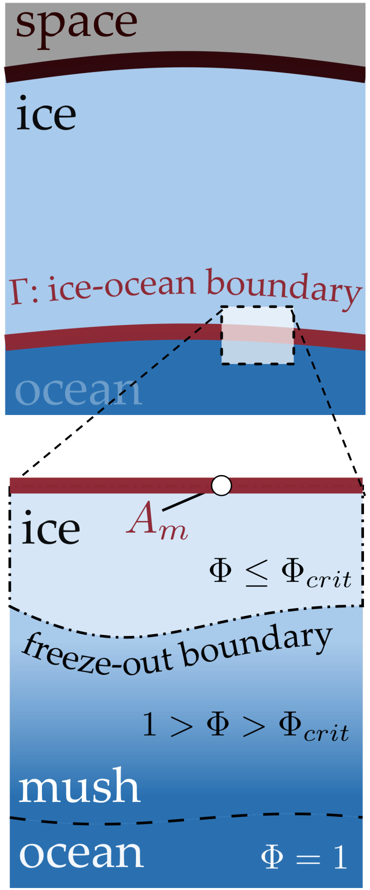
Spatial Coupling

Macro

flux is meso-surface average:

$$\nabla T_{\text{macro}} \hat{=} \frac{1}{|A_m|} \int_{A_m} \nabla T_{\text{meso}} \cdot \mathbf{n} \, \mathrm{d}A$$

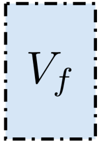




Meso

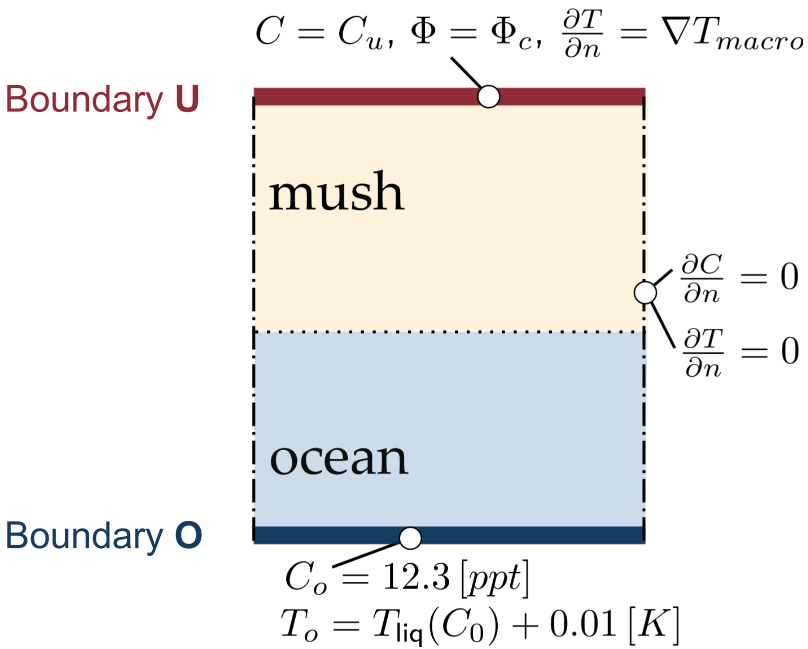
state is volume-averaged over freeze-out volume

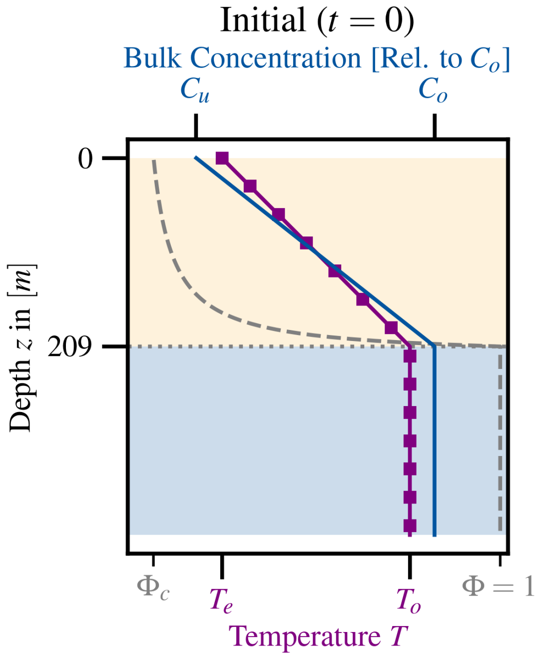
e.g. $T_{\text{macro}} = \frac{1}{V_f} \int_{V_f} T_{\text{meso}} \, \mathrm{d}V$



