Towards a very High-Resolution Global Gravity Field Model up to degree and order 10800 based on Forward Modelling of the Earth's Topography

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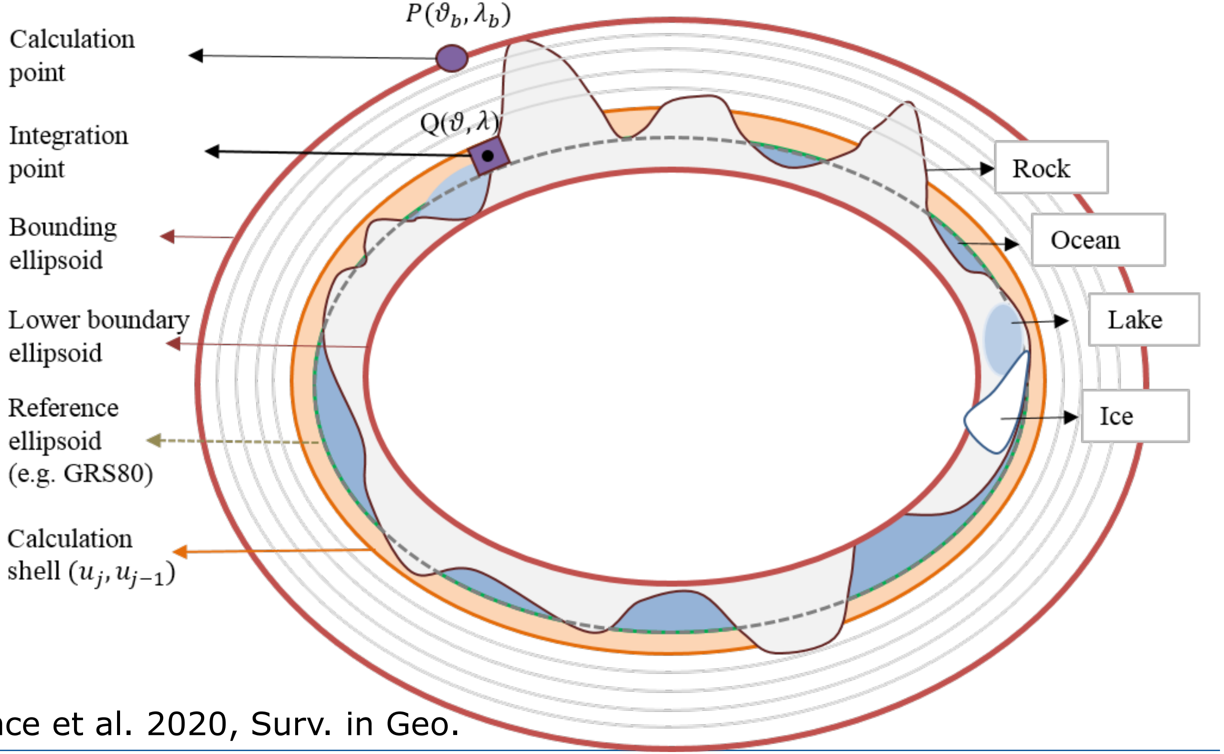
Gravity Forward Modelling (GFM)

- Computation of global gravitational field model using source mass distribution (geometry and density)
- Can be realized in spatial or spectral domain, both require global coverage of the mass-density information

Purpose / Use of GFM

- To construct high frequency components of the gravitational field (i.e. to reduce the omission error)
- To interpolate or predict gravity values in the regions that have limited or no gravity measurements
- To reduce gravity measurements for the topographical effect.

ROLI Topographic Gravity Field Model



Ince et al. 2020, Surv. in Geo.

- The calculation of the potential in terms of ellipsoidic harmonics is performed outside of all the masses (on the bounding ellipsoid).
- The Earth crust is devided in confocal ellipsoidal shells
- The integration is computed for each shell starting from a defined lower boundary ellipsoid. In the computations, 4 different density values are considered (for rock, ocean, lake and ice = „**ROLI**“)
- The gravitational potential is expanded for each shell and then summed up to represent the whole effect of the topographic potential.
- Finally, transformation into spherical harmonics is done.

