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Inferring ocean transport statistics with probabilistic neural networks

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Surface transport and the Global Drifter Program

Surface currents are responsible for the transport of bouyant material such as plankton, plastic and other pollutants. The Global Drifter Program (NOAA PhOD) has collected time series data from satellite-tracked drifting buoys (drifters) since 1969 [1]. These data have been used to estimate spatially varying statistics such as mean velocity and diffusivity [2] and to build transition matrix models of drifter motion [3]. We propose a new data-driven model — a type of probabilistic neural network — for the transition probability density function (pdf) of drifters. The transition pdf provides a comprehensive description of drifter dynamics allowing for the simulation of drifter trajectories and the estimation of a wealth of dynamical statistics without the need to revisit the raw data [4].

Transition probability density functions

The transition pdf $p(\boldsymbol{X}_{n+1} \mid \boldsymbol{X}_n)$ denotes the pdf for the position of a drifter at time t_{n+1} given that it was $\boldsymbol{X}_n$ at time t_n. Assuming Markovianity, the transition pdf determines the evolution of a drifter's pdf via

$$p\left(\boldsymbol{X}_{n+1}\right) = \int_\Omega p\left(\boldsymbol{X}_{n+1} \mid \boldsymbol{X}_n\right) \, p\left(\boldsymbol{X}_n\right) \, \mathrm{d}\boldsymbol{X}_n.$$

Over large spatial and temporal scales the drifter pdf $p(\boldsymbol{X}_n)$ can represent the concentration of a buoyant tracer.

Probabilistic neural networks

While standard neural networks model deterministic functions $\boldsymbol{f}(\boldsymbol{x})$, probabilistic neural networks represent parameters $\boldsymbol{\theta}(\boldsymbol{X})$ for a conditional model $p(\boldsymbol{Y} \mid \boldsymbol{X}) = \rho(\boldsymbol{Y}\,;\,\boldsymbol{\theta}(\boldsymbol{X}))$. We model the transition density in the form $p(\Delta\boldsymbol{X} \mid \boldsymbol{X}_0)$. The parametric form ρ that we use is a Gaussian mixture distribution,

$$p(\Delta\boldsymbol{X} \mid \boldsymbol{X}_0) = \sum_{i=1}^{M} \alpha_i(\boldsymbol{X}_0) \det\left(2\pi C_i\left(\boldsymbol{X}_0\right)\right)^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\Delta\boldsymbol{X} - \boldsymbol{\mu}_i\left(\boldsymbol{X}_0\right)\right)^{\mathrm{T}} C_i^{-1}\left(\boldsymbol{X}_0\right)\left(\Delta\boldsymbol{X} - \boldsymbol{\mu}_i\left(\boldsymbol{X}_0\right)\right)\right],$$

with $\sum_i \alpha_i = 1$. In this case $\boldsymbol{\theta} = \{\alpha_i,\, \boldsymbol{\mu}_i,\, C_i\}_{i=1}^{M}$ — see Figure 1.

The parameters of the neural network, $\boldsymbol{w}$, are optimised to maximise the likelihood of observed drifter trajectories, $\rho\left(\Delta\boldsymbol{X}\,;\,\boldsymbol{\theta}(\boldsymbol{X}_0\,;\,\boldsymbol{w})\right)$.

Features

- Provides estimates of a range of dynamical statistics simultaneously.
- Can be used to emulate drifter release experiments to study surface transport.
- Continuous in space representation — avoids the need to bin data.
- Flexible model — captures non-Gaussian statistics.