First Simple Problem Setup

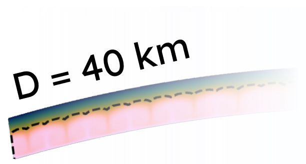
- Test influence of different heat flux boundary conditions from macro scale model on meso scale model (i.e. mushy layer growth, thickness)



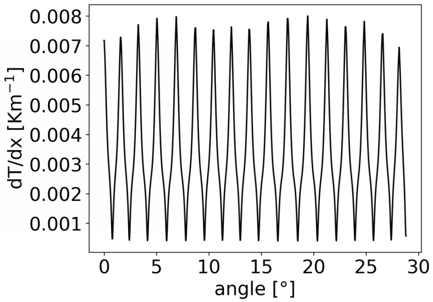


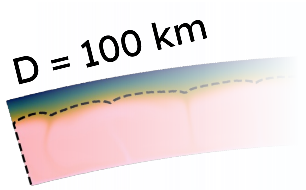
Preliminary Test Results (dT/dz = 0.005 [K/m])

$G = 1\ \mathrm{mm}\ (\eta_{ref} = 10^{14}\ \mathrm{Pa\ s})$

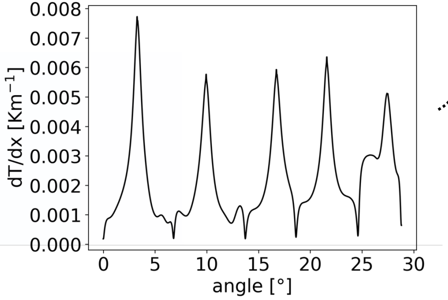


$Ra \approx 3 \cdot 10^6$

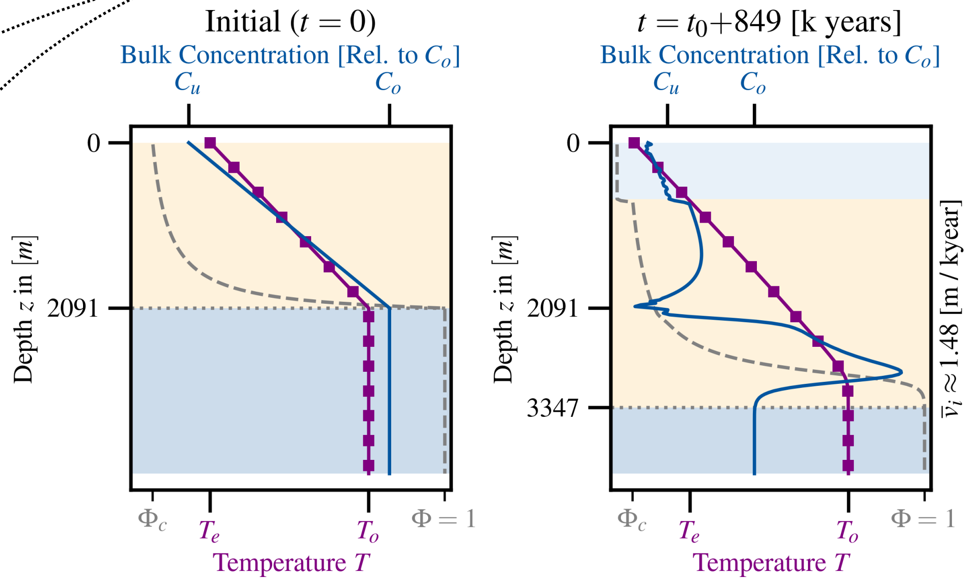




$Ra \approx 3 \cdot 10^5$



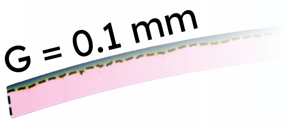
$\frac{\partial T}{\partial n} = 0.005\,[K/m], C_u = \Phi_c \cdot C_l(T_e)$



