

# A hybrid method to tackle conditional systematic errors of hydrological models

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# What is a hydrological model

- Definition 1: A mathematical formula  $O(t) = f(x_1(t), x_2(t), \dots)$ , where  $O$  the dependent (discharge), and  $x_1, x_2, \dots$  the independent (the stresses) time-varying variables.
- Definition 2: A stochastic function  $\underline{O} = f(\underline{x}_1, \underline{x}_2, \dots)$ , where  $\underline{O}, \underline{x}_1, \underline{x}_2 \dots$  stochastic variables.
- Definition 3: A mutual information function  $I[\{\underline{x}_1, \underline{x}_2, \dots\}, \underline{O}]$ .

# Uncertainty

- Aleatory – lack of knowledge (unknown errors in measurements, unknown physical processes, ... )
- Epistemic – insufficient model (simplistic structure, poor calibration, ...)

Note: this is not the most common definition.

# Modelling uncertainty

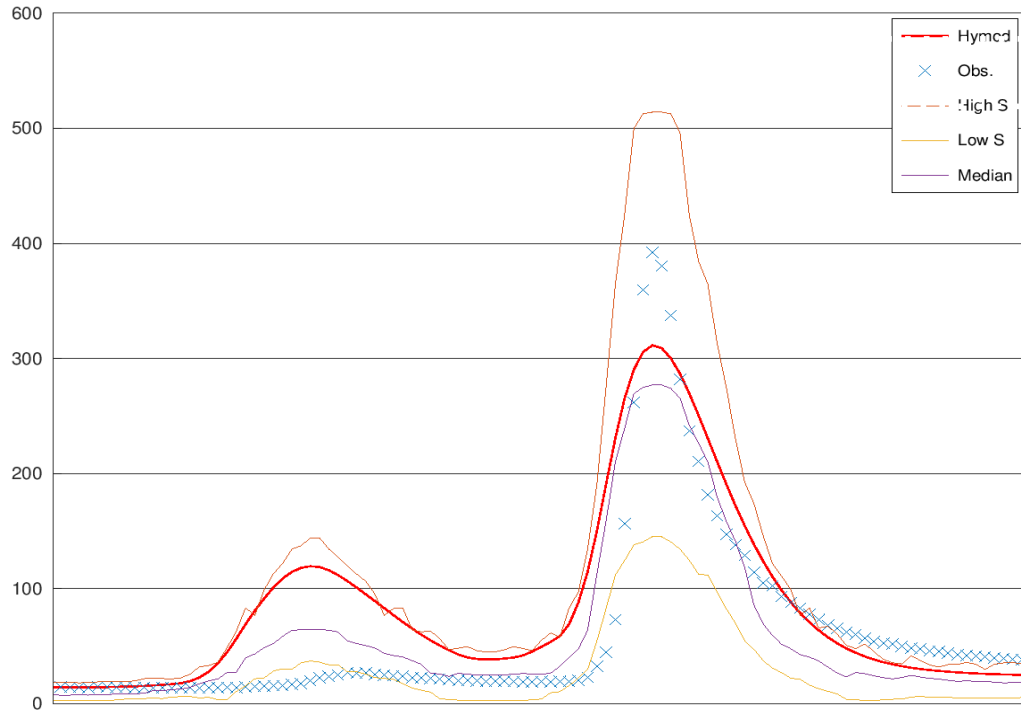
AR1 / Residual / Error

- Definition 1:  $O(t) = f(x_1(t), x_2(t), \dots) + \rho \delta(t-1) + \varepsilon(t)$   
(epistemic aleatory) (Schaepli et al., 2007).
- Definition 2:  $\rightarrow \dots$
- Definition 3:  $I[\{\underline{x}_1, \underline{x}_2, \dots\}, \underline{O}] - I[\underline{O}, \hat{O}]$   
(aleatory  $\Phi[\underline{O} | \{\underline{x}_1, \underline{x}_2, \dots\}]$ ) (Findanis and Loukas, 2022).

