Conversion of IP data and its uncertainty from Time-Domain to Frequency-Domain using Debye decomposition

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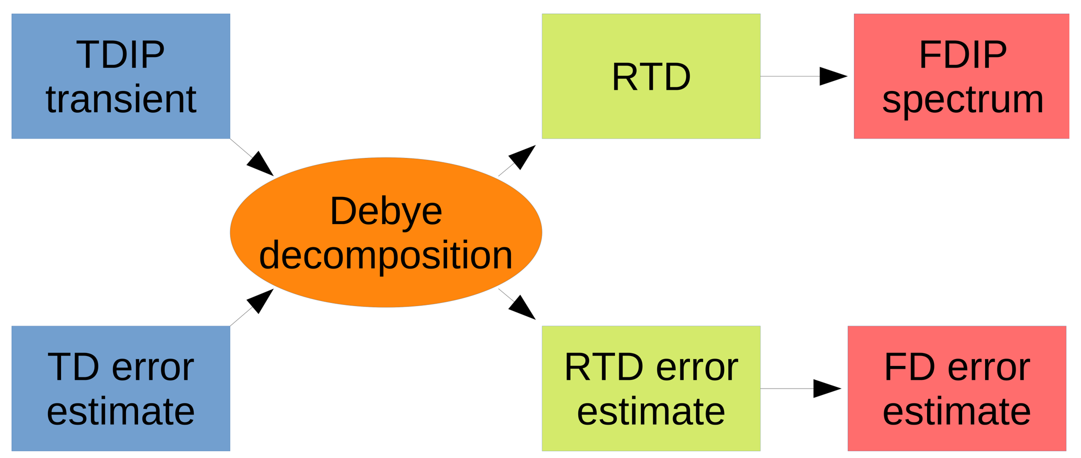
Introduction

The time-domain (TD) induced polarization (IP) method is used as an extension to direct-current resistivity measurements to capture information on the ability of the subsurface to develop electrical polarization. In the TD, the transient voltage decay is measured after the termination of the current injection. In order to invert tomographic TD IP data sets into frequency-domain (FD) models of complex electrical resistivity, a suitable approach for the conversion of TD IP transients and their corresponding uncertainties into the FD is essential. In order to apply existing FD inversion algorithms to TD IP measurements, a conversion approach must transform the measured decay curves into FD impedances and also propagate the corresponding measurement uncertainty from TD into FD. Here we present such an approach based on a Debye decomposition (DD) of the decay curve into a relaxation-time distribution (RTD) and calculation of the equivalent spectrum.



Debye decomposition

We use the Debye decomposition to calculate a FD equivalent to the measured transient. The spectrum of the complex electrical impedance is given by a superposition of Debye relaxation terms, scaled by γ_k:

$$Z(\omega) = R_0 - \sum_{k=1}^{M} \gamma_k \left(1 - \frac{1}{1 + i\omega\tau_k}\right). \qquad (1)$$

Plotting γ_k against τ_k yields the RTD. To estimate the values γ_k we invert the measured transient using the TD forward operator that is equivalent to Equation (1):

$$\eta(t) = \frac{1}{R_0} \sum_{k=1}^{M} \gamma_k \exp\left(-\frac{t}{\tau_k}\right). \qquad (2)$$

To restrict γ_k from taking non-physical values below 0, as well as to achieve a higher consistency in the inversion results, we use a logarithmic parameterization:

$$m_k = \ln(\gamma_k)\,, \qquad (3)$$

with $\gamma_0 = 1\,\Omega$. We solve the resulting non-linear inverse problem using a Pseudo-Newton optimization scheme:

$$\mathbf{m}_{q+1} = \mathbf{m}_q + \alpha\Delta\mathbf{m} = \mathbf{m}_q - \alpha\left(\mathbf{J}^T\mathbf{C}_D^{-1}\mathbf{J} + \lambda\mathbf{R}\right)^{-1}\left(\mathbf{J}^T\mathbf{C}_D^{-1}\left(\mathbf{f}(\mathbf{m}_q) - \mathbf{d}\right) + \lambda\mathbf{R}\mathbf{m}_q\right). \qquad (4)$$

Measurement errors are accounted for in the data precision matrix $\mathbf{C}_D^{-1}$. To make the solution unique we use a smoothing operator $\mathbf{R}$. The regularization strength λ is optimized during the inversion to yield a root-mean square error ε near 1.

