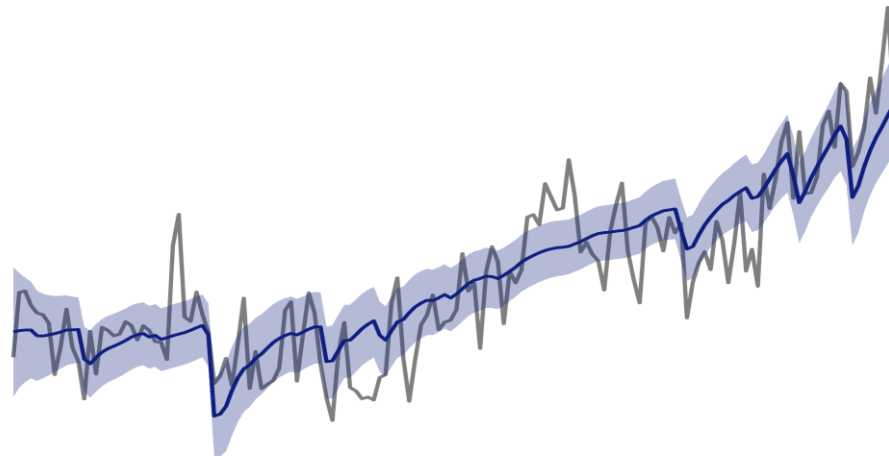


# Emulating internal and external components of global temperature variability

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# Temperature variability

Temperature variability  
(observed / simulated)