Our preliminary topographic gravity field model: ROLI_EllApprox_SphN_3660

- The Earth's relief model Earth2014 (Hirt and Rexer, 2015) was used
  - **Input:** 1'x 1' global grids for SUR (surface), BED (bedrock),TBI (Topography, bedrock and ice), ICE (Major ice sheets) and Landtypes mask
  - This model is published in the IAG's ICGEM (International Centre for Global Earth Models) Service (Abrykosov et al. 2019)
- Extensive validations confirmed the validity of our approach (Ince et al. 2020)
  - W.r.t similar models in the literature (Grombein et al. 2016; Rexer et al. 2016)
  - W.r.t ground truth measurements
- The outcome product "topographic GGM represented in terms of spherical harmonic coefficients" has been tested to augment/complement GFZ's EIGEN-6C4 for the short wavelength components from degree 2190 up to 3660.

Contribution of ROLI_SphN_3660 to EIGEN-6C4

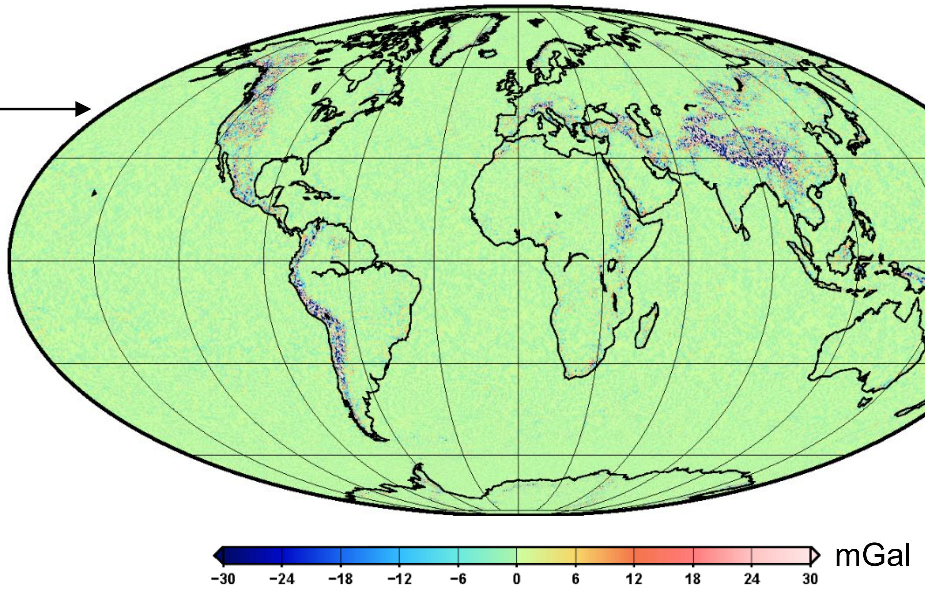
Blended Test Model

EIGEN-6C4.2000.2100.ROLI_SphN_3660

The blended model is augmented using ROLI with a transition range at the coefficient level

Promising reduction of omission error in future generation EIGEN series

What can we do better?



Difference between the enhanced EIGEN-6C4 expanded up to degree 3660 and original EIGEN-6C4

The project GRAV4GEO

GRAVitational field modelling of Earth's topography **For GEO**detic and **GEO**physical applications

- Based on our previous work we are going to to expand our global gravity field forward modelling up to degree and order 10800 using an optimum shell thickness (about few meters)
- This forward modelling will be done by using state-of-the-art high resolution laterally varying density and elevation models (with dedicated validation and merging).
- The project is funded over three years by the German Research Foundation (Deutsche Forschungsgemeinschaft – DFG, grand No. 505165206)

New DEMs and density models

Instead of using Earth2014 only, the listed DEM and density models are considered in the new analyses.