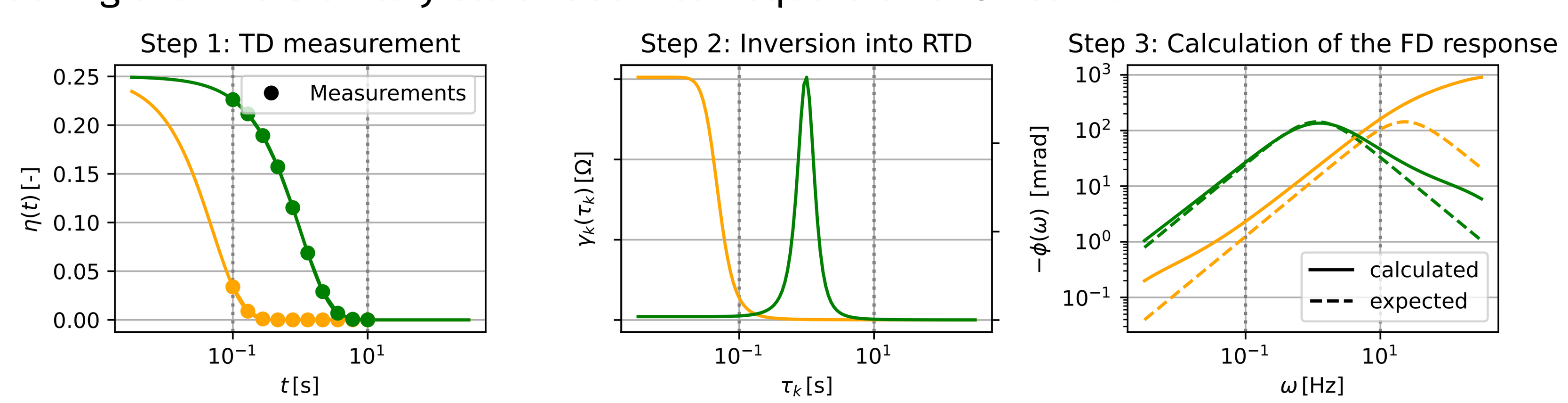


Figure: Conversion of transients to spectra.

Accuracy of phase estimate

To estimate the **accuracy of the phase estimate** we:

1. generated transients with varying relaxation time τ_s,
2. converted the transients into phase estimates at 1 Hz,
3. compared the phase estimates against the expected phases.