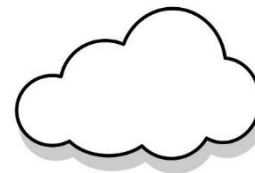
forced



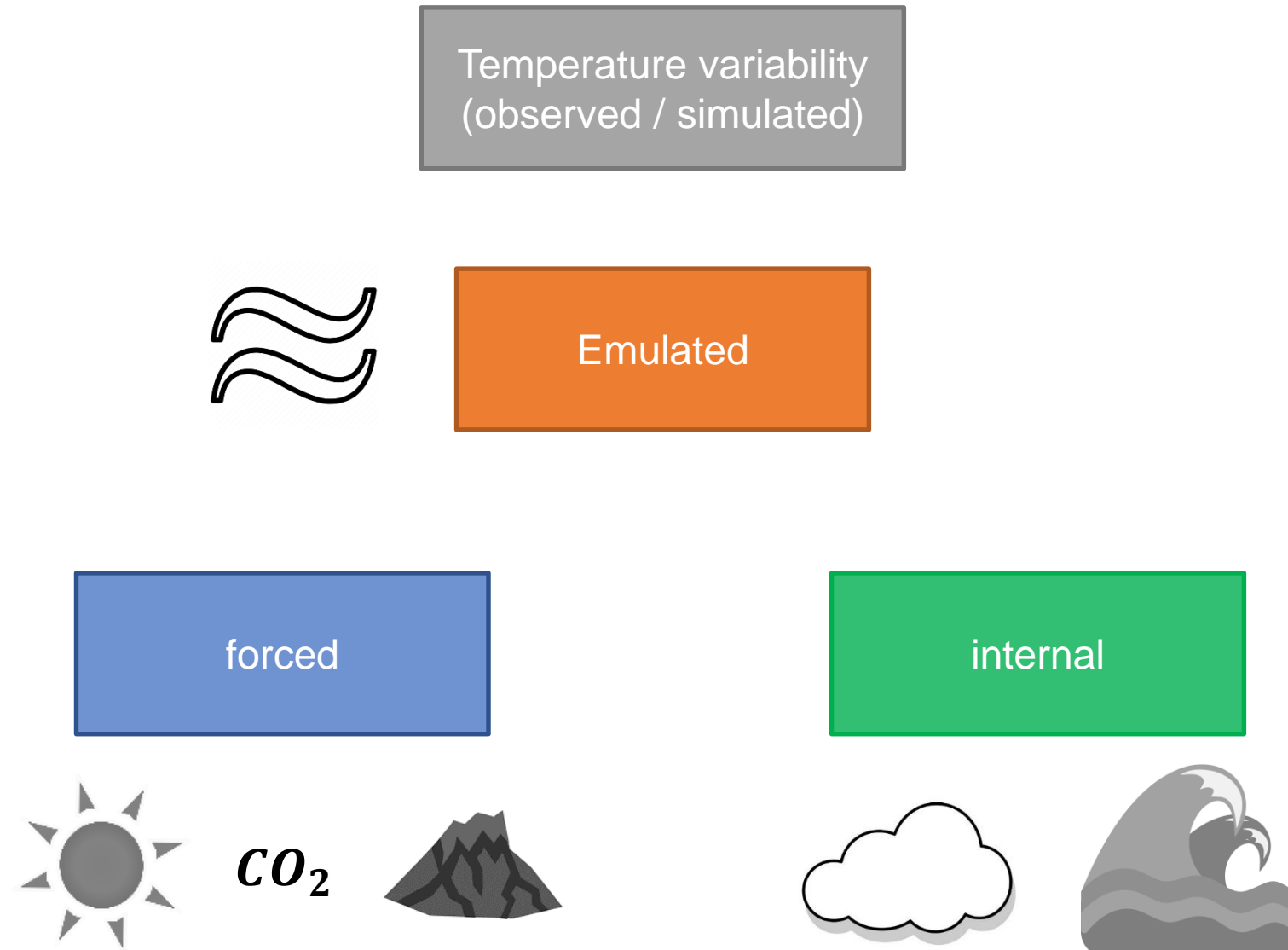
*CO<sub>2</sub>*



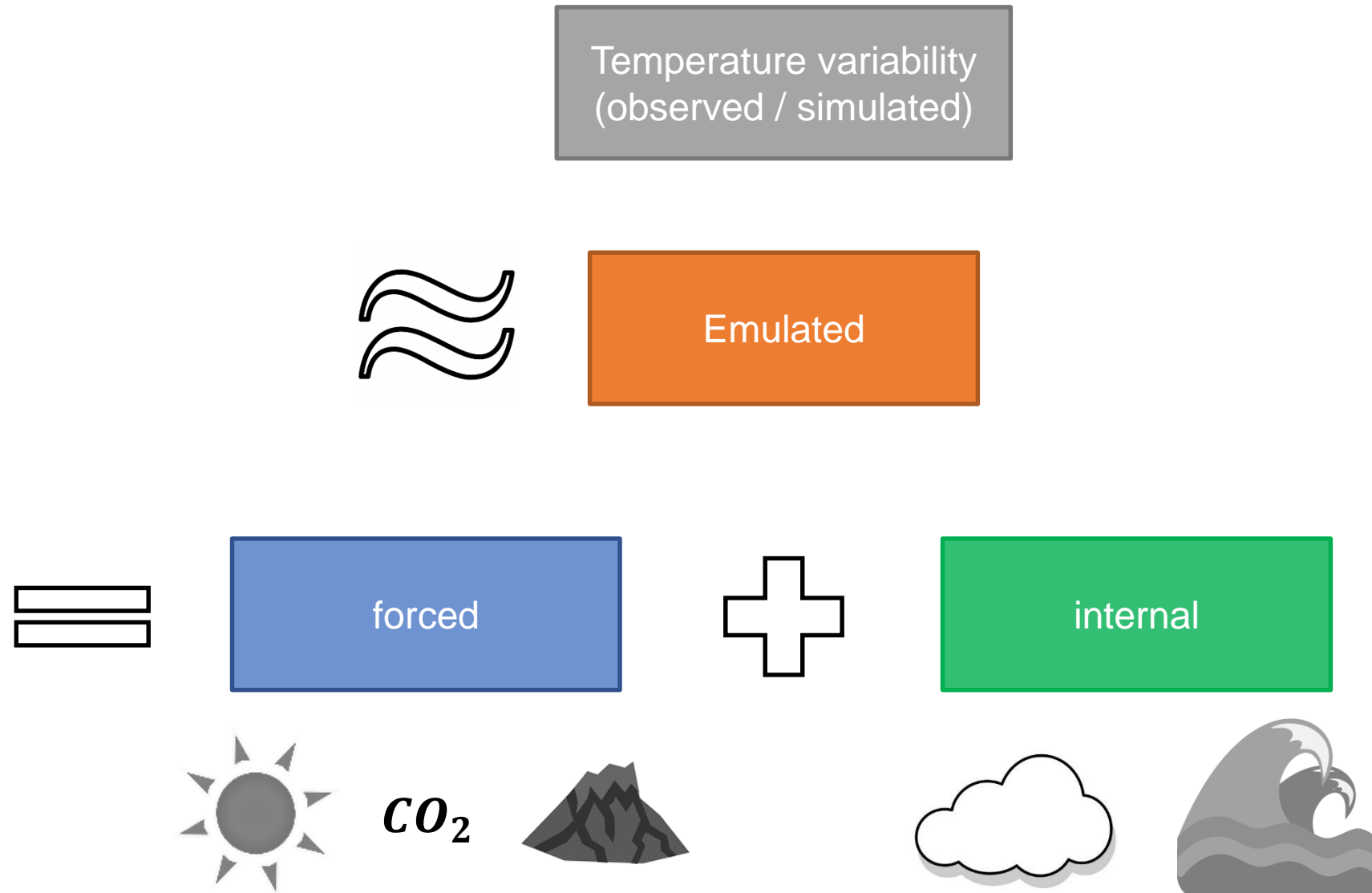
internal



# Temperature variability



# Temperature variability



# Separating forced from internal variability

1

Linear stochastic EBM

$$T(t) = \textit{forced} + \textit{internal}$$

Data

temperature

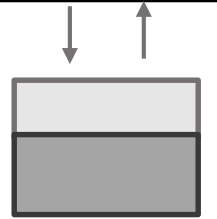
Bayesian Inference

Output

Best estimate of forced and internal temperature variations

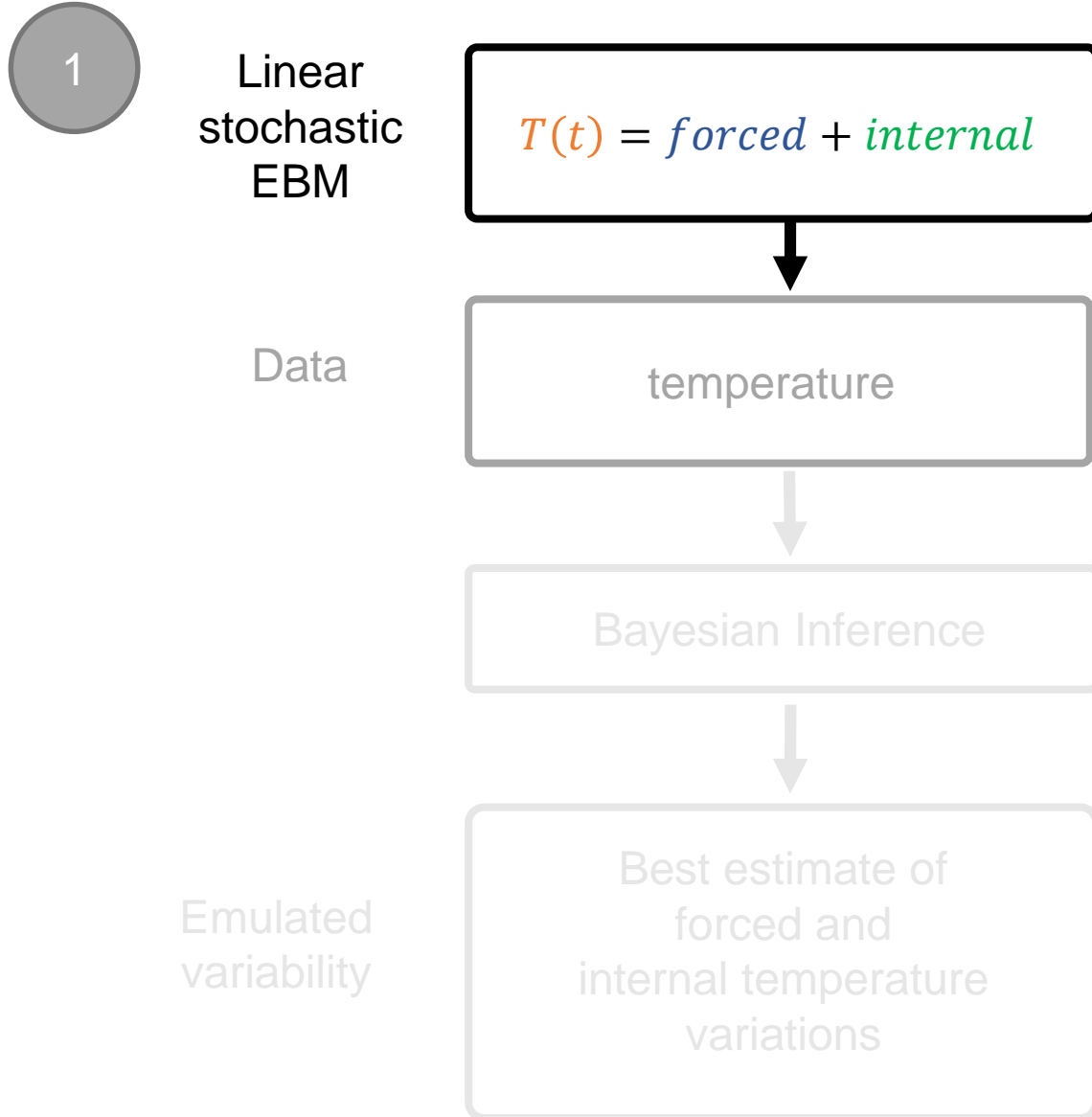
Linear stochastic two-box Energy Balance Model