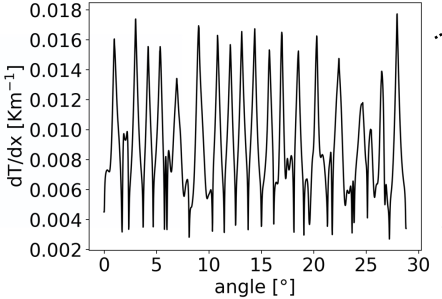
- Growth rate mushy layer: 1.48 m/kyrs
- Thickness of mushy layer: kms

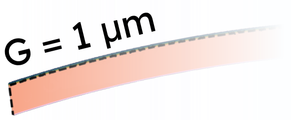
Preliminary Test Results (dT/dz = 0.05 [K/m])

$D = 30$ km



$\eta_{ref} \approx 4 \cdot 10^{12}$
$Ra \approx 10^{8}$





$\eta_{ref} \approx 4 \cdot 10^{8}$
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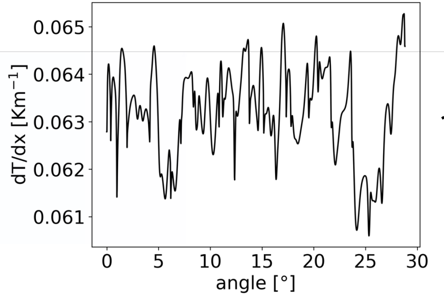
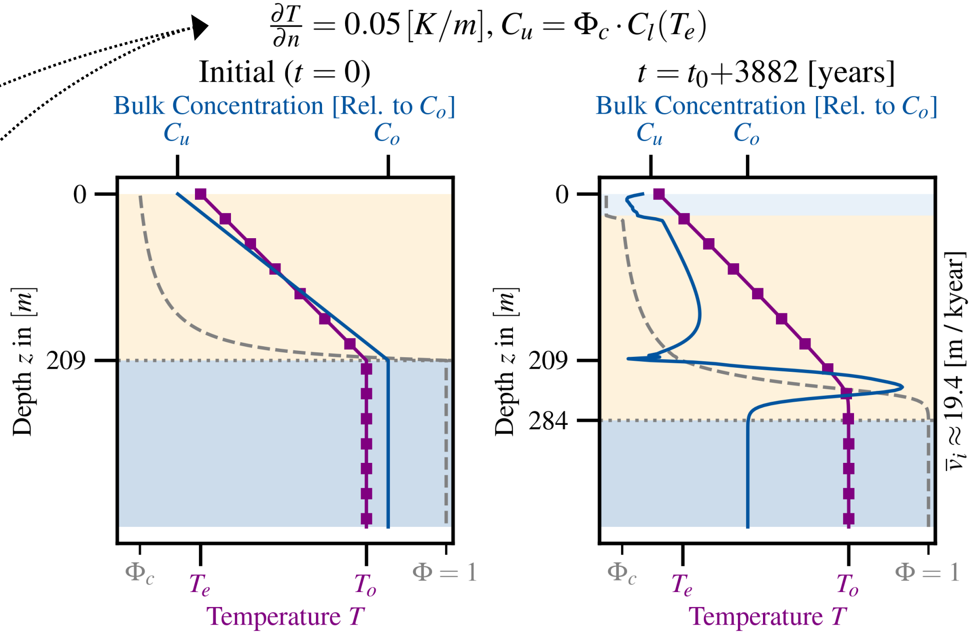




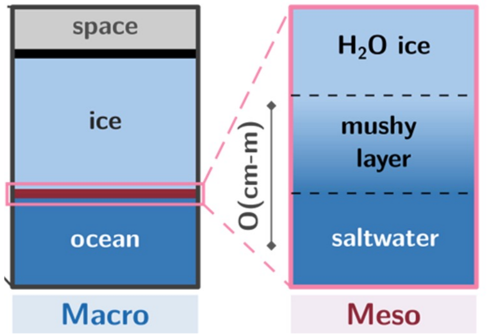
- Growth rate mushy layer: 19.4 m/kyrs
- Thickness of mushy layer: 10^2 m

Conclusion

- Salt uptake in ice-ocean system is a multiscale problem
  - Mushy layer: cm to m, days to years
  - Ice shell: m to km, year to hundreds/thousands of years