- TanDEM-X 90m DEM , ETOPO2022, GLOBE 30, GEBCO2022, IceBridge BedMachine Greenland v5, MEaSUREs BedMachine Antarctica v3

- UNB_TopoDensT_2v02, CRUST 1.0, WINTERC-G

- Improved masking for ocean, land, lakes and ice covered areas

Ongoing GRAV4GEO activities

- Digital elevation and global and regional density models are acquired from different sources and assessed for our forward modelling purposes
- Assessment and evaluation of very-high-resolution DEMs are on-going. A dedicated merged DEM model will be created for the project and for the public use
- Corresponding density-depth-thickness information for different crustal layers will be made compatible with each other and with our approach
- Datasets for validation purposes such as other topographic GGMs as well as ground point measurements will be collected

Summary and Outlook

- We are going to develop a global topographic gravity field model up to degree and order 10800 based on a multi layer approach in the spectral domain
- This model will be based on the state-of-the-art high resolution laterally varying density and elevation models.
- Our product will be provided in terms of spherical harmonic coefficients incl. uncertainty estimations
- This model can be used to accurately calculate any gravity field functional at any location on the Earth surface reaching a spatial resolution of ~2km.

References

Abrykosov O, Ince ES, Förste C, Flechtner F (2019): ROLI topographic gravity field model, from four-layer Earth decomposition. GFZ Data Serv. https://doi.org/10.5880/ICGEM.2019.011

Grombein T, Seitz K, Heck B (2016): The rock–water–ice topographic gravity field model RWI_TOPO_2015 and its comparison to a conventional rock-equivalent version. Surv Geophys 37(5):937–976. https://doi.org/10.1007/s10712-016-9376-0

Hirt C, Rexer M (2015): Earth 2014 1 arc-min shape, topography, bedrock and ice-sheet models—available as gridded data and degree-10,800 spherical harmonics. Int J Appl Earth Obs Geoinf 39:103 – 112. https://doi.org/10.1016/j.jag.2015.03.001

Ince, E. S., Abrykosov, O., Förste, C., Flechtner, F. (2020): Forward Gravity Modelling to Augment High-Resolution Combined Gravity Field Models. - Surveys in Geophysics, 41, 767-804. https://doi.org/10.1007/s10712-020-09590-9

Rexer M, Hirt C, Claessens S, Tenzer R (2016): Layer-based modelling of the Earth's gravitational potential up to 10-km scale in spherical harmonics in spherical and ellipsoidal approximation. Surv Geophys 37(6):1035–1074. https://doi.org/10.1007/s10712-016-9382-2

Appendices

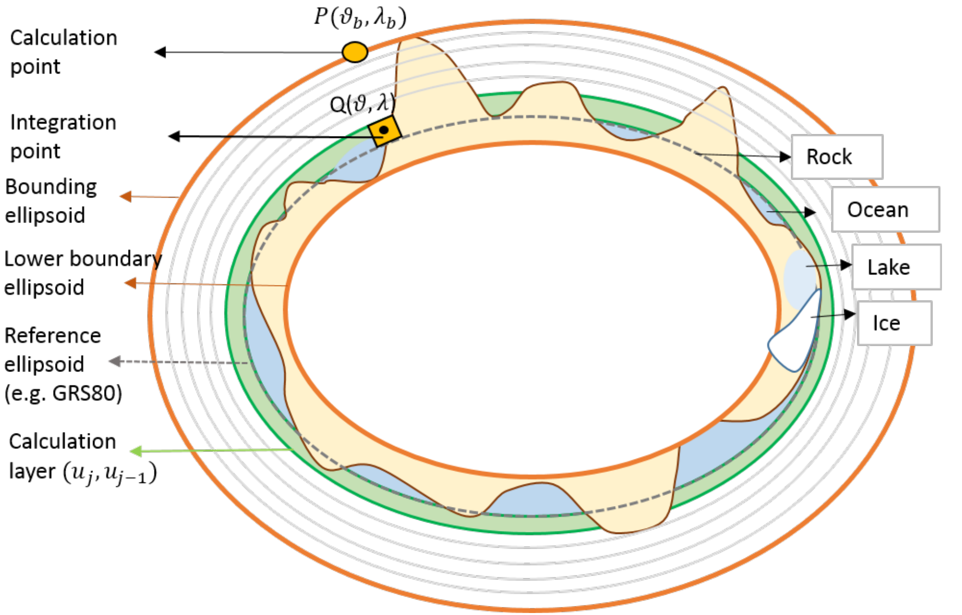
1. Our approach - ROLI Topographic Gravity Field Modelling
2. DEMs and density models to be used