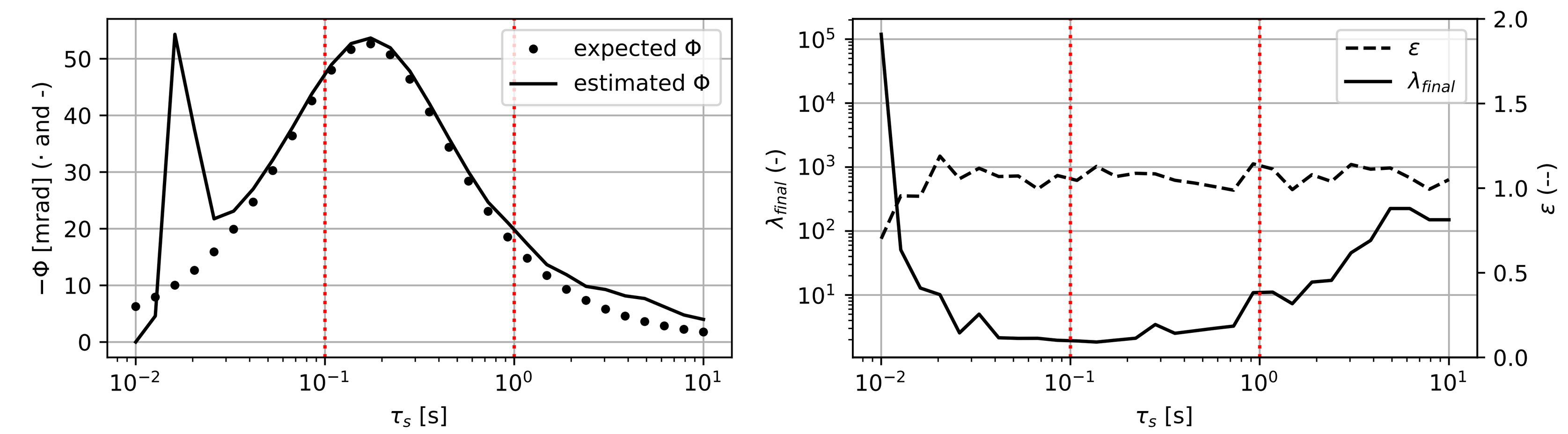


Figure: Phase estimates (left) and corresponding estimates for λ_{final} (right). Within the bounds of the sampled time frame (indicated by the vertical red lines), the estimates are **in agreement with the expected values**.

Accuracy of error propagation

- Conversion of the data error is done using Gaussian-error propagation.
- The model-domain uncertainty is described by using a covariance matrix that isolates the mapping of the data errors (6) (Gubbins, 2004).

$$\mathbf{C}_M = \left(\mathbf{J}^T\mathbf{C}_D^{-1}\mathbf{J} + \lambda\mathbf{R}\right)^{-1} \qquad (5) \qquad\qquad \mathbf{C}_E = \mathbf{C}_M\mathbf{J}^T\mathbf{C}_D^{-1}\mathbf{J}\,\mathbf{C}_M. \qquad (6)$$

To **validate the error propagation** we:

1. calculated the phase estimates for 10 000 noise realizations of the same transient,
2. used the scatter of the phase estimates to calculate a reference standard deviation,
3. compared the results of the error propagation, performed for different noise realizations of the same transient, against the reference-standard deviation.

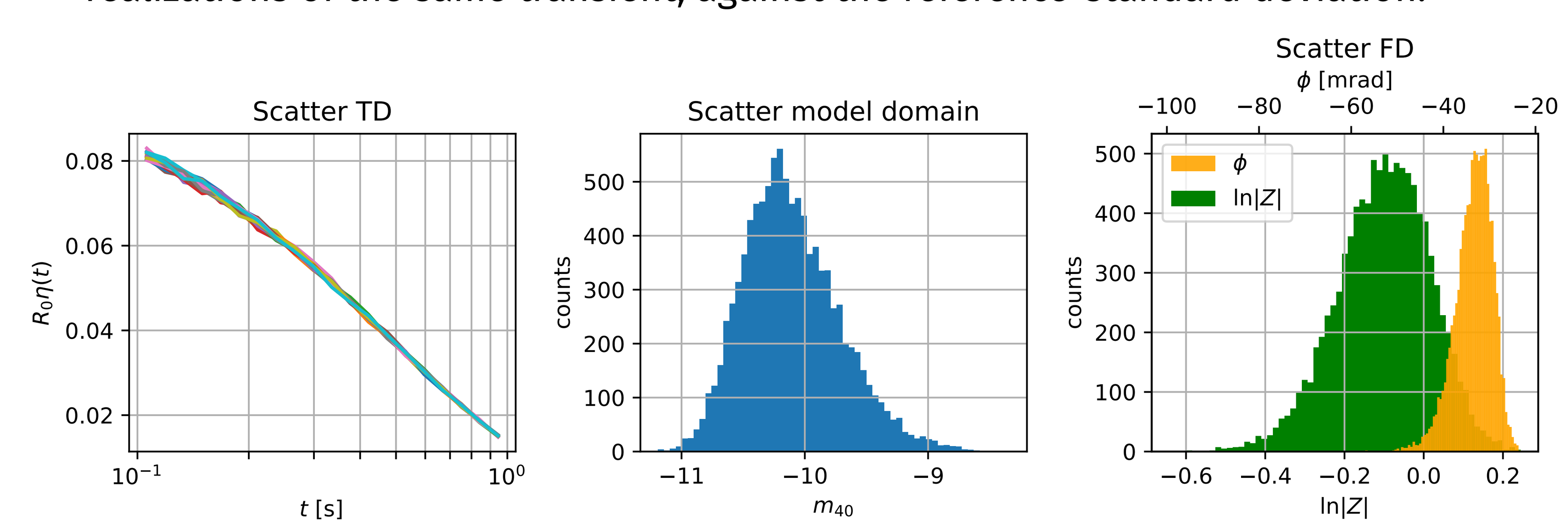


Figure: Left: noise realizations of input transient. Center: Scatter of example-model parameter m_{40}. Right: Scatter of FD estimates.