# Modelling uncertainty – stochastic approach

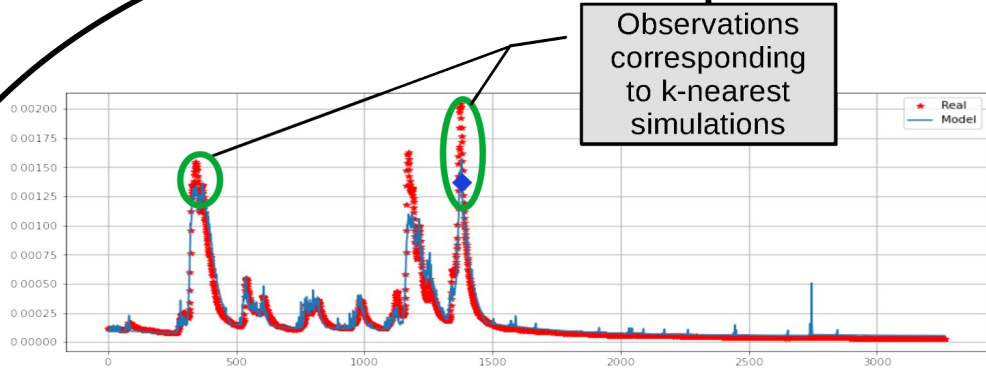
- $F_{\underline{O}|\hat{O}}(O|\hat{O}) \approx P \{ \underline{O} \leq O \mid \hat{O} - \Delta\hat{O}_1 \leq \hat{O} \leq \hat{O} + \Delta\hat{O}_2 \}$

epistemic  
+  
aleatory

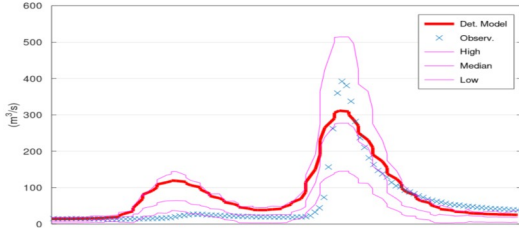


See BlueCat  
(Koutsoyiannis and  
Montanari, 2022)

# Stochastic approach – implementation



This is actually KNN, see Rozos et al., (2022)



