Footprint parameterization derived from a graded multilayer semi-analytical model valid in homogeneously driven boundary layers described by Monin-Obukhov theory

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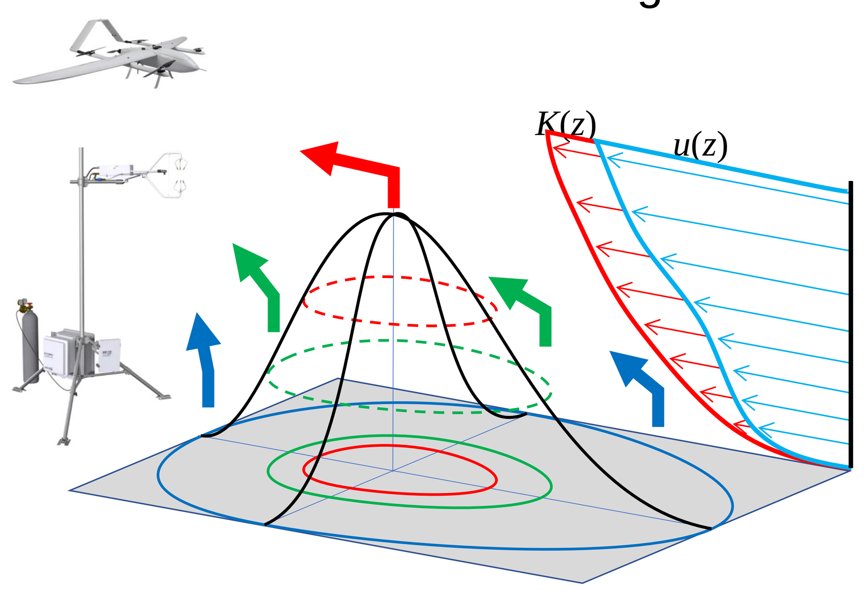
1-Introduction

Source area/footprint : the area of influence of a concentration/flux measurement performed at height (e.g. eddy covariance, scintillometry) = sensor field of view.
Footprint function : integration kernel linking the distribution of sources/sinks on the terrain surface and the sensor signal.



Influence of :

- sensor height z_m
- atmospheric turbulence field

mean wind direction and speed profile $u(z)$

- roughness length z_0
- friction velocity u_*
- atmospheric stability (Obukhov length L)

Lagrangian stochastic models are asymptotically accurate, versatile, but time expensive → analytical parameterizations (HKC, 2000; KCRS, 2015)

Analytical footprint models (Eulerian framework + K-theory → eddy-diffusivity profile $K(z)$) are fast, but all assume, at best, power-law profiles for $u(z)$ and $K(z)$ (e.g. KM, 2001)

A new (semi-)analytical footprint model was developped, which is fully compliant with arbitrary profiles, e.g. Monin-Obukhov profiles in the ASL (KK, 2023)

2-Liouville transformation, new scale and new parameter

Variable change : depth $z \rightarrow \xi(z) \equiv \int_{z_0}^{z} \sqrt{u(z')/K(z')}\,dz'$