$$C \frac{dT}{dt}(t) = K T(t) + F(t) + \xi(t)$$

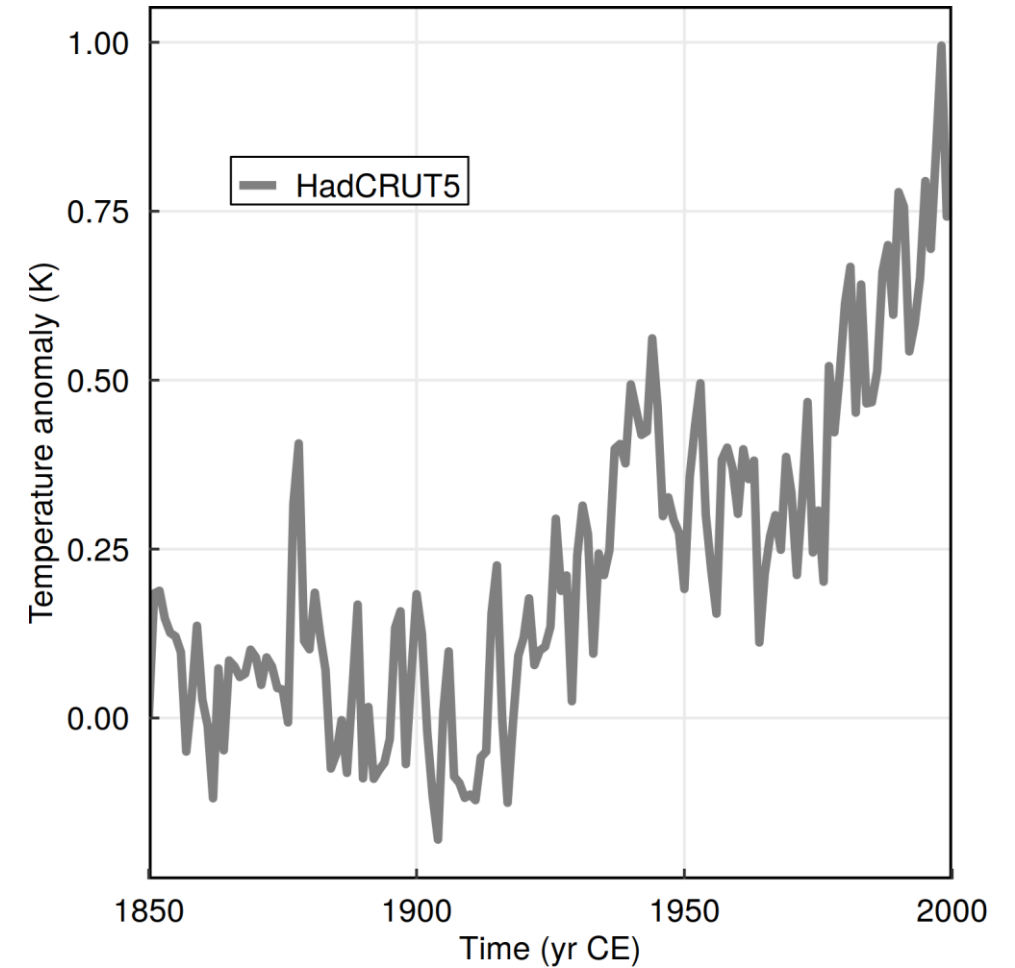


$$T(t) = \int R(t-s)(\underbrace{F(s)ds}_{\textit{forced}} + \underbrace{dW(s)}_{\textit{internal}})$$

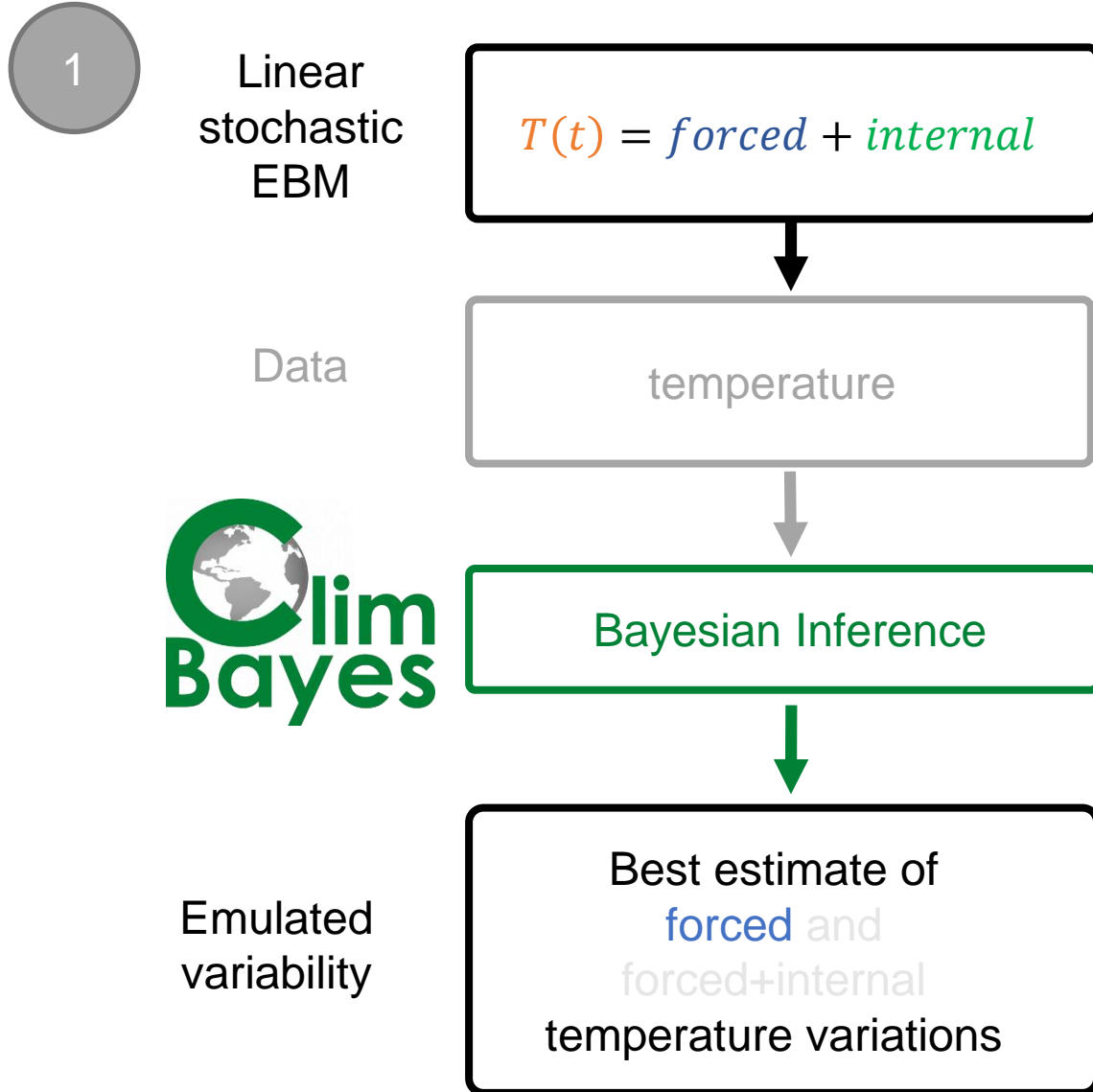
# Separating forced from internal variability