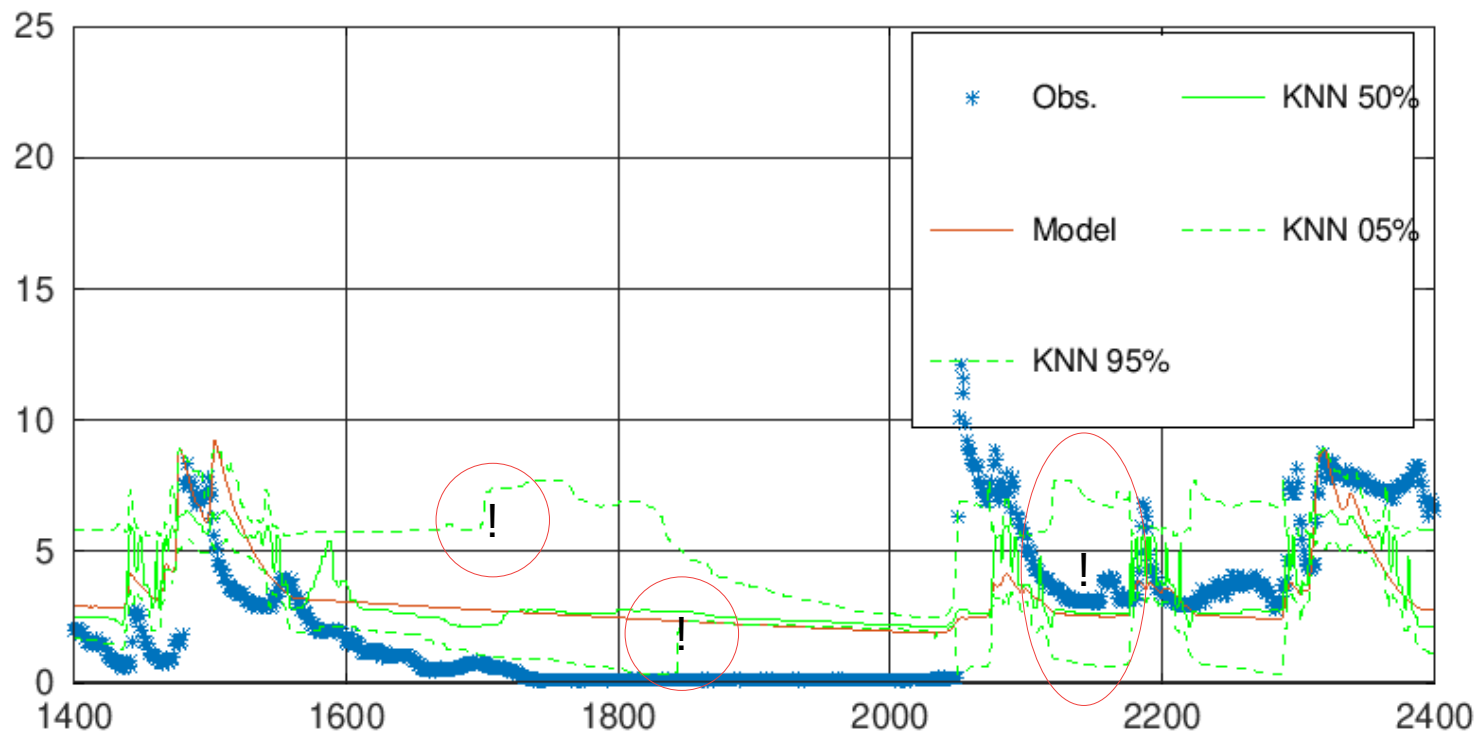
# Case studies

How do the different types of errors manifest?

Where to get the tools:

- KNN C code for MATLAB or command prompt from **hydronoa.gr** (software → ... uncertainty with KNN).
- BlueCat R code from Alberto Montanari's github (just search for **hymodbluecat**).