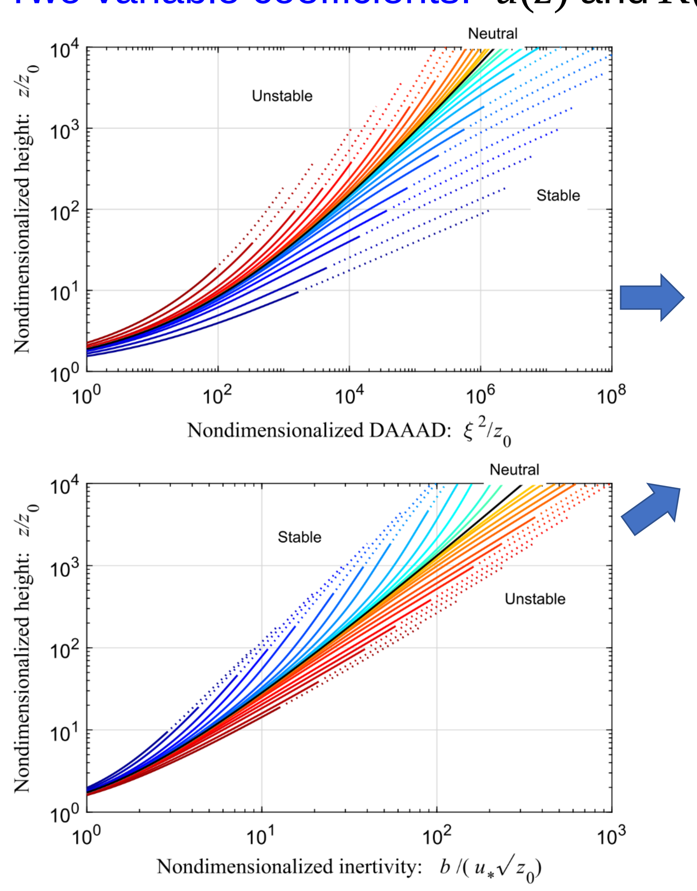
$\xi^2(z)$ is called the Diffusion-Ascent-Associated Advection Distance (DAAAD)

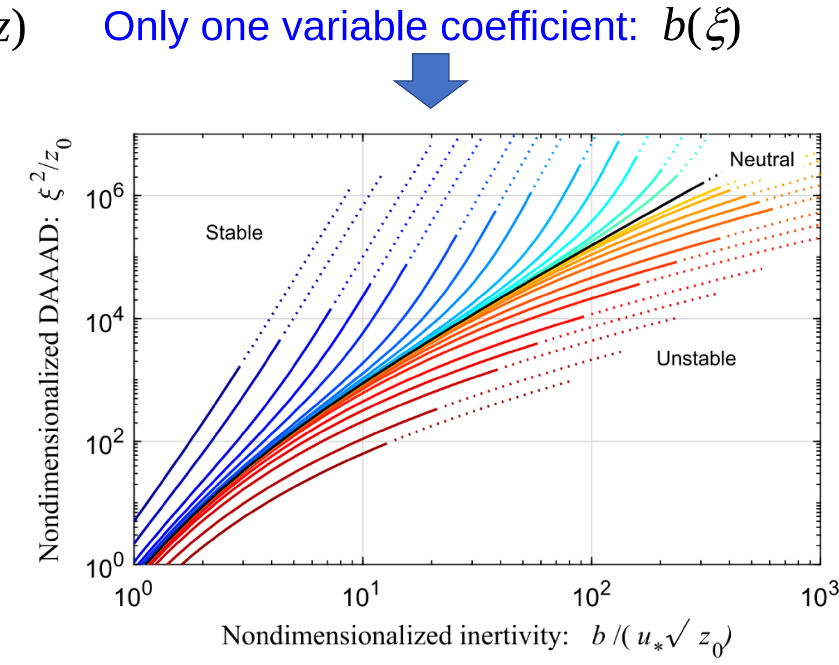
$b(z) \equiv \sqrt{u(z)K(z)}$ is called the atmosphere inertivity (~ inertia to any change of state)

2D advection-diffusion equation for the Reynolds-averaged crosswind-integrated concentration or flux:

In x-z space:
Two variable coefficients: $u(z)$ and $K(z)$

In x-ξ (Liouville-)space:
Only one variable coefficient: $b(\xi)$





MOST profiles issued from Businger-Högström parameterization with z_0/L between -10^{-5} and -0.1 (unstable; orange to dark red) and between $+10^{-5}$ and +0.1 (stable; light blue to dark blue). The neutral case ($z_0/L = 0$) in black. z_m/L restricted between -2 (unstable) and +1 (stable) in continuous lines, with extension to -20, resp. +10 in dotted lines.