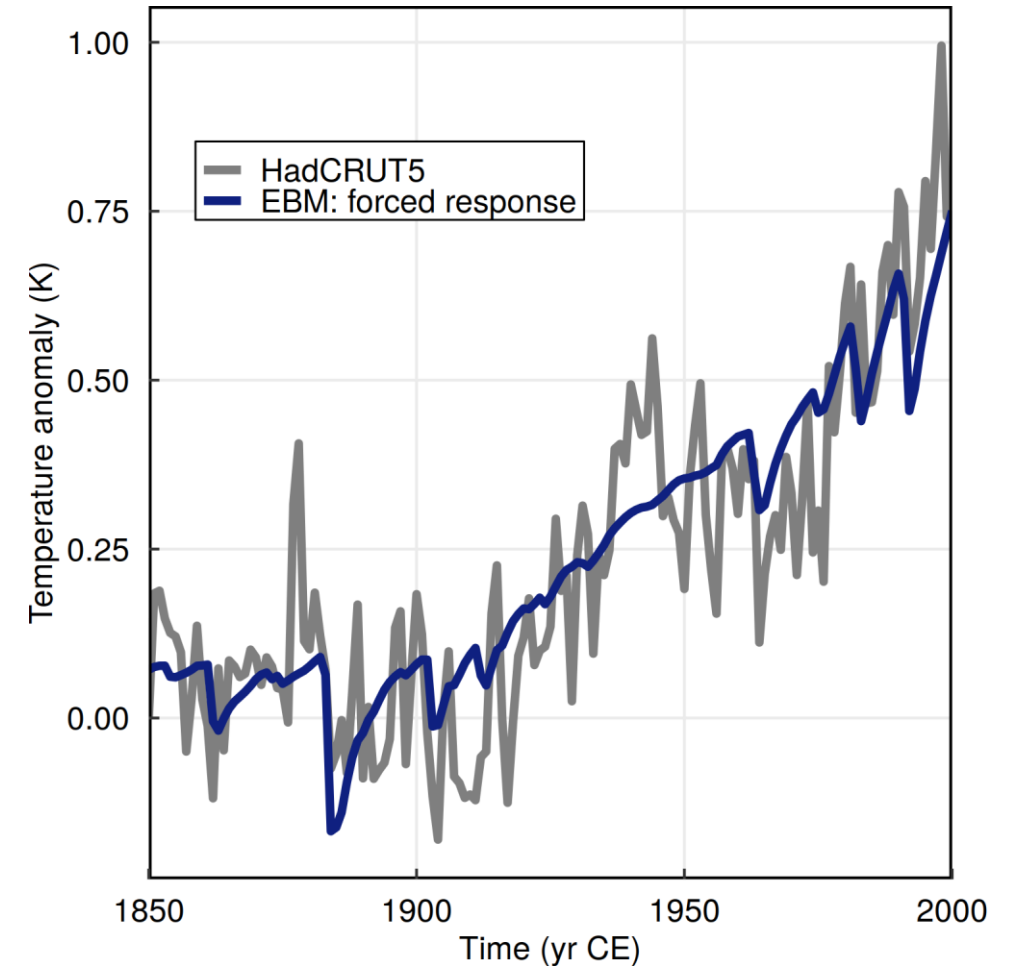
## Example: historical data



# Separating forced from internal variability



## Example: historical data



# Separating forced from internal variability

1

Linear stochastic EBM

$$T(t) = \textit{forced} + \textit{internal}$$

Data

temperature

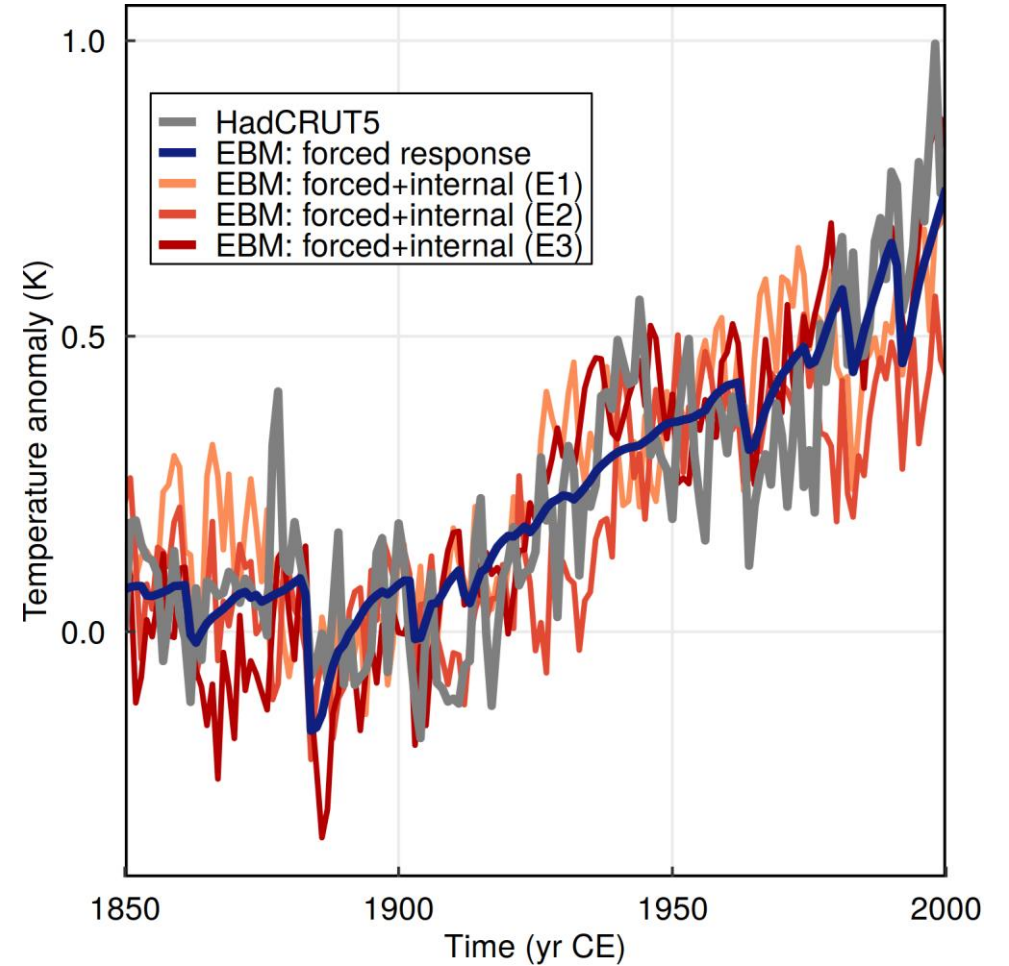