Our approach:
ROLI Topographic Gravity Field Modelling (1)

- **ROLI (Rock Ocean Lake Ice)** model is developed based on multi layer approach and it represents the topographic gravity field model up to maximum d/o
- Newton's integral is expressed for the masses located between the bounding ellipsoid (external to all source masses) and the lower most boundary ellipsoid

$$V(b,\vartheta_b,\lambda_b) = G\iiint \frac{\rho(u,\vartheta,\lambda)}{\ell(b,\vartheta_b,\lambda_b,u,\vartheta,\lambda)}dv$$

$$dv = \left(u^2 + E^2\cos^2\vartheta\right)\sin\vartheta d\vartheta d\lambda du$$



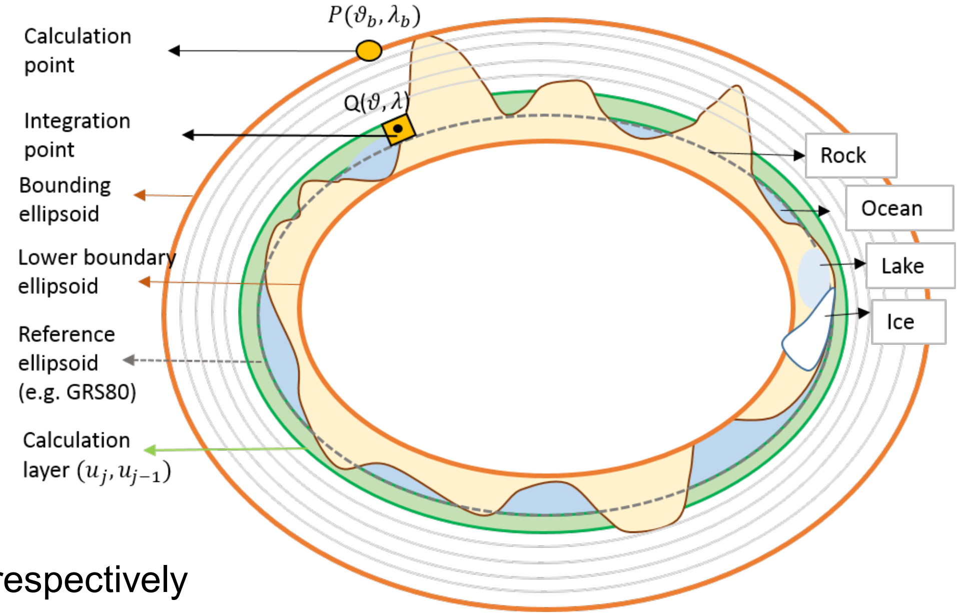
ROLI Topographic Gravity Field Modelling (2)