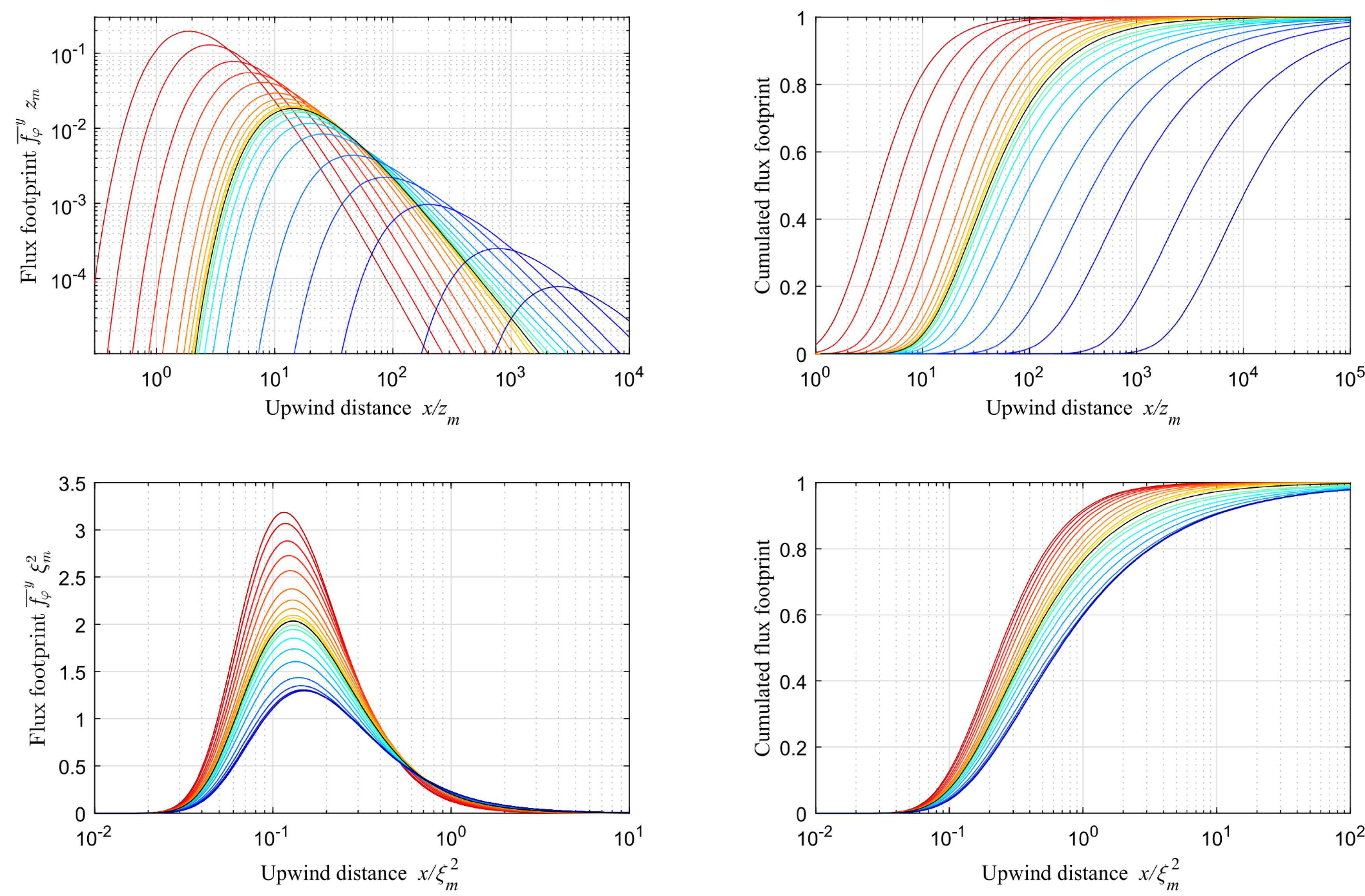
3-Semi-analytical solution

ADGMM: Advection-Diffusion Graded Multilayer Method

- Splitting of the boundary layer (e.g. ASL) into N sublayers
- Piecewise fitting of the (e.g. MOST) profile $b(\xi)$ with a « solvable » inertivity profile, viz. leading to an exact analytical solution in the Laplace-Liouville space (here, a combination of two shifted Euler-Power-law profiles - EPL)
- Assembly of the corresponding analytical quadrupoles
- Numerical Laplace inversion

With an error tolerance of 0.001 on inertivity fitting, less than 10 EPL sublayers are necessary; the estimated error on the footprint is less than $5 \cdot 10^{-5}$, the computation time is less than 0.5 s.