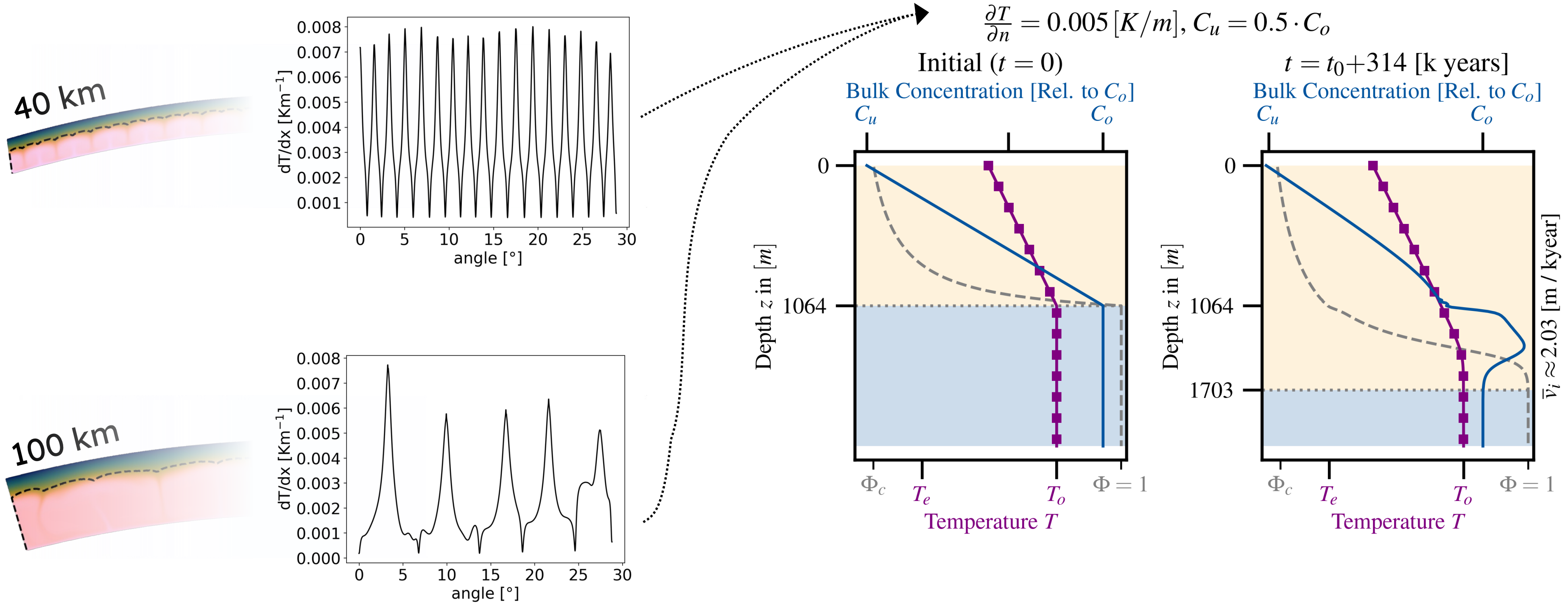
- Mushy layer determines how much salt is incorporated into ice shell
- Ice shell governs cooling rate of the mushy layer

- **Preliminary results (testing influence of heat flux boundary conditions provided by macro scale):** Higher heat fluxes yield …
  - Faster growth of the mushy layer
  - Thinner mushy layers



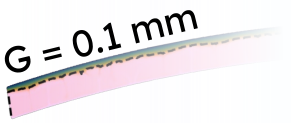
Backup Slides

Multi-Scale Coupling: Results (dT/dz = 0.005 [K/m])



Multi-Scale Coupling: Results (dT/dz = 0.05 [K/m])

$D = 30$ km



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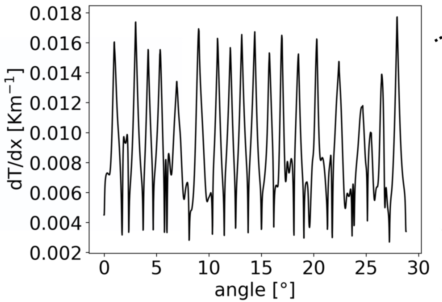