# Case studies – Aleatory uncertainty

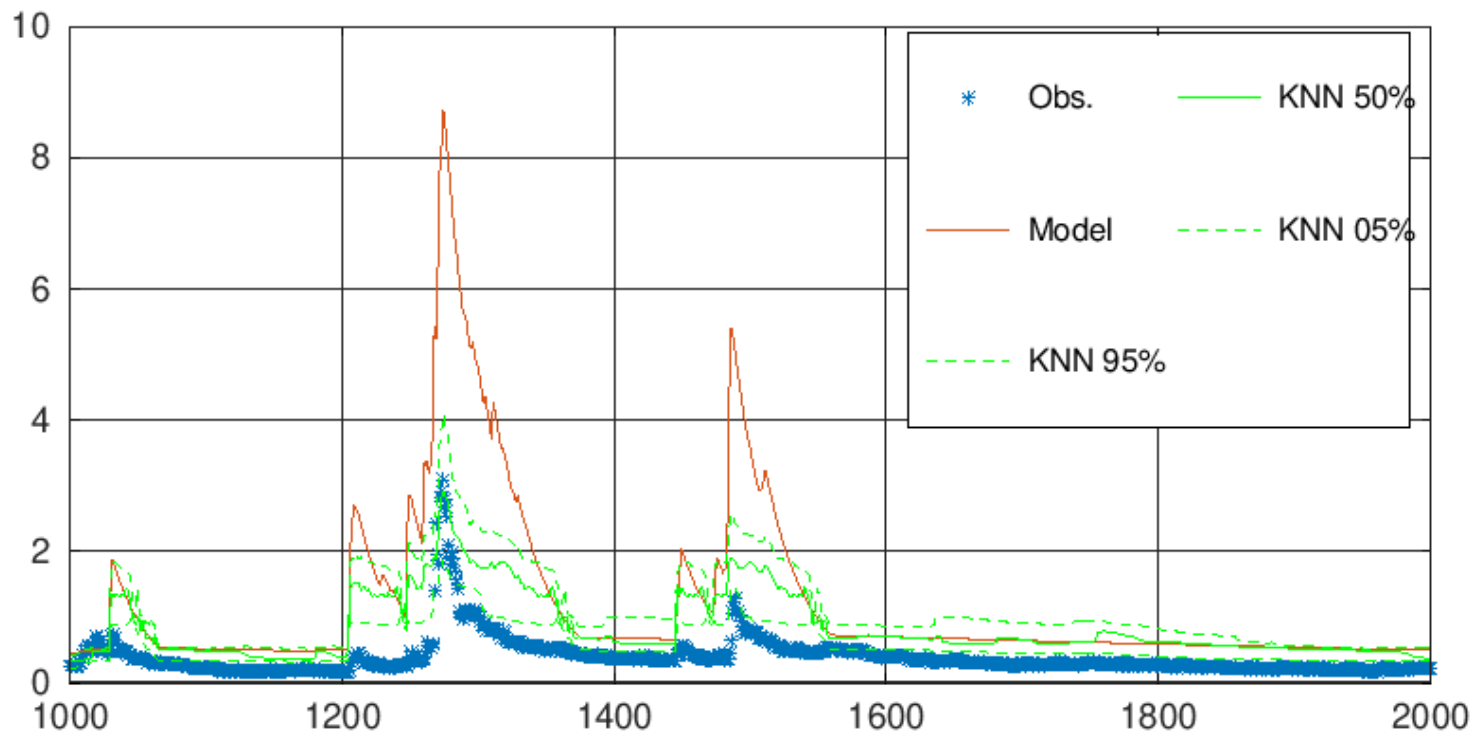


Unknown  
physical  
process

(Rozos, 2023)



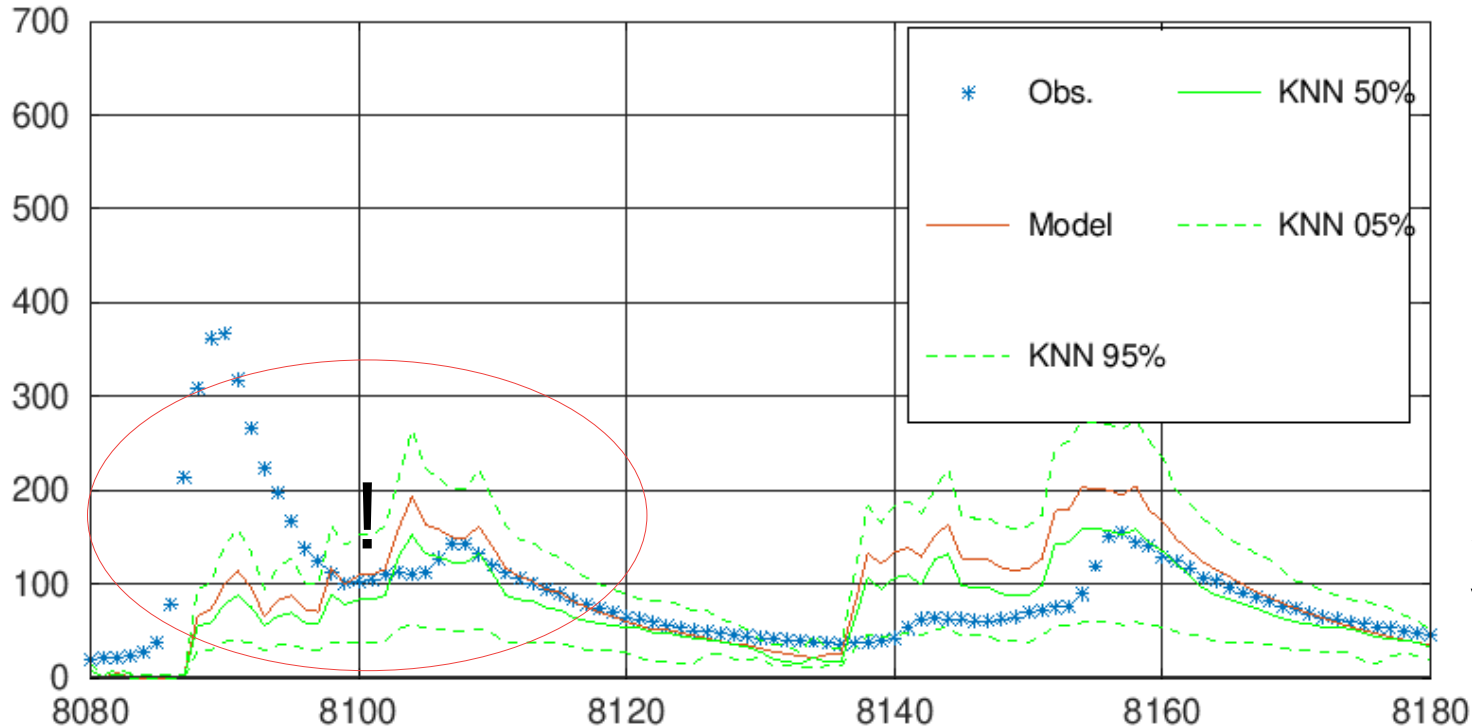
# Case studies – Epistemic uncertainty



Poorly calibrated model, systematic error

(Rozos, 2023)