4-Flux footprint (crosswind integrated)



Flux footprint (left) and cumulated flux footprint (right). Nondimensionalization with sensor height z_m (top) or with the DAAAD $\xi_m^2 = \xi^2(z_m)$ (bottom). MOST profiles issued from Businger-Högström parameterization with z_0/L between -0.01 and +0.01 (unstable in red tone, neutral in black, stable in blue tone). Illustration for the case of a sensor at height $z_m/z_0 = 1000$.

5-Surrogate models (concentration and flux)

Inverse Gamma distribution

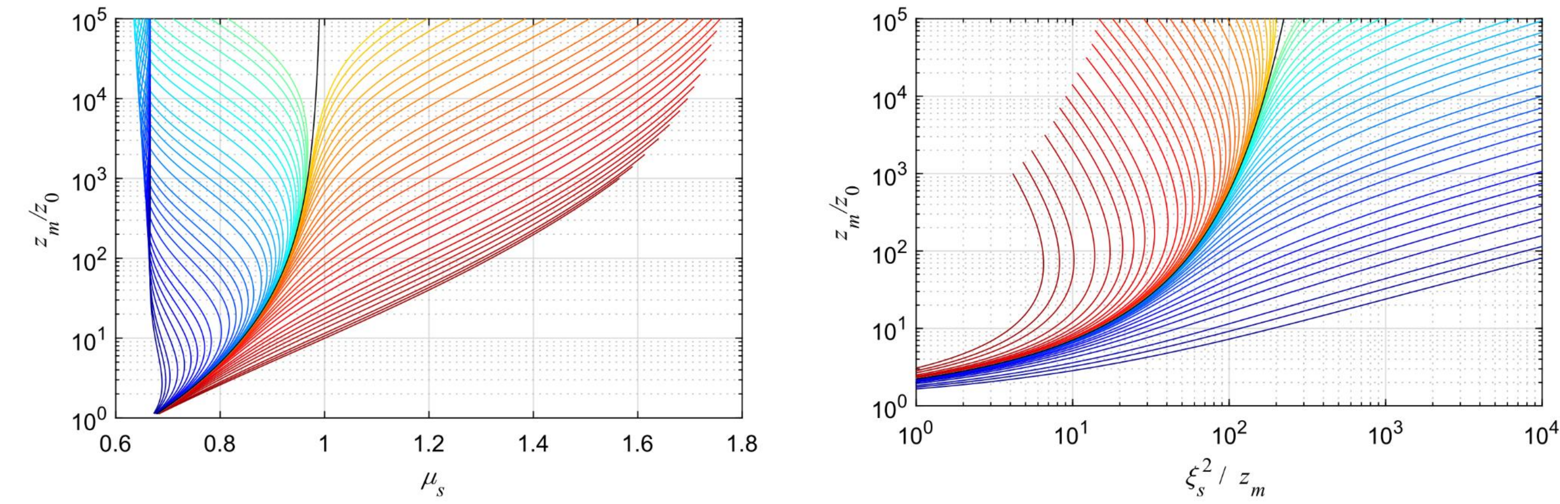
Flux: $\overline{f_\varphi}^y(x,z_m) \approx f\left(x,\mu_s,\frac{\xi_s^2}{4}\right) \equiv \frac{4}{\xi_s^2\Gamma(\mu_s)}\left(\frac{\xi_s^2}{4x}\right)^{\mu_s+1}\exp\left(-\frac{\xi_s^2}{4x}\right)$ ➡ Two parameters: μ_s, ξ_s

Concentration: $\overline{f_\chi}^y(x,z_m) \approx \frac{\xi_s}{2b_s}\frac{\Gamma(\mu_s-1)}{\Gamma(\mu_s)} f\left(x,\mu_s-1,\frac{\xi_s^2}{4}\right)$ ➡ Three parameters: μ_s, ξ_s, b_s