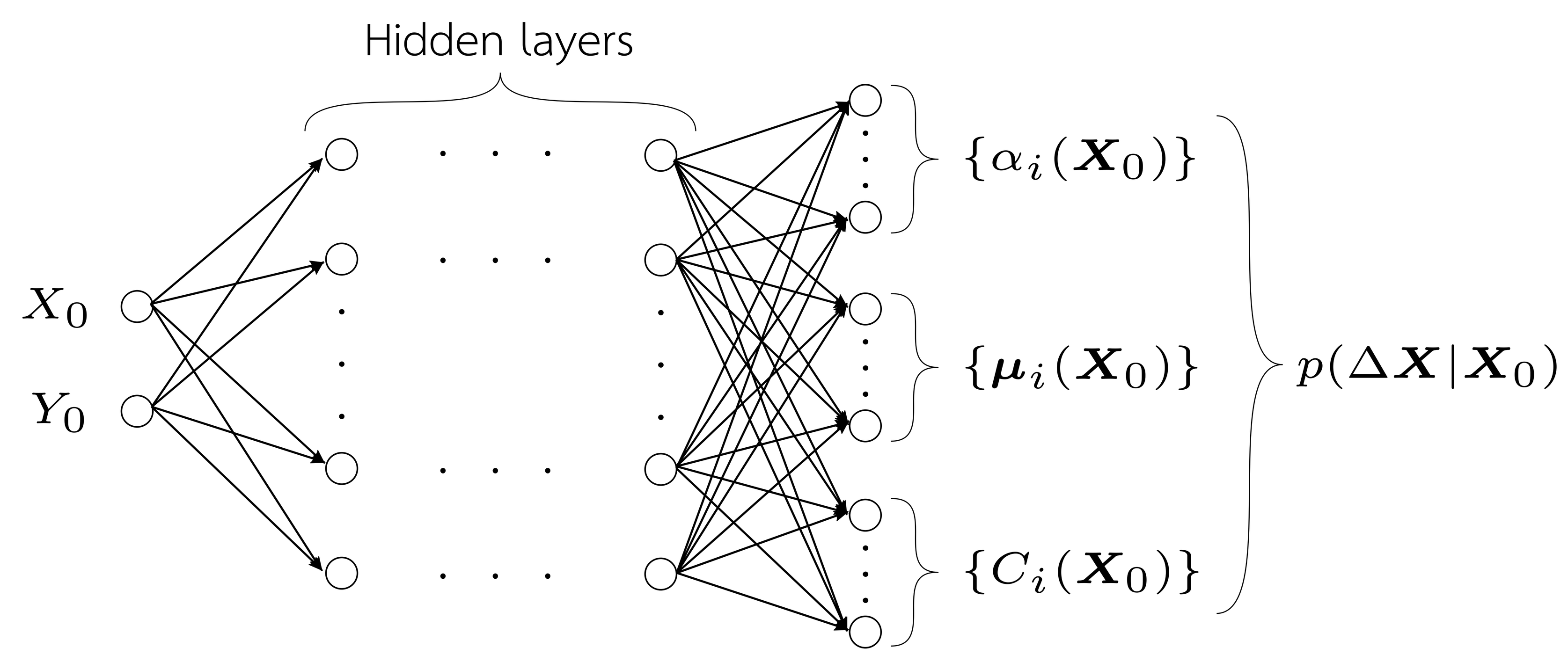

Figure 1. Schematic of the probabilistic neural network used, known as a mixture density network [5].

Diagnosing dynamics: spatially-varying statistics

We show examples of the statistics captured by the model. Figure 2 illustrates non-Gaussianity in transition density; Figure 3 shows mean displacements as function of initial position; and Figure 4 shows a derived estimate of lateral diffusivity.