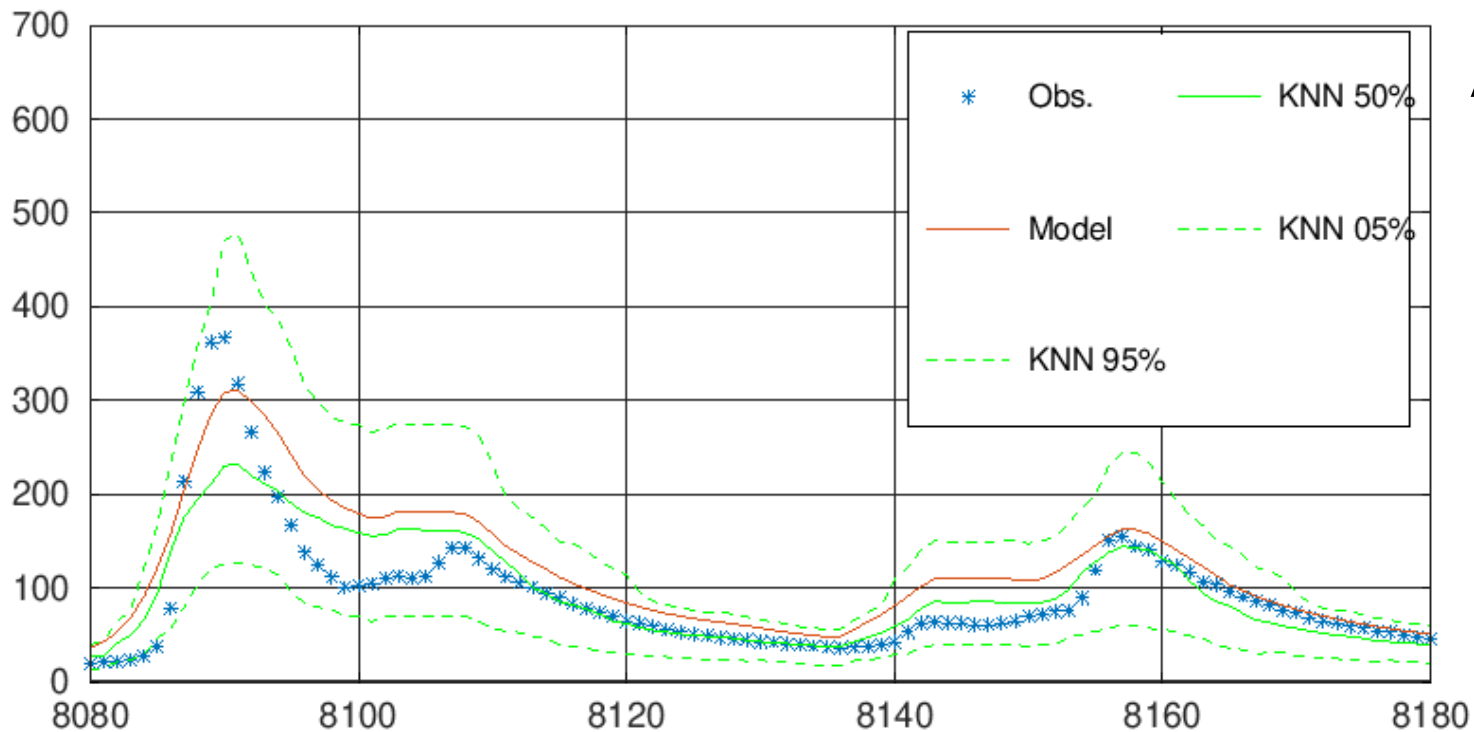
# Case studies – Epistemic uncertainty



Insufficient structure, conditional errors

It may not show in verification!

# Case studies – Epistemic uncertainty



Another  
model  
(Rozos, 2023)

# How to cope with aleatory uncertainty

- Get more information
- Evaluate the reliability of the available data
- More independent variables
- Monte Carlo simulations