The surrogate models reach the following performances for: $\begin{cases} z_0/L \in [-0.1,+0.1] \\ z_m/z_0 \in [1,10^5] \\ z_m/L \in [-100,+100] \end{cases}$

	Flux	Concentration
Root mean square error	< 1%	< 1.2%
Maximum absolute error	< 2%	< 2.1%

6-Optimal parameters for the surrogate model for the flux footprint



Two options for feeding the surrogate model:

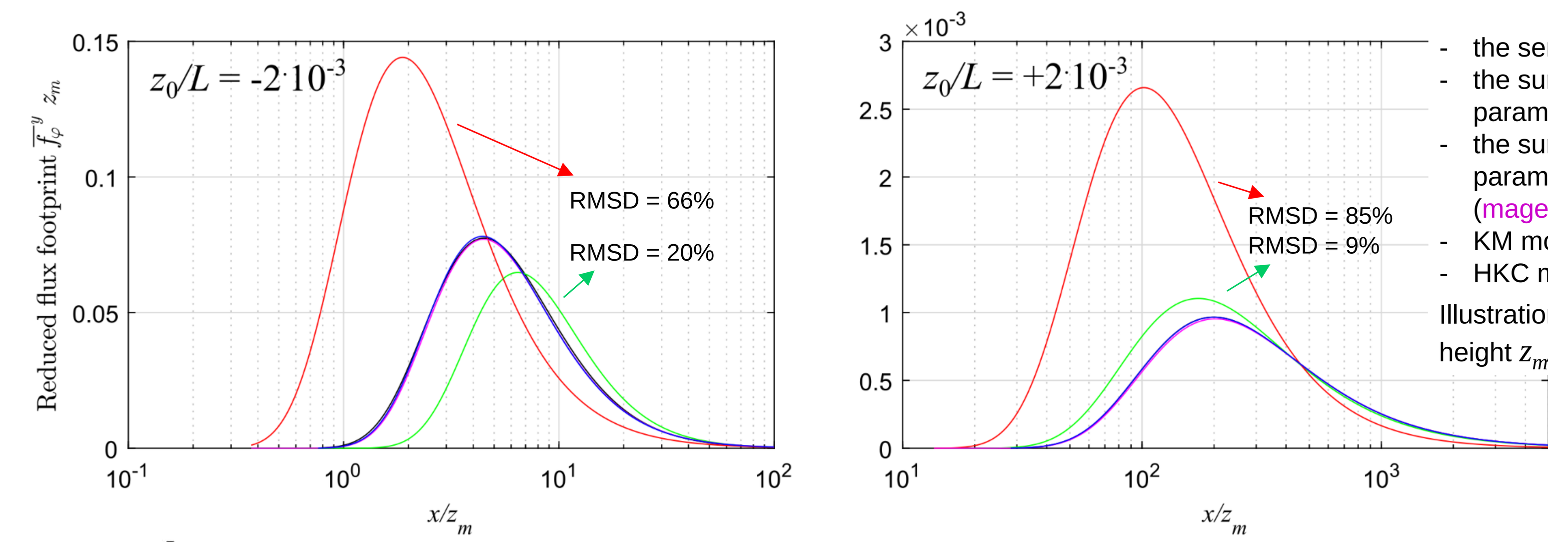
1-interpolation in the databases of optimal values for μ_s and ξ_s^2/z_m as functions of z_0/L and z_m/z_0

2-leverage analytical parameterizations of $\mu_s(z_0/L, z_m/z_0)$ and $\xi_s^2/z_m(z_0/L, z_m/z_0)$ (see KK, 2023)

Performances for $\begin{cases} z_0/L \in [-0.07,+0.02] \\ z_m/z_0 \in [6,10^5] \\ z_m/L \in [-100,+4] \end{cases}$