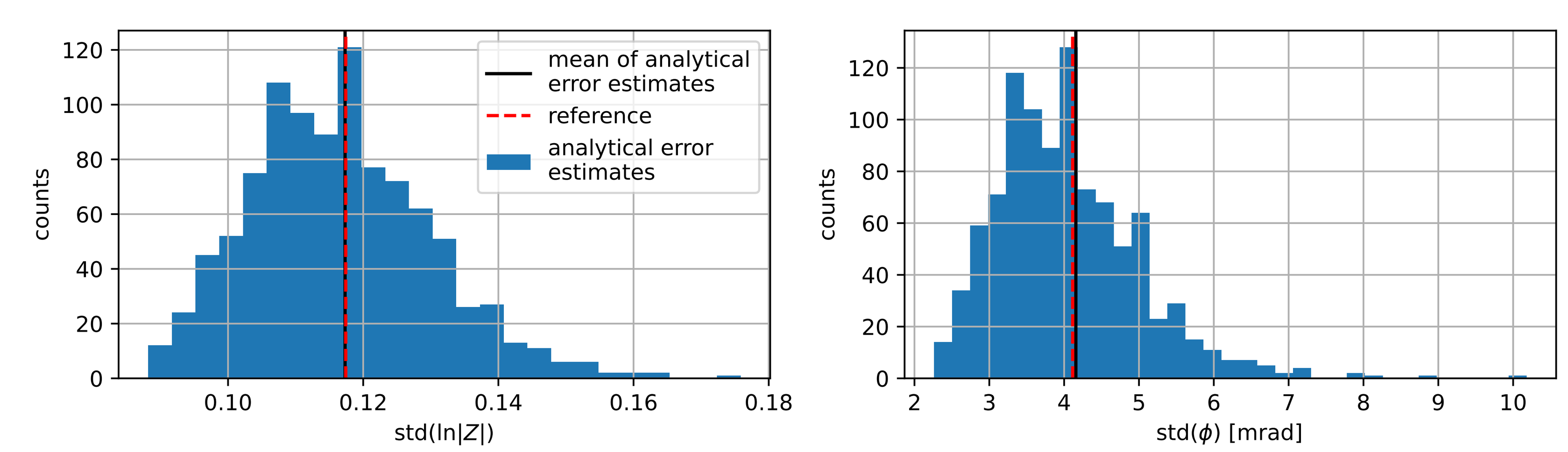


Figure: Estimated standard deviations are in **agreement with the reference values**.

Field application

Here we **present our workflow** for the application of our approach to a tomographic TD IP data set measured in Kamchatka:

1. Fitting of simple power-law decays to the transients to filter non-physical measurements.
2. Estimation of the TD error on the transients.
3. Conversion of the transients to the FD.
4. Propagation of the TD error to the FD.
5. Inversion of the FD data into a subsurface model of complex resistivity.