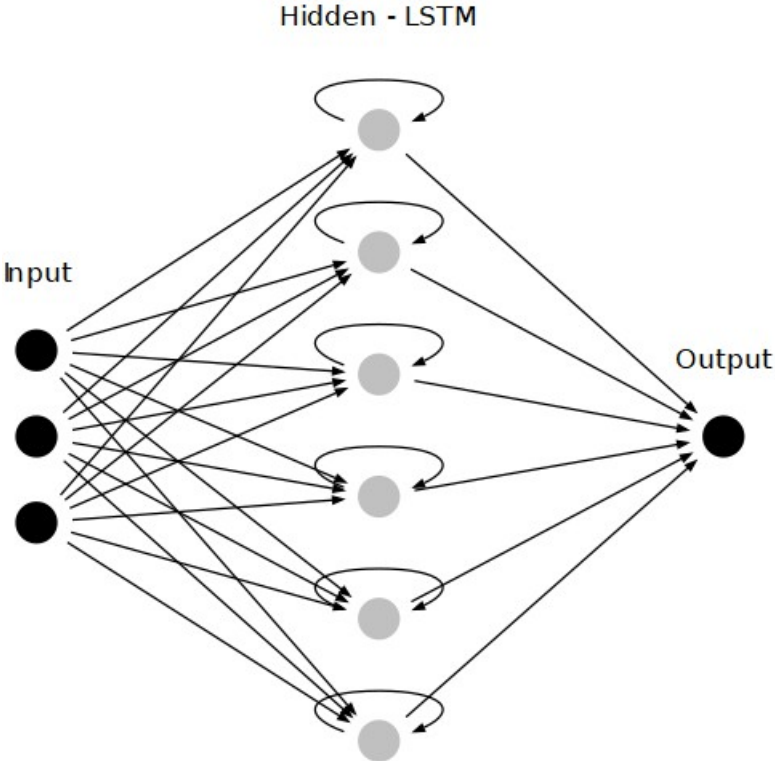
# How to cope with epistemic uncertainty

- Recalibrate the model
- Try another model (latent conditional errors)
- Model ensembles

Here a hybrid approach is suggested that combines a feedforward neural network with model ensembles.