	Flux
Root mean square error	< 1.6%
Maximum absolute error	< 4%

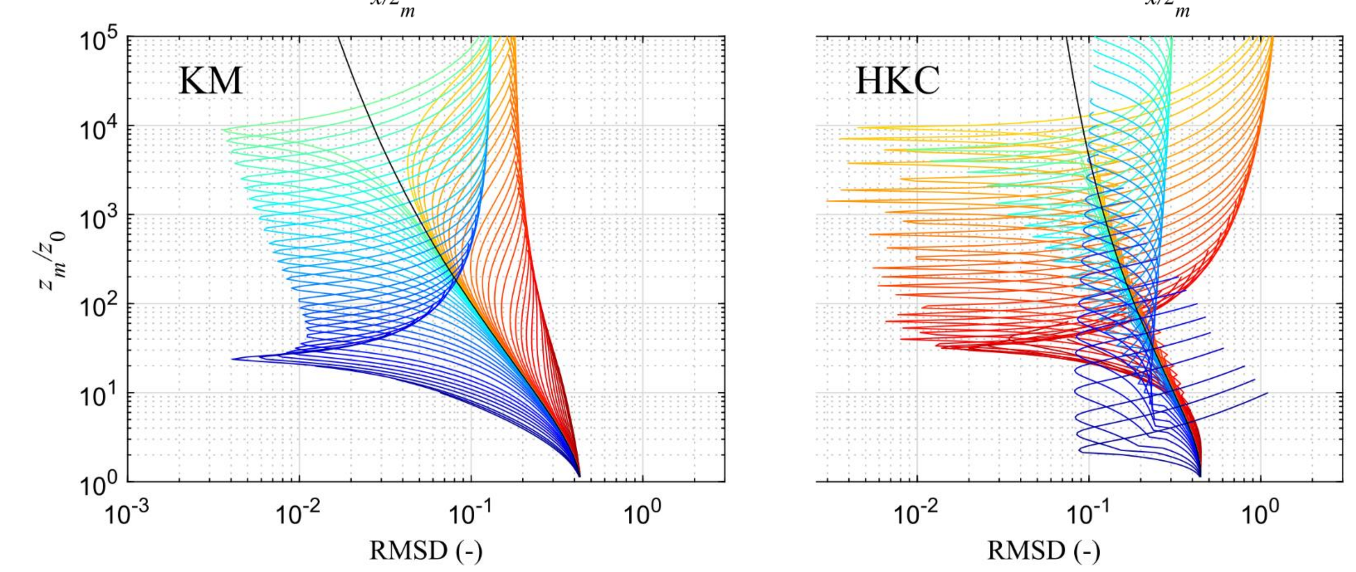
7-Comparison with KM and HKC models



Flux footprint obtained with:

- the semi-analytical model (**black**)
- the surrogate model with optimal parameters (blue)
- the surrogate model with parameterized parameters (magenta)
- KM model (green)
- HKC model (red)

Illustration for a sensor at height $z_m/z_0 = 1000$



Root mean square difference between the present semi-analytical model and:

- KM model (left)
- HKC model (right)

z_0/L between -0.01 (unstable-red) and +0.01 (stable-blue), neutral in black; z_m/L between -2 and +1

8-Summary and outlook

- Mathematically efficient (semi-)analytical footprint model (accurate, fast)
- Compliant with arbitrary atmospheric stratification (illustration was given for MOST profiles in the ASL; future extension down into the roughness layer and up to the BL height)
- 3D extension with admissible plane heterogeneity : $\begin{cases} u(y,z) = u_2(y)u_3(z) \\ K_y(x,y,z) = K_{y1}(x)K_{y2}(y)u_3(z) \\ K_z(x,y,z) = K_{z1}(x)u_2(y)K_{z3}(z) \end{cases}$

References

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KCRS, 2015: Kljun, N., Calanca, P., Rotach, M. W., & Schmid, H. P. (2015). *Geoscientific Model Development*, *8*(11), 3695-3713.

KM, 2001: Kormann, R., & Meixner, F. X. (2001). *Boundary-Layer Meteorology*, *99*, 207-224.

KK, 2023: Krapez, J.-C., & Ky, G. A. (2023). *Boundary-Layer Meteorology*, *186*(4), 1-49.