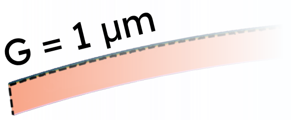

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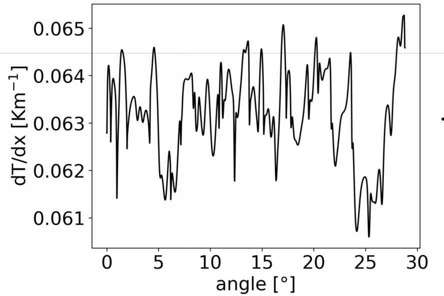
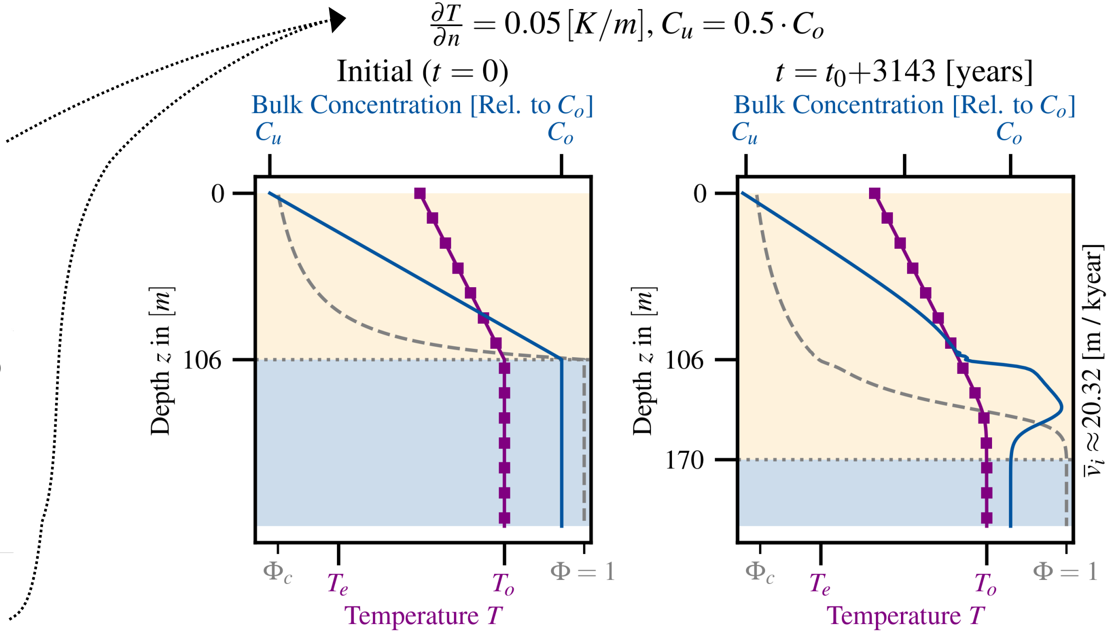


$\frac{\partial T}{\partial n} = 0.05\,[K/m], C_u = 0.5 \cdot C_o$



Meso-Scale Model: Equations

Two Phase Energy Conservation

$$\text{(T1)} \quad \frac{\partial(\overline{H})}{\partial t} + \nabla \cdot (\mathbf{q}\, H_l - \overline{\kappa}(\Phi)\nabla T) = 0$$

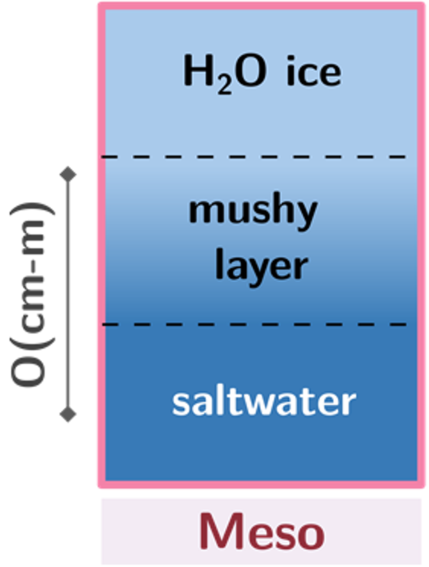
Solute (*Salt*) Transport

$$\text{(T2)} \quad \frac{\partial \overline{C}}{\partial t} + \nabla \cdot (\mathbf{q}\, C_l' - D_l(\Phi)\nabla C_l') = 0$$