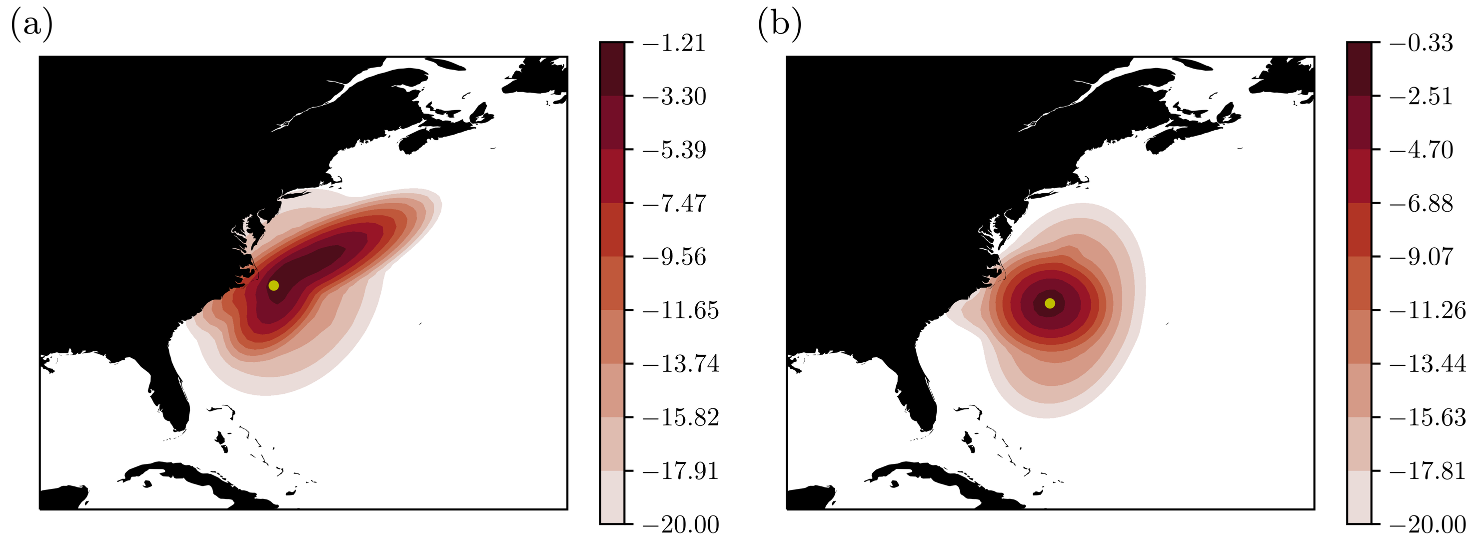

Figure 2. Maps of the log transition density, $\log p(\Delta\boldsymbol{X} \mid \boldsymbol{X}_0)$, for initial positions, $\boldsymbol{X}_0$, (a) in the Gulf Stream, and (b) adjacent to the Gulf Stream, derived from the MDN model with $t_{n+1} - t_n = 4$ days. Yellow dots indicate $\boldsymbol{X}_0$.

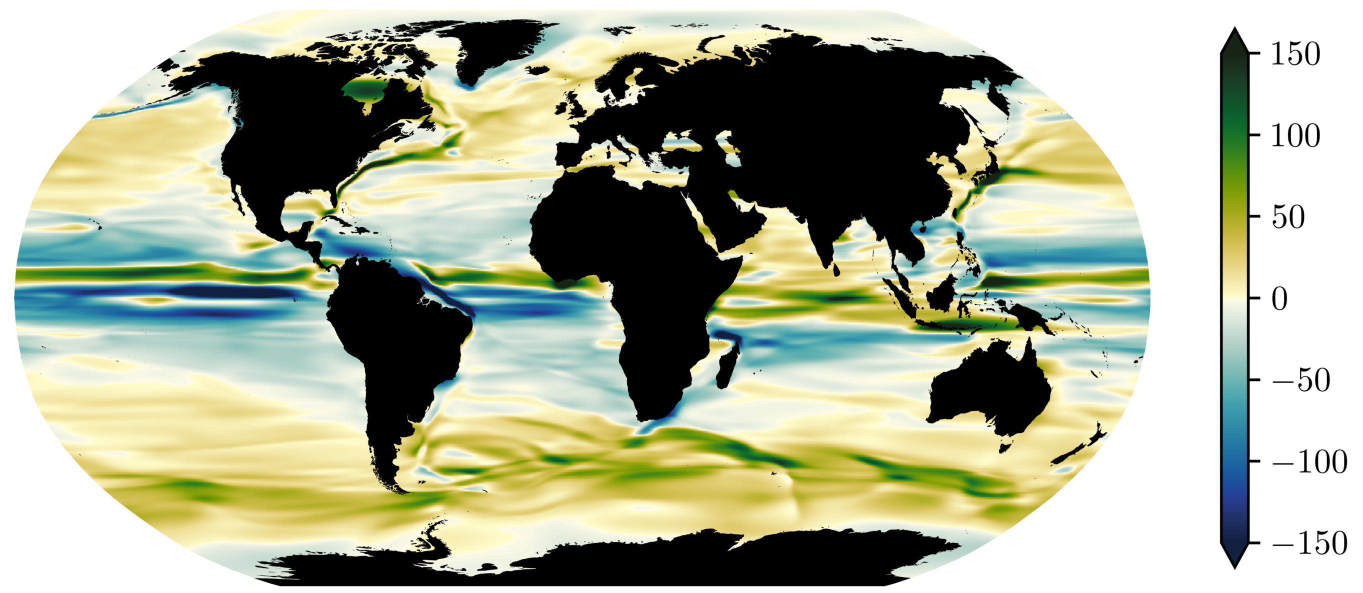

Figure 3. Mean of 4-day zonal displacement (km) as a function of initial position $\boldsymbol{X}_0$.