Bayesian Inference

Emulated variability

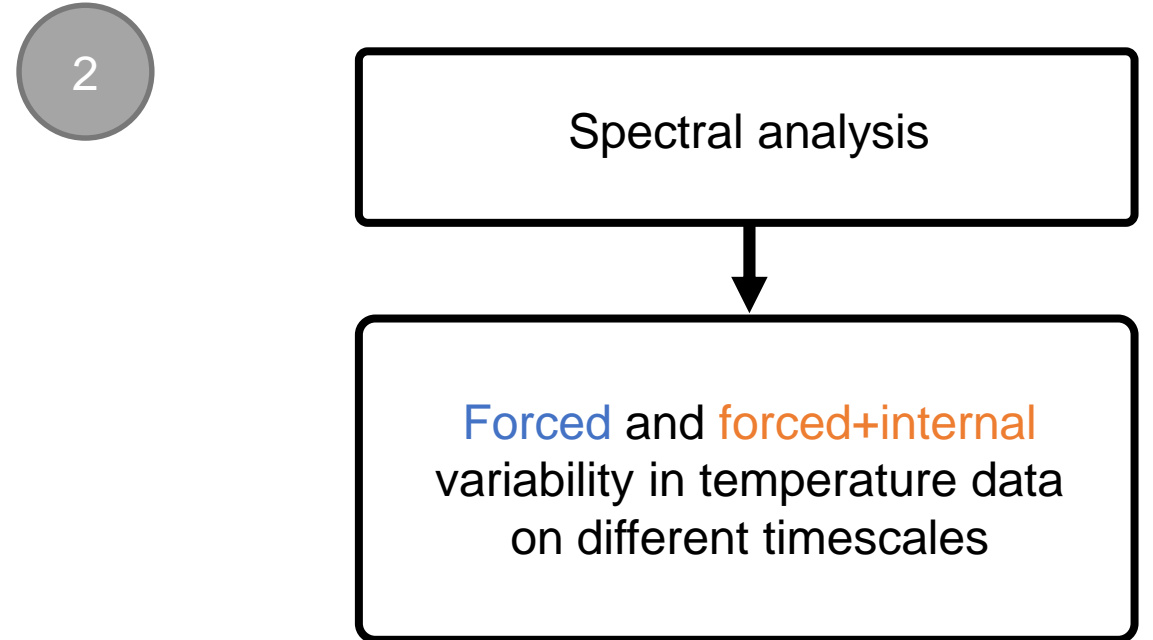
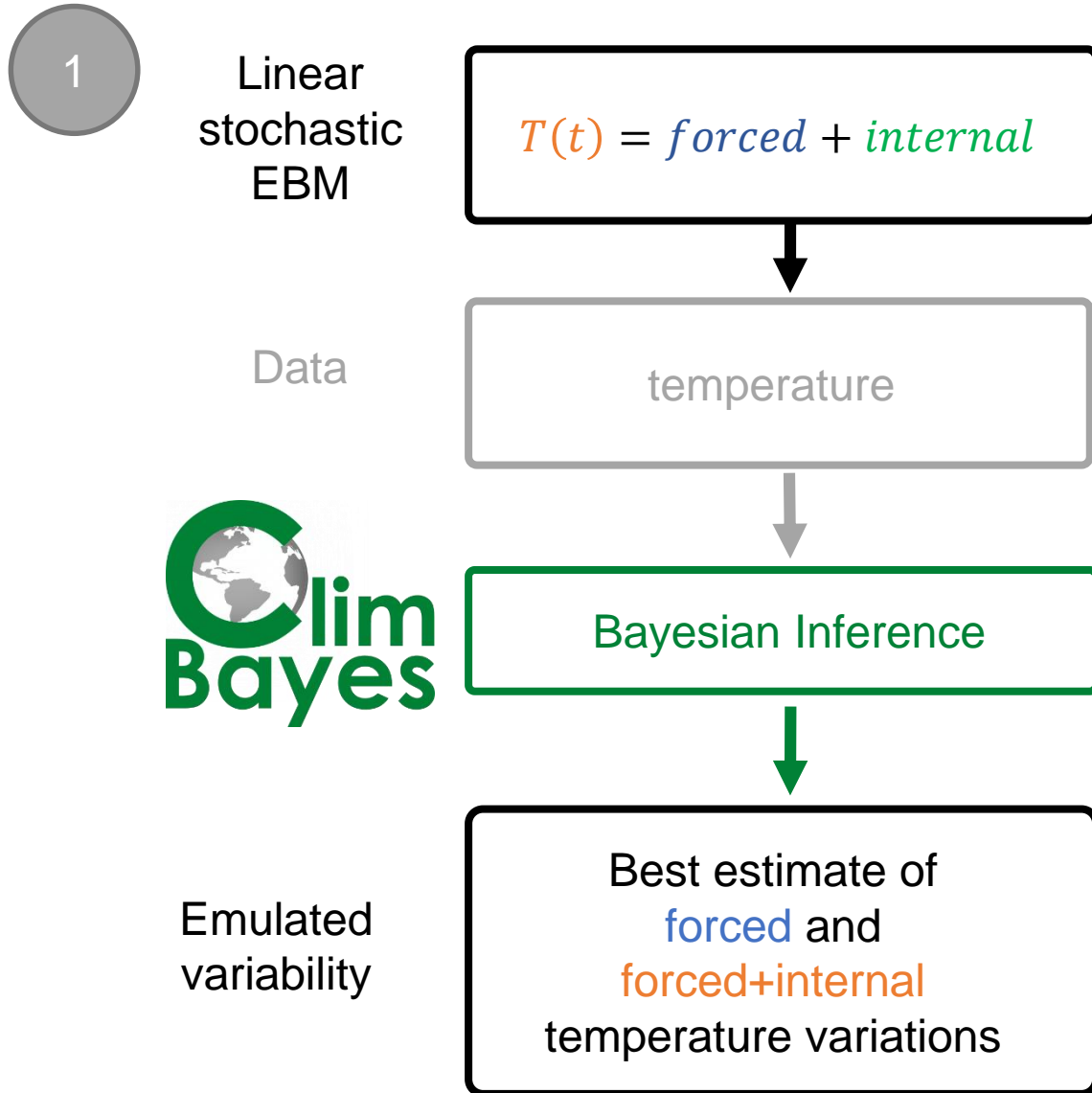
Best estimate of **forced** and **forced+internal** temperature variations

### Example: historical data

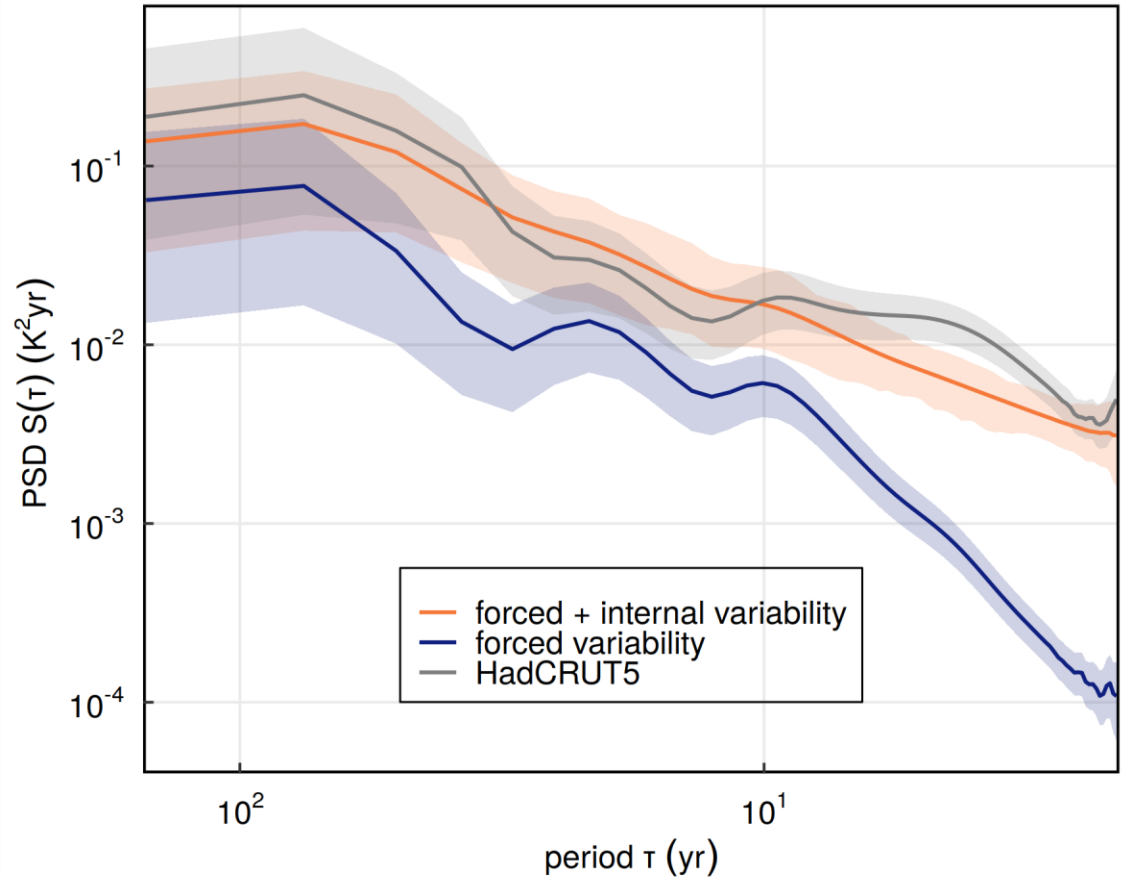
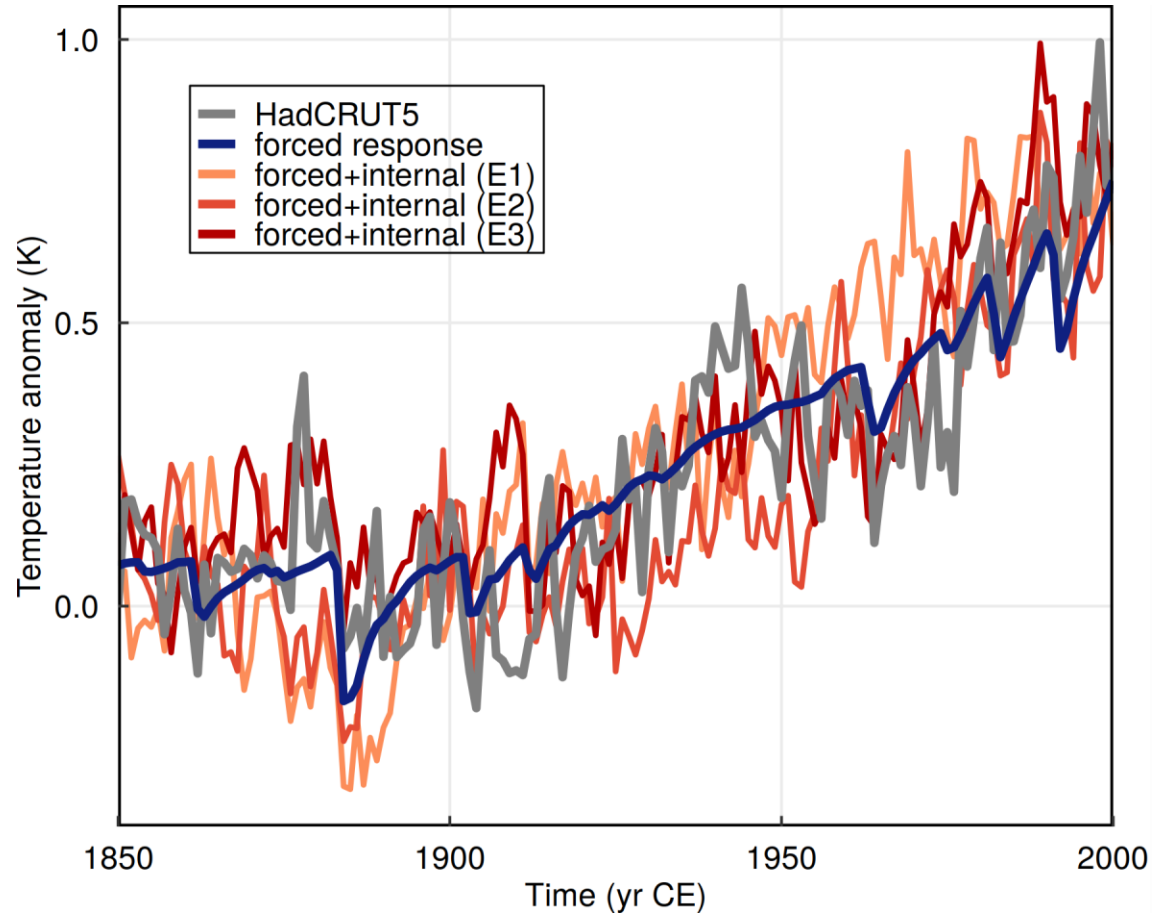