Closure / Mixture Relations

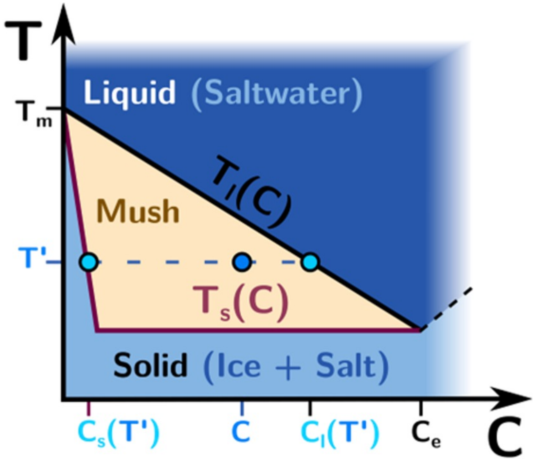
$$\text{(T3)} \quad \Phi = \frac{\overline{C} - C_s}{C_l - C_s}, \quad \overline{\kappa} = \Phi\kappa_l + (1-\Phi)\,\kappa_s$$

$$\overline{H} = \Phi\, H_l + (1-\Phi)\, H_s\,, \quad H_l = \rho_s\, c_{p,l}\, T + \rho_l\, L\,, \quad H_s = \rho_s\, c_{p,s}\, T$$



https://doi.org/10.1016/j.jcpx.2019.100043
https://doi.org/10.1029/2020JE006741

Meso-Scale Model: Equations



Solute (*Salt*) Transport

$$\text{(T2)} \quad \frac{\partial \overline{C}}{\partial t} + \nabla \cdot (\mathbf{q}\, C_l' - D_l(\Phi) \nabla C_l') = 0$$

Nonlinear Constraint

$$C_l' \equiv C_l'[\overline{C}, T] = \begin{cases} 0 & T < T_e \\ C_l(T) & T_e <= T <= T_l(\overline{C}) \\ \overline{C} & T > T_l(\overline{C}) \end{cases}$$

Meso-Scale Model: Equations

Two Phase Mass Conservation

$$\text{(M1)} \quad \left(1 - \frac{\rho_s}{\rho_l}\right)\frac{\partial \Phi}{\partial t} + \nabla \cdot \mathbf{q} = 0 \qquad \mathbf{q} = \Phi\, \hat{\mathbf{v}}_l$$

Assumptions

no advection in the solid ($v_s \equiv 0$)
phase-wise constant densities ρ_l, ρ_s

Conservation of Momentum (*Darcy-Brinkman Equation*)

$$\text{(M2)} \quad \rho_l\left(\frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} \cdot \nabla\left(\frac{\mathbf{q}}{\Phi}\right)\right) = -\Phi\, \nabla \hat{p}_l + \eta_l \nabla^2 \mathbf{q} + \Phi\, \tilde{\rho}_l\, \mathbf{g} - \underbrace{\eta_l\, \Phi\, \underline{\underline{\Pi(\Phi)}}^{-1}\, \mathbf{q}}_{\text{Darcy Term}}$$

Macro-Scale Model: Equations

Conservation Equations

$$\nabla \boldsymbol{u} = 0$$

$$\underbrace{\nabla p}_{\text{pressure}} + \underbrace{\nabla \cdot [\eta(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T)]}_{\text{viscous}} = \underbrace{RaTe_r}_{\text{buoyancy}}$$

$$\frac{\partial T}{\partial t} + \underbrace{\boldsymbol{u} \cdot \nabla T}_{\text{heat advection}} = \underbrace{\nabla^2 T}_{\text{heat diffusion}}$$

$$\frac{\partial C}{\partial t} + \boldsymbol{u} \cdot \nabla C = 0\,(Le \lll 1)$$

Rheology

$$\frac{1}{\eta_{eff}} = \frac{1}{\eta_{diff}} + (\eta_{gbs} + \eta_{bs})^{-1} + \frac{1}{\eta_{dis}}$$

$$\eta_i = \frac{1}{2A_i^{\frac{1}{n_i}}} \dot{\epsilon}^{\frac{1-n_i}{n_i}} d^{\frac{m_i}{n_i}} \exp\left(\frac{E_i + PV_i}{n_i RT}\right)$$