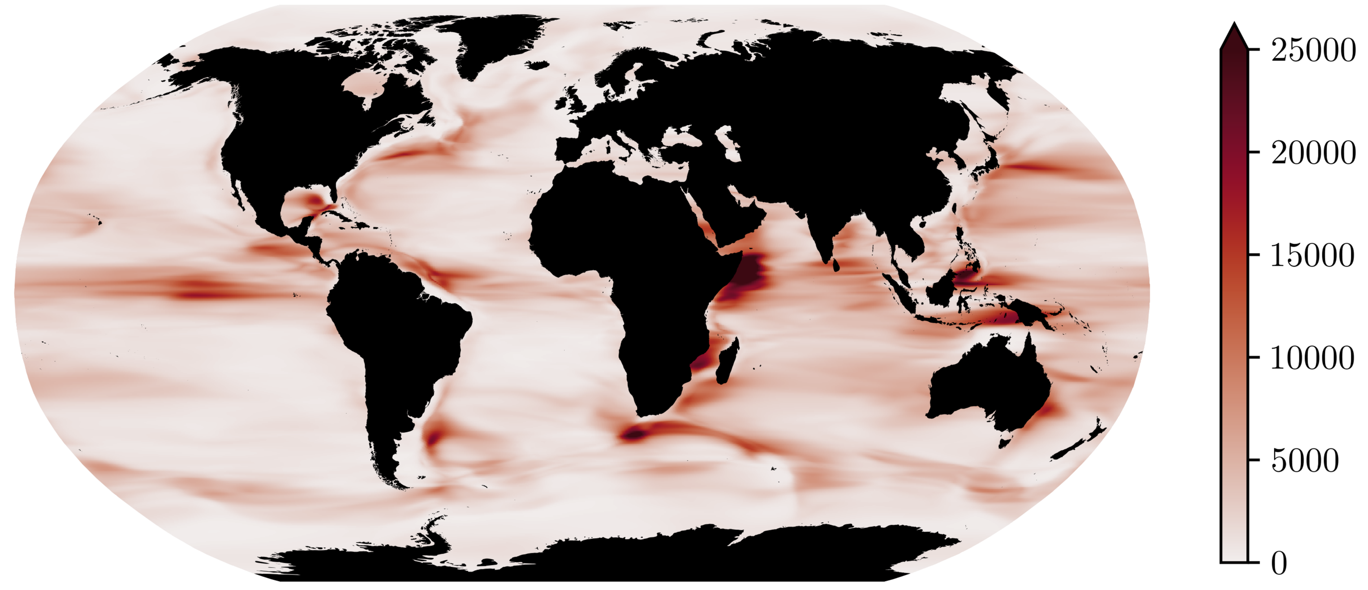

Figure 4. Scalar estimate of lateral diffusivity, K ($\mathrm{m^2s^{-1}}$), following the recipe of Zhurbas and Oh [6].

Surrogate modelling: drifter release experiment

In Figure 5 we show results of a simulation of drifters initially distributed on a uniform longitude–latitude grid. Displacements for each drifter are drawn from the transition density independently at each time step. Drifters are seen to cluster in subtropical gyres, as in [3].