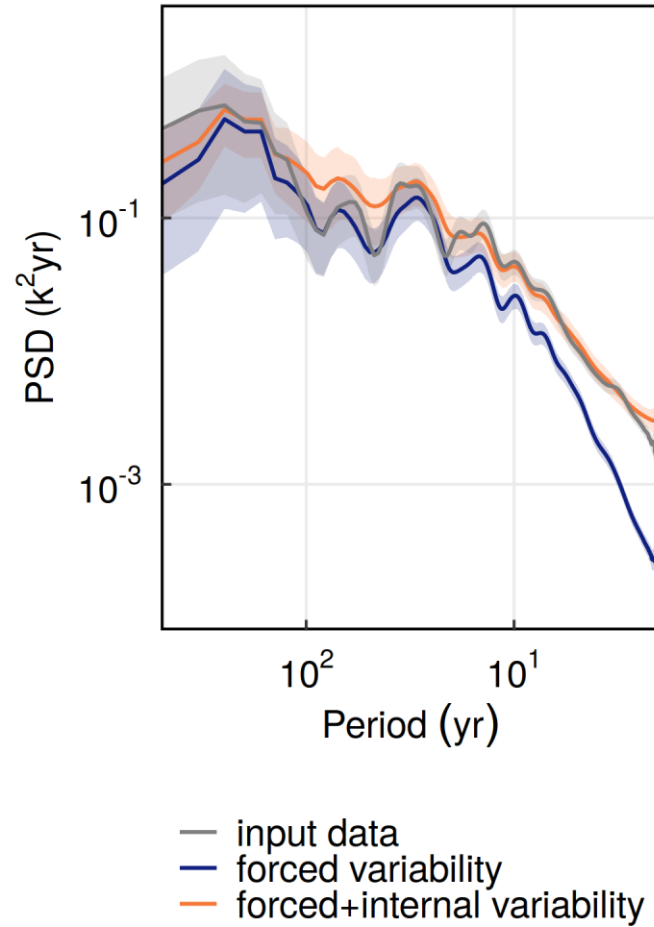
# Separating forced from internal variability