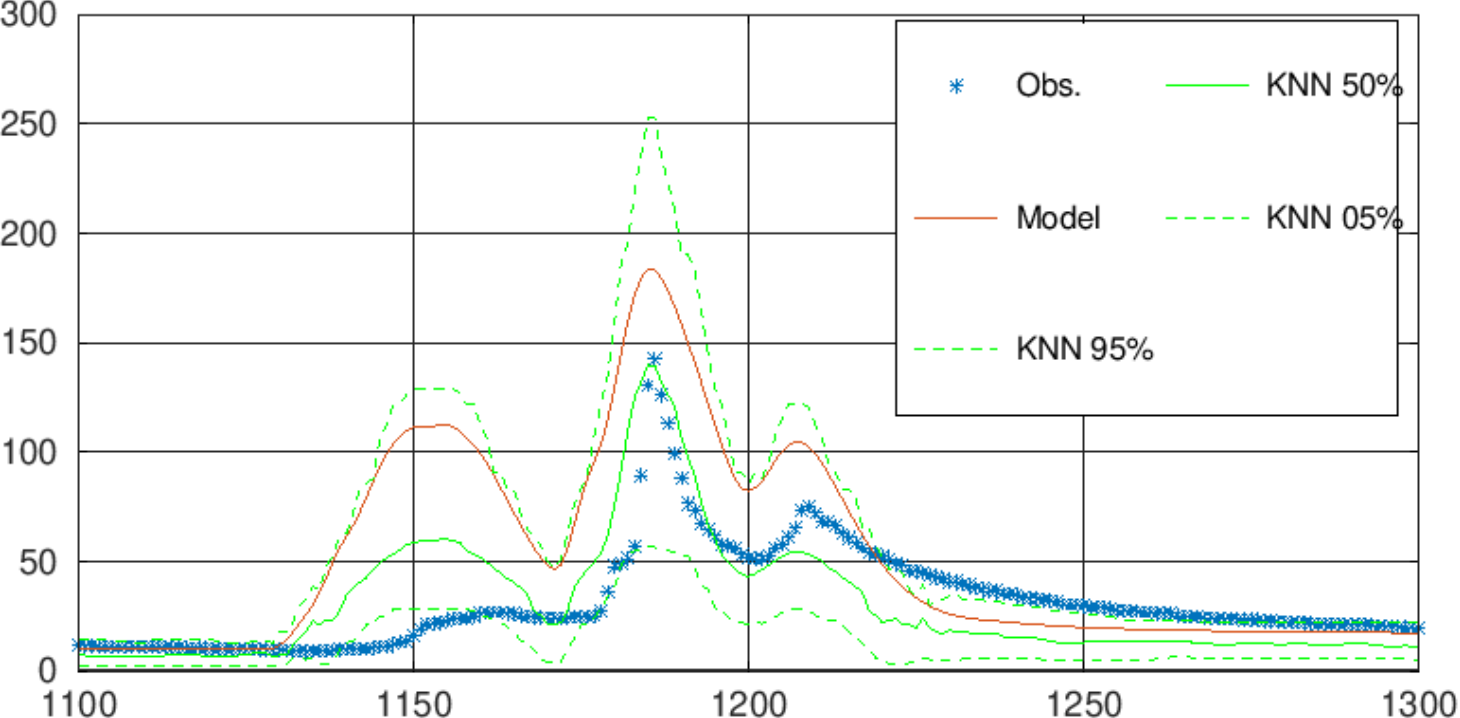
# Hybrid approach

Stresses  
and models  
outputs



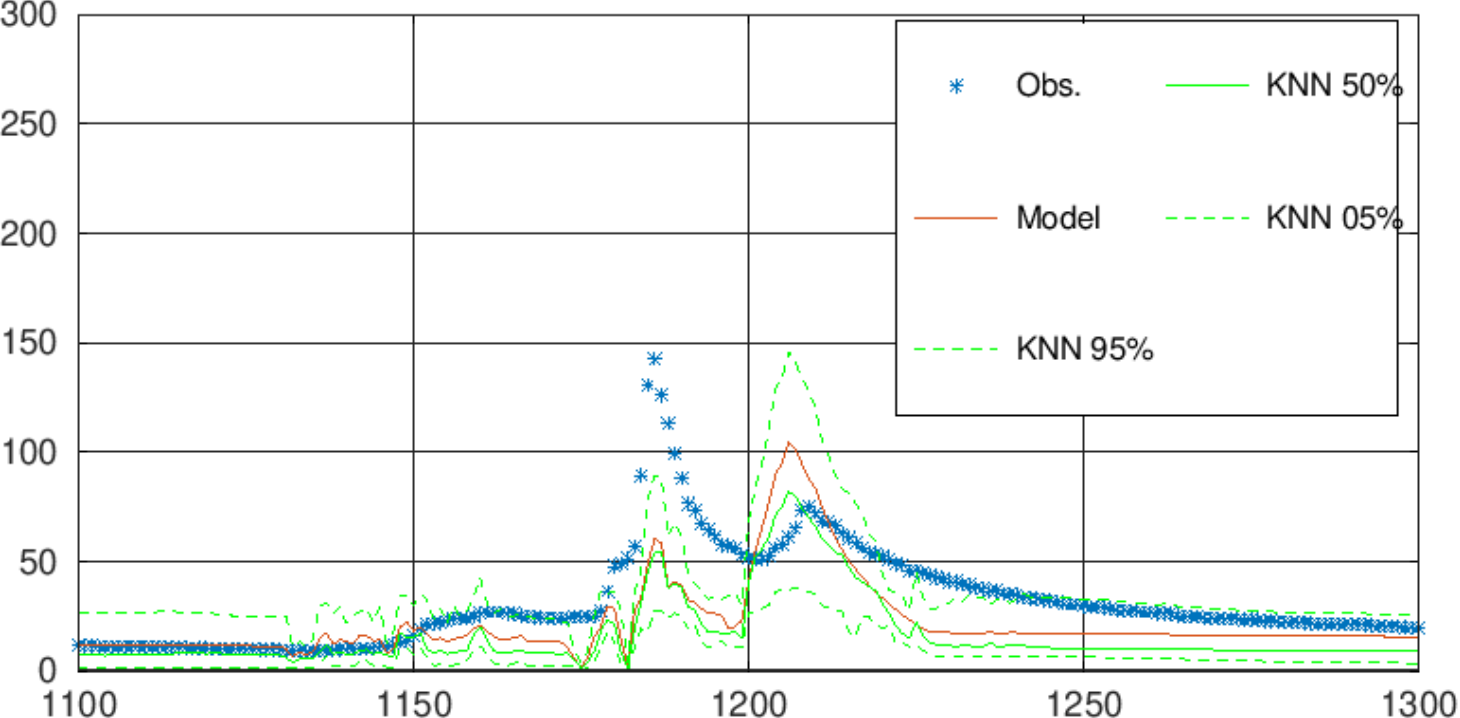
Simulated  
time series

# Results: Model A



Model A  
systematic  
overestimation  
of flows

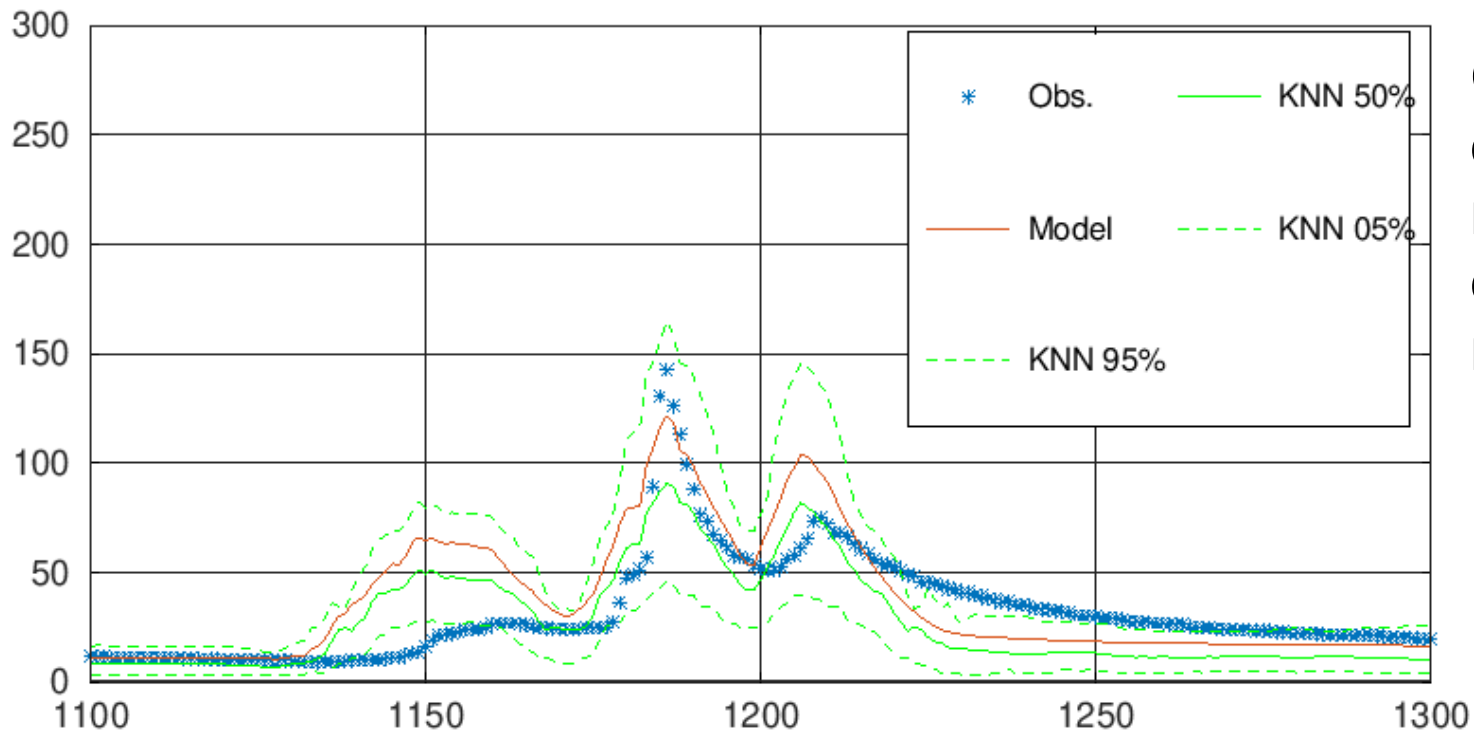
# Results: Model B



Model B  
conditional  
error



# Results: model A + model B $\rightarrow$ FNN



Conditional error eliminated, over-estimation minimized

# Conclusions

- A stochastic approach can be employed to analyze both aleatory and epistemic uncertainties in hydrological models.
- Aleatory and epistemic uncertainties manifest as characteristic patterns in the plot of the confidence intervals; however, conditional errors may remain hidden.
- A hybrid approach, combining multi-model ensembles with FNN, can be employed to cope with epistemic uncertainties, especially in addressing latent conditional errors.

# End of presentation



Thank You!

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# References

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