The reciprocal distance between the calculation and integration points is expanded in terms of elementary ellipsoidal surface harmonics, where the calculation point is located on the bounding ellipsoid that is outside of all the masses and the integration point that is located within the layers inside the topography

$$\frac{1}{\ell}=\frac{1}{E}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{1}{2n+1}q_{nm}(\sigma_b)p_{nm}(\sigma)\times$$
$$\left[\bar{R}_{nm}(\vartheta_b,\lambda_b)\bar{R}_{nm}(\vartheta,\lambda)+\bar{S}_{nm}(\vartheta_b,\lambda_b)\bar{S}_{nm}(\vartheta,\lambda)\right]$$

$$\left.\begin{matrix}\bar{R}_{nm}(\vartheta,\lambda)\\ \bar{S}_{nm}(\vartheta,\lambda)\end{matrix}\right\}=\bar{P}_{nm}(\cos\vartheta)\left\{\begin{matrix}\cos m\lambda\\ \sin m\lambda\end{matrix}\right. \qquad \sigma_b=\frac{b}{E},\quad \sigma=\frac{u}{E}$$

$q_{nm}(\sigma_b)$ and $p_{nm}(\sigma)$ are real functions proportional to Legendre functions of 2nd and 1st kind respectively

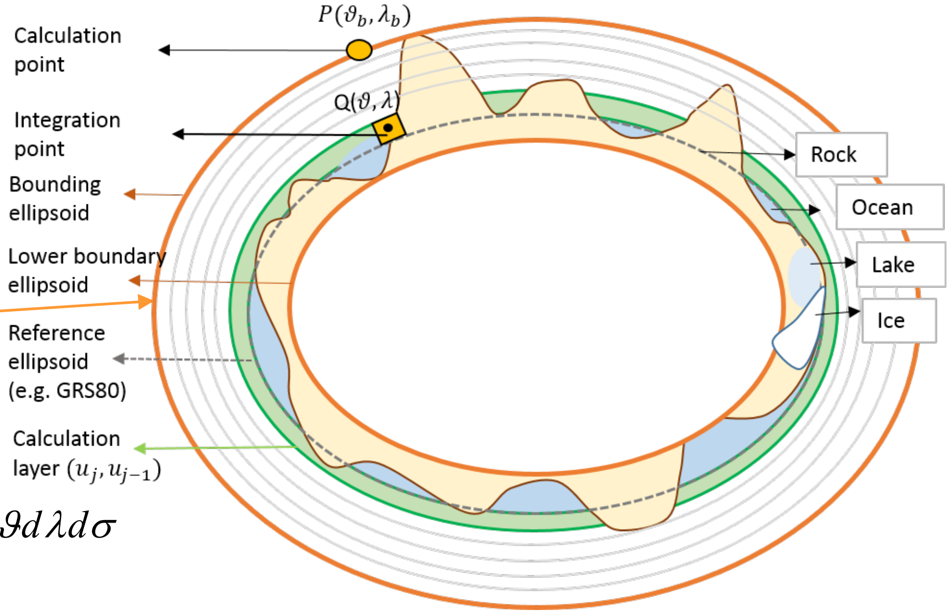


b semi minor axis of bounding ellipsoid, u semi minor axis of the ellipsoid of integration point, E linear eccentricity.

ROLI Topographic Gravity Field Modelling (3)

- The shells between two confocal ellipsoids are represented that are independent of the vertical density variation
- The gravitational potential of the topographic masses is expressed in terms of ellipsoidal harmonics

$$V = GE^3 \sum_{n=0}^{N} \sum_{m=0}^{n} \left[\bar{A}_{nm} \bar{R}_{nm}(\vartheta_b, \lambda_b) + \bar{B}_{nm} \bar{S}_{nm}(\vartheta_b, \lambda_b) \right]$$



where the coefficients are volume integrals

$$\bar{A}_{nm} = \frac{q_{nm}(\sigma_b)}{2n+1} \iiint \rho p_{nm}(\sigma) \bar{R}_{nm}(\vartheta, \lambda) \left(\sigma^2 + \cos^2 \vartheta\right) \sin \vartheta d\vartheta d\lambda d\sigma$$

$$\bar{B}_{nm} = \frac{q_{nm}(\sigma_b)}{2n+1} \iiint \rho p_{nm}(\sigma) \bar{S}_{nm}(\vartheta, \lambda) \left(\sigma^2 + \cos^2 \vartheta\right) \sin \vartheta d\vartheta d\lambda d\sigma$$