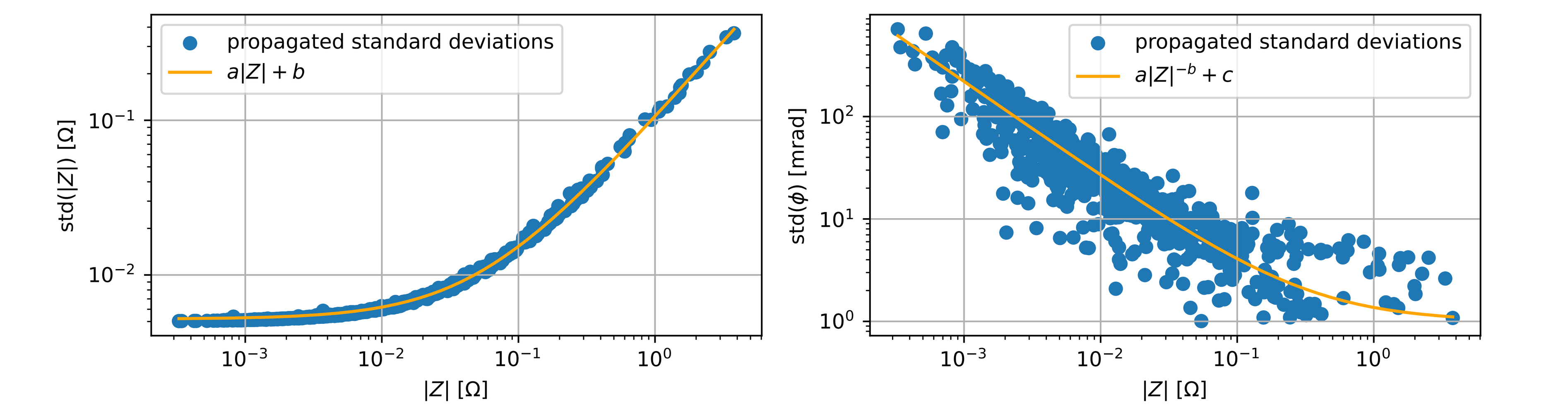


Figure: Systematic behavior of the error estimates. Note that the error models are only fitted for the purpose of illustration. During the tomographic inversion we used the individual error estimates.

Results:

- The propagated error estimates for magnitude and phase exhibit the expected behavior. The error of the magnitude behaves linearly, while the error of the phase follows an inverse power law.
- Tomographic-inversion results at 1 Hz and 20 Hz show negligible differences in the magnitude but a significant frequency dependence of the phase.

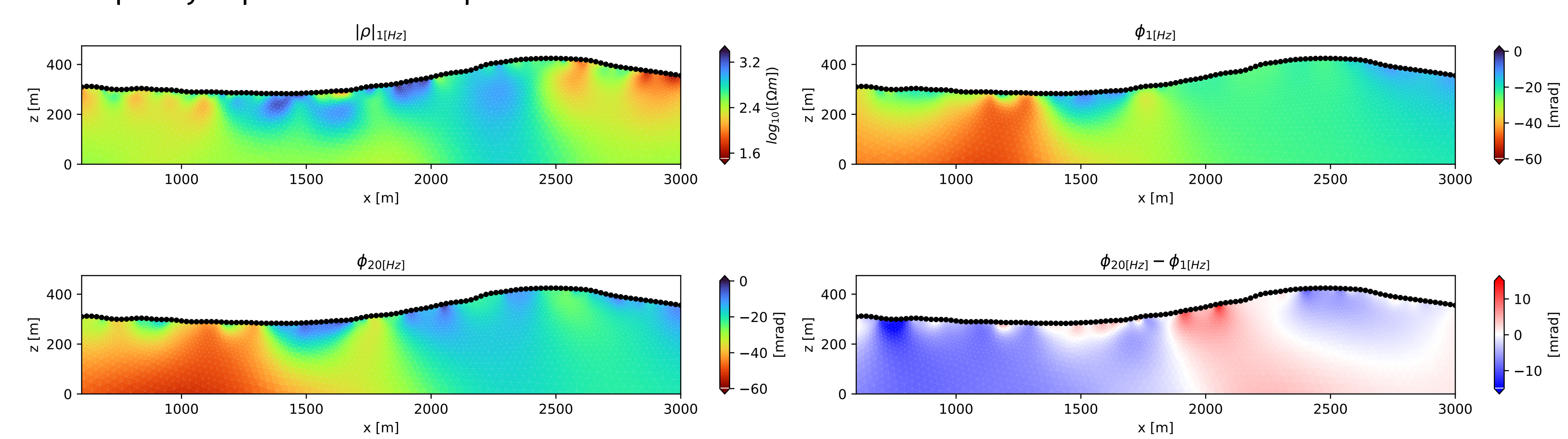


Figure: Inversion results at 1 Hz and 20 Hz compared.

Conclusion

The conversion approach:

- provides accurate estimates of FD parameters and their uncertainties,
- is feasible for the application to large tomographic data sets,
- is able to recover spectral characteristics from tomographic TD IP measurements.

Main **limitations** are:

- Sparse sampling of transients leads to high phase error.
- Narrow sampling-time frame leads to narrow sensitivity range in the FD.

References

Gubbins, D. (2004). "Time series analysis and inverse theory for geophysicists." *Cambridge University Press.*

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