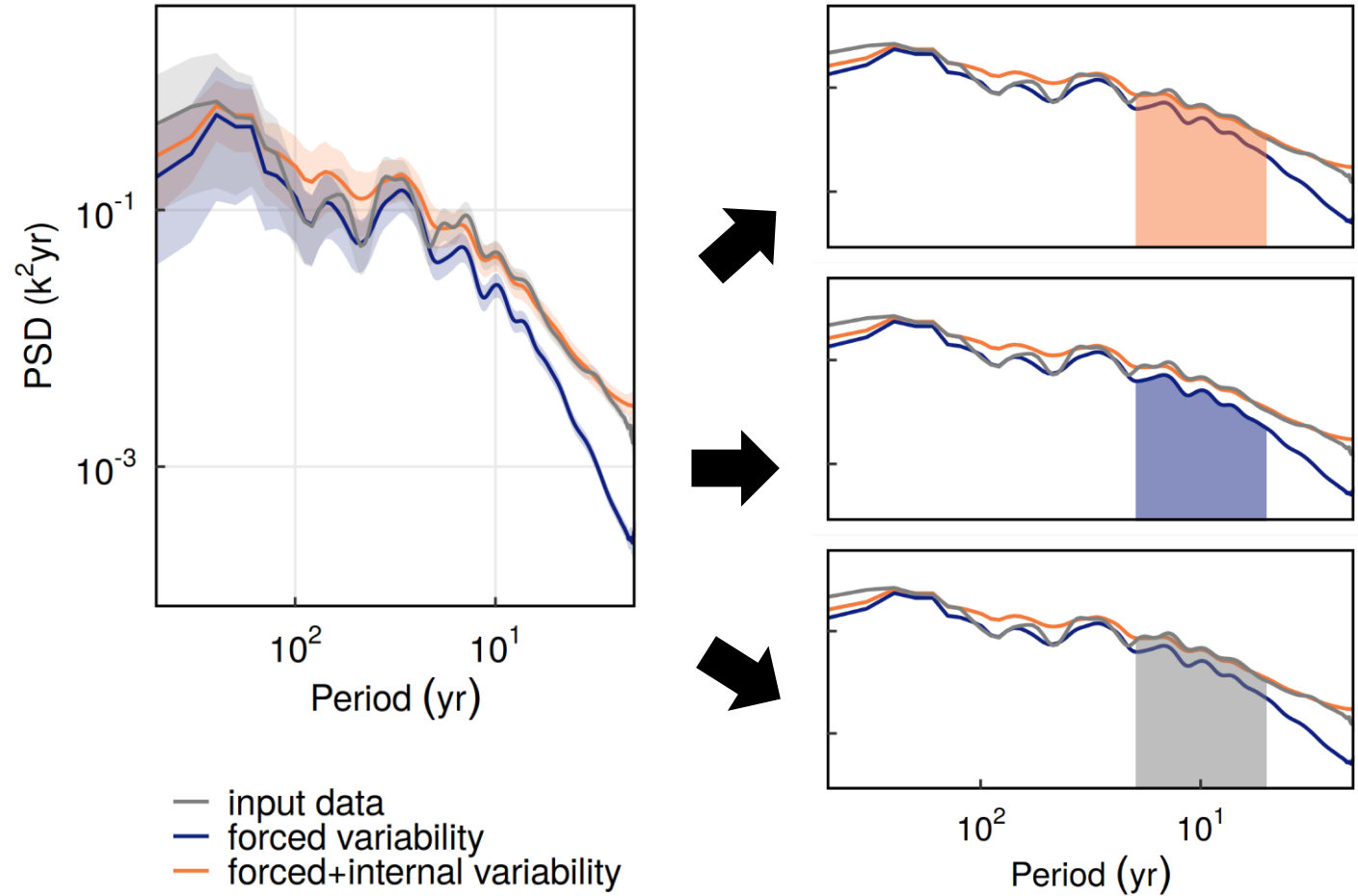
# Step 2: Spectral analysis



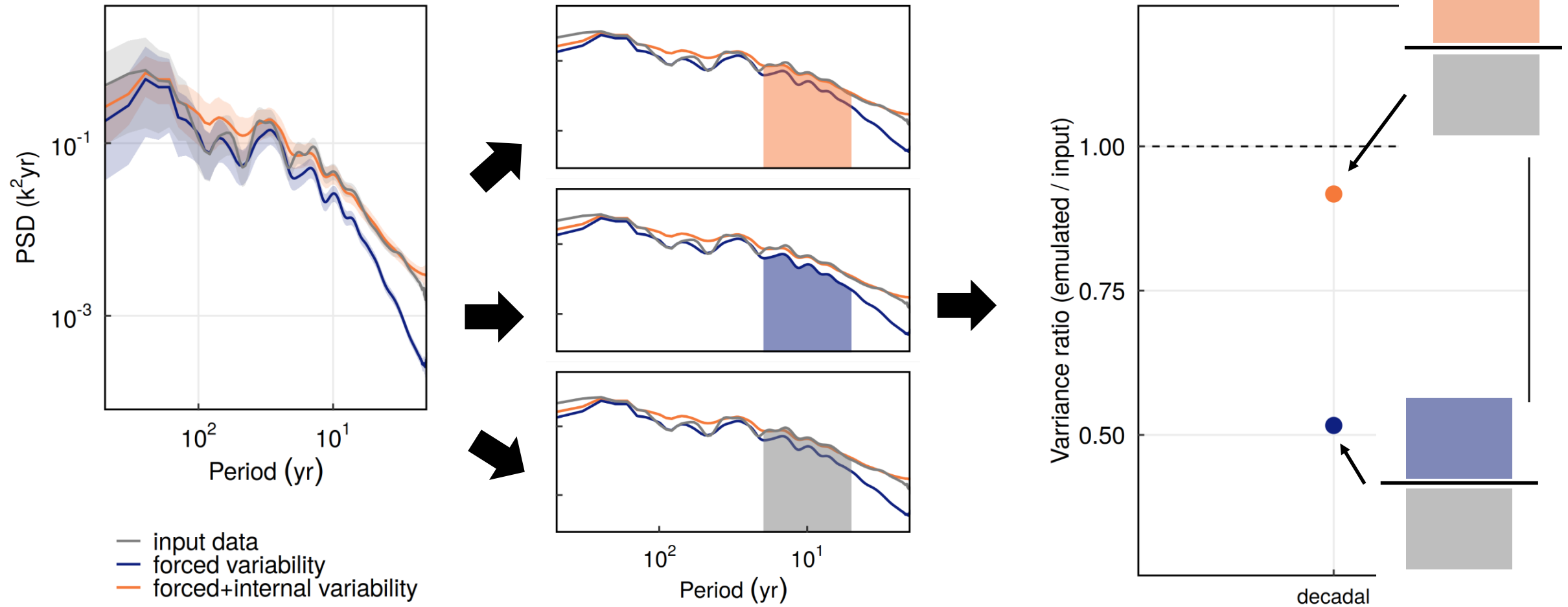
# From the power spectrum to spectral ratios



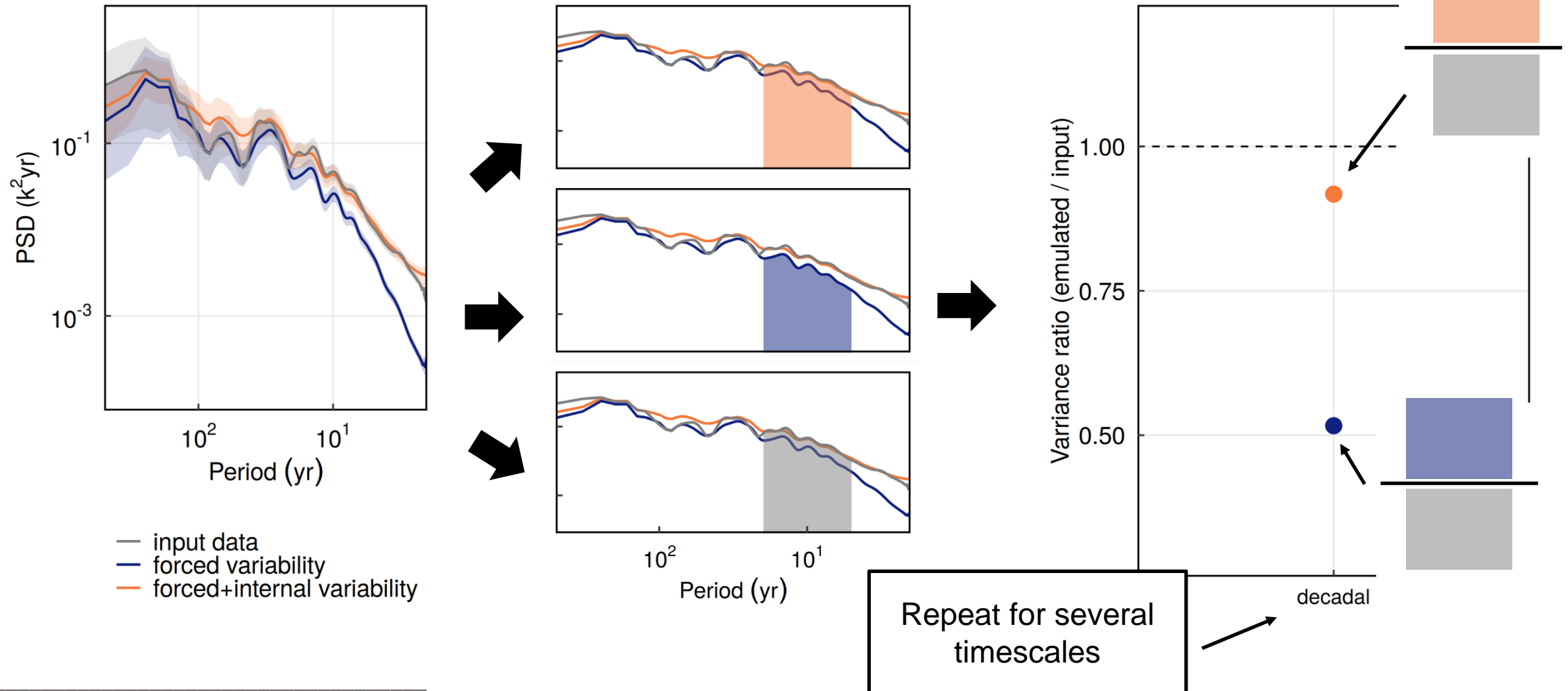
# From the power spectrum to spectral ratios