ROLI Topographic Gravity Field Modelling (4)

- The coefficients are represented as sums of potential coefficients corresponding to each shell:

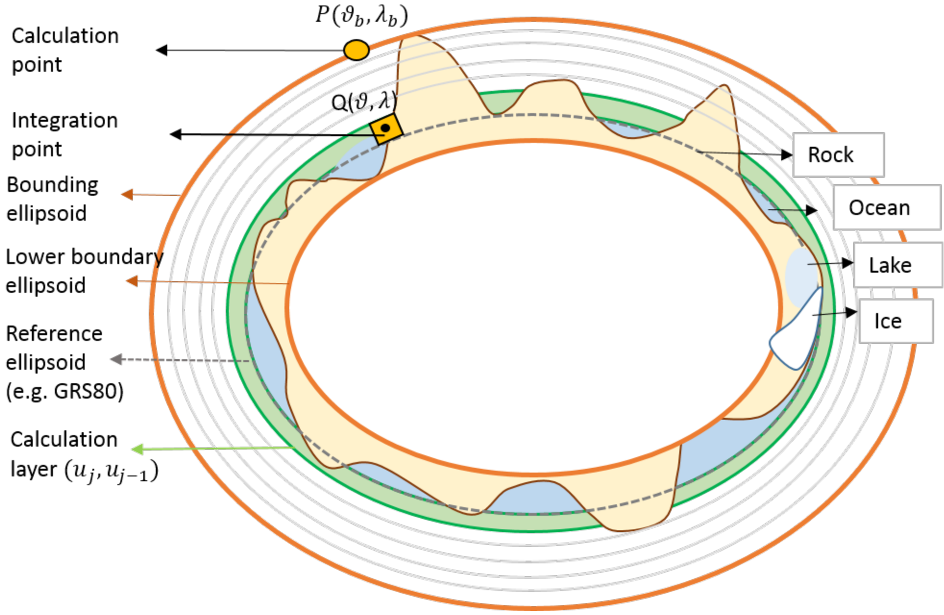
$$\bar{A}_{nm} = \sum_j \bar{A}_{nm}^{j-1,j} \qquad \bar{B}_{nm} = \sum_j \bar{B}_{nm}^{j-1,j}$$

- Finally, we put all the coefficients above the maximum degree to be equal to zero:

$$\{\bar{A}_{nm} = \bar{B}_{nm} = 0\}_{n>n_{max}}$$

and apply Jekeli's transformation to obtain spherical harmonic coefficients

$$\{\bar{C}_{nm}, \bar{S}_{nm}\}_{n\leq n_{max}}$$



DEMs and density models to be used

Digital elevation models

- ETOPO2022: https://www.ncei.noaa.gov/products/etopo-global-relief-model
- GLOBE 30: https://www.ngdc.noaa.gov/mgg/topo/globe.html
- GEBCO2022: https://www.gebco.net/data_and_products/gridded_bathymetry_data/
- BedMac. Gre. v5: https://sites.uci.edu/morlighem/dataproducts/bedmachine-greenland/
- MEaSUREs BedMachine Antarctica v3: https://nsidc.org/data/NSIDC-0756/versions/2
- Earth2014 (or updated version if released): http://ddfe.curtin.edu.au/models/Earth2014/
- TanDEM-X 90m DEM: https://geoservice.dlr.de/web/dataguide/tdm90

Global density models:

- UNB_TopoDensT_2v02 density model: https://www.unb.ca/fredericton/engineering/depts/gge/resources.html
- CRUST 1.0: https://igppweb.ucsd.edu/~gabi/crust1.html
- Earth's Spectral Crustal Model (ESCM180, available upon request)
- WINTERC-G