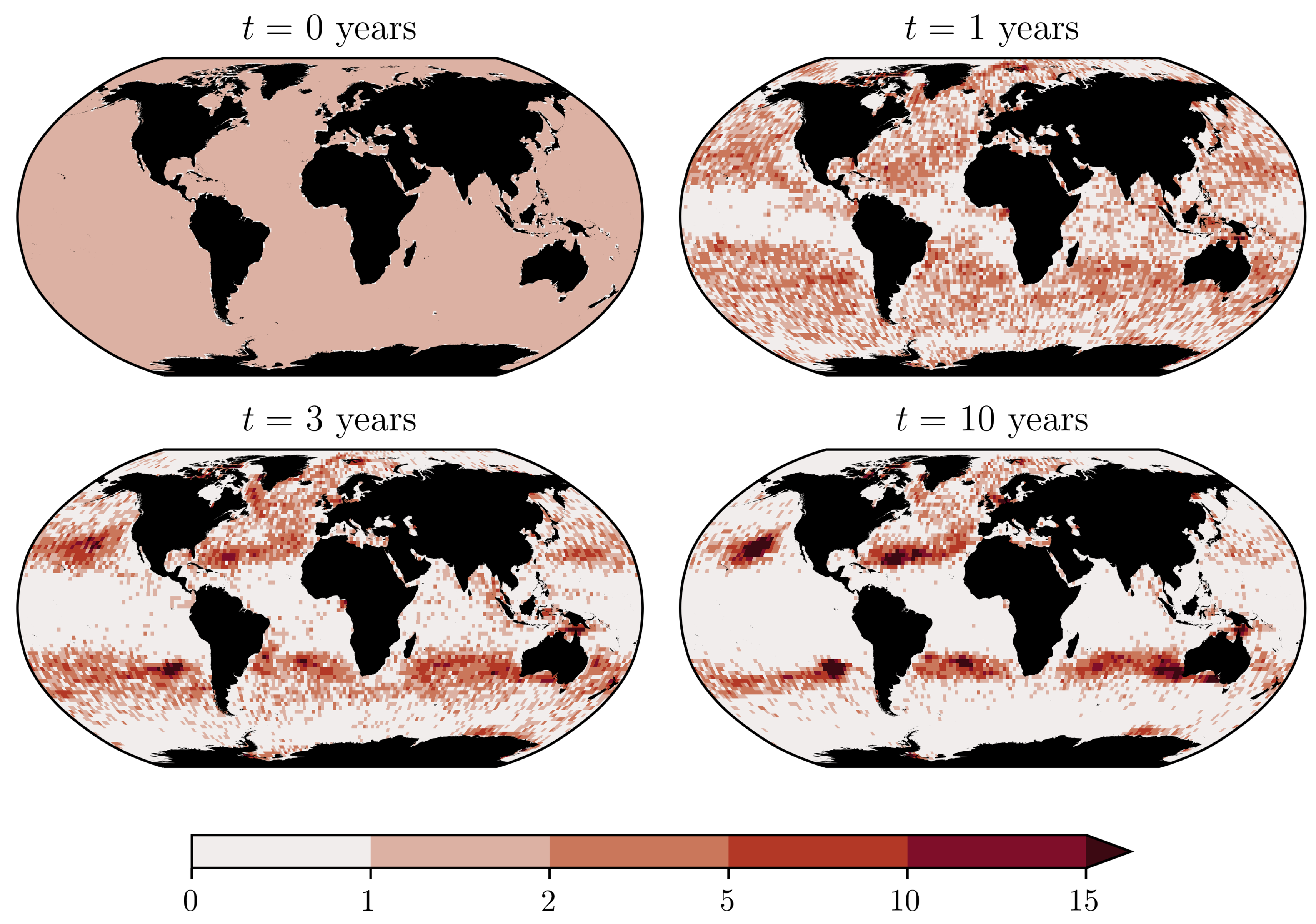

Figure 5. Histograms of simulated drifters initially and after one, three, and ten years of evolution under the model, respectively.

Other applications

Many statistics of interest in fluid dynamics are described by conditional pdfs. Flexible probabilistic models can be used to infer these from data, following the approach demonstrated here. Some examples are:

- Multi-point Eulerian statistics: e.g. structure functions via $p(\delta\boldsymbol{u} \mid r)$;
- Multi-point Lagrangian statistics: e.g. two-particle transition density $p(\boldsymbol{X}_{n+1}^{(1)}, \boldsymbol{X}_{n+1}^{(2)} \mid \boldsymbol{X}_n^{(1)}, \boldsymbol{X}_n^{(2)})$;
- Stochastic parameterisations of subgrid processes: by sampling from p(subgrid forcing | coarsely resolved variables).

References

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