# From the power spectrum to spectral ratios

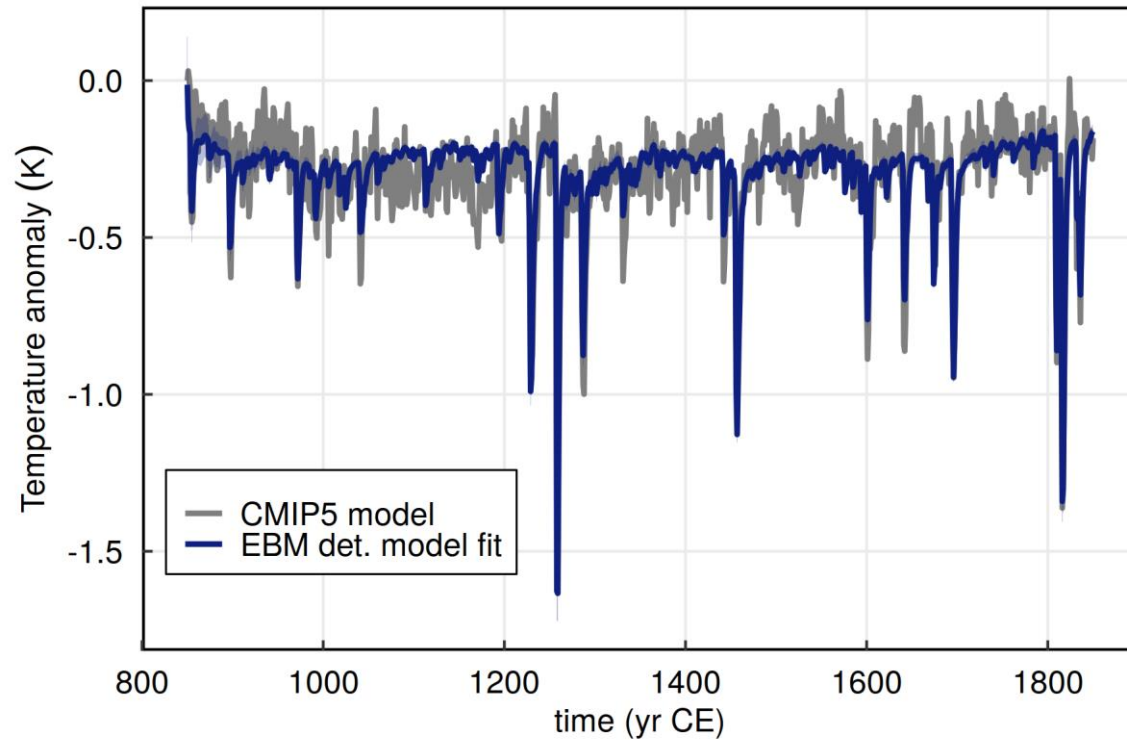


# From the power spectrum to spectral ratios

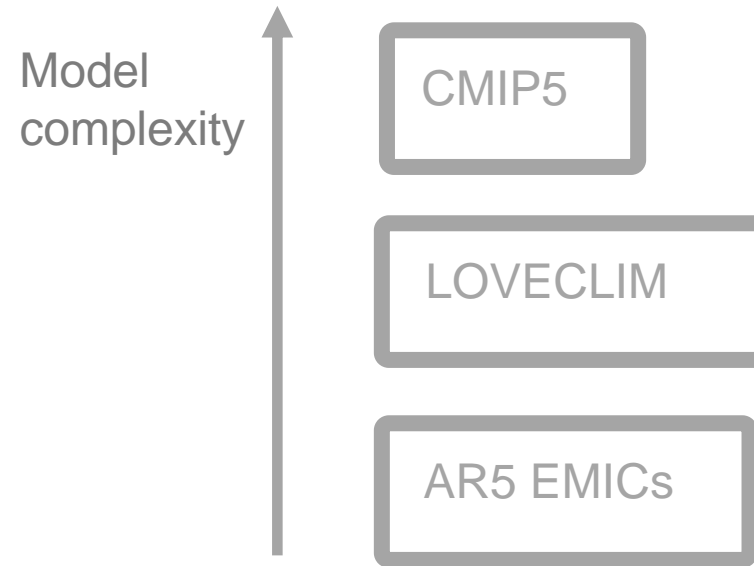


# Input data for main analysis

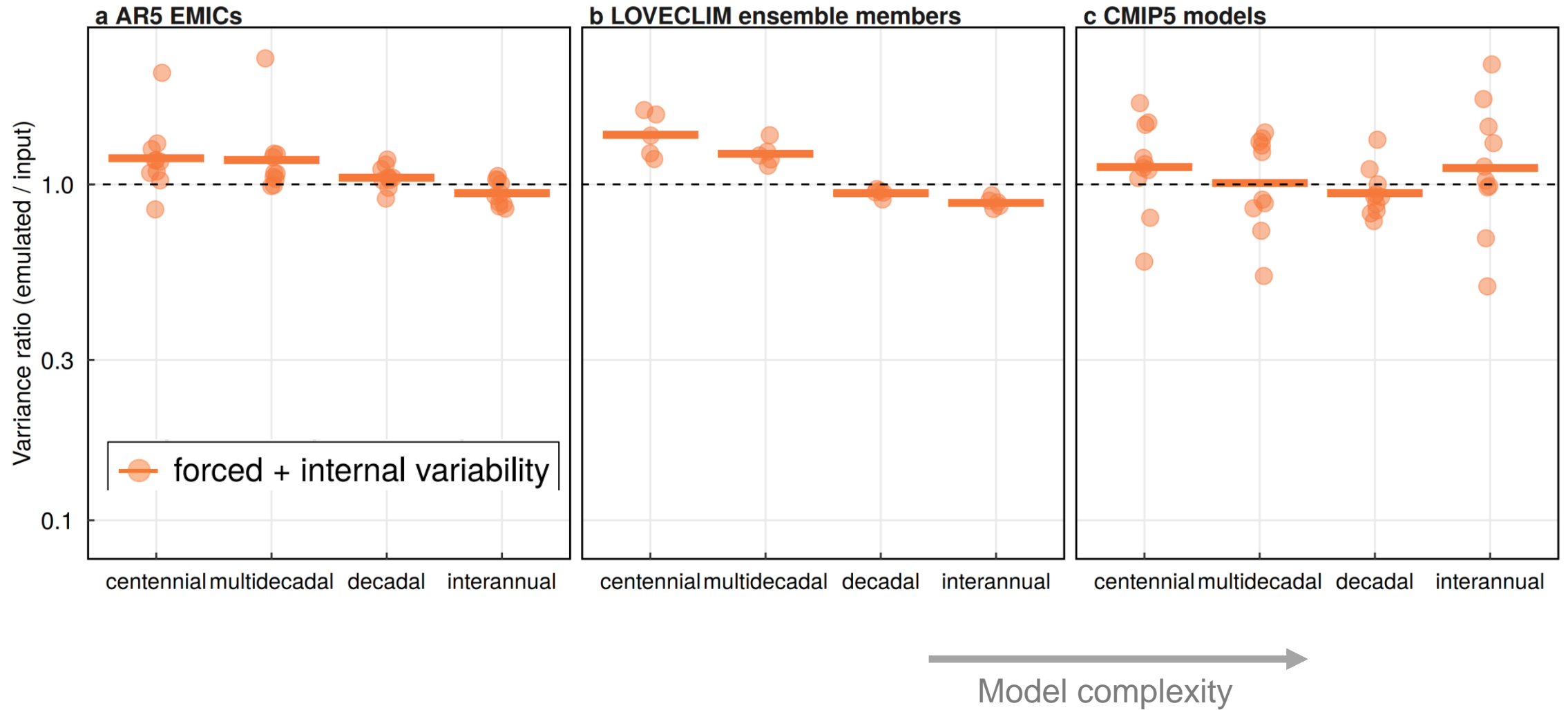
Data from Last Millennium (850-1850)



Models of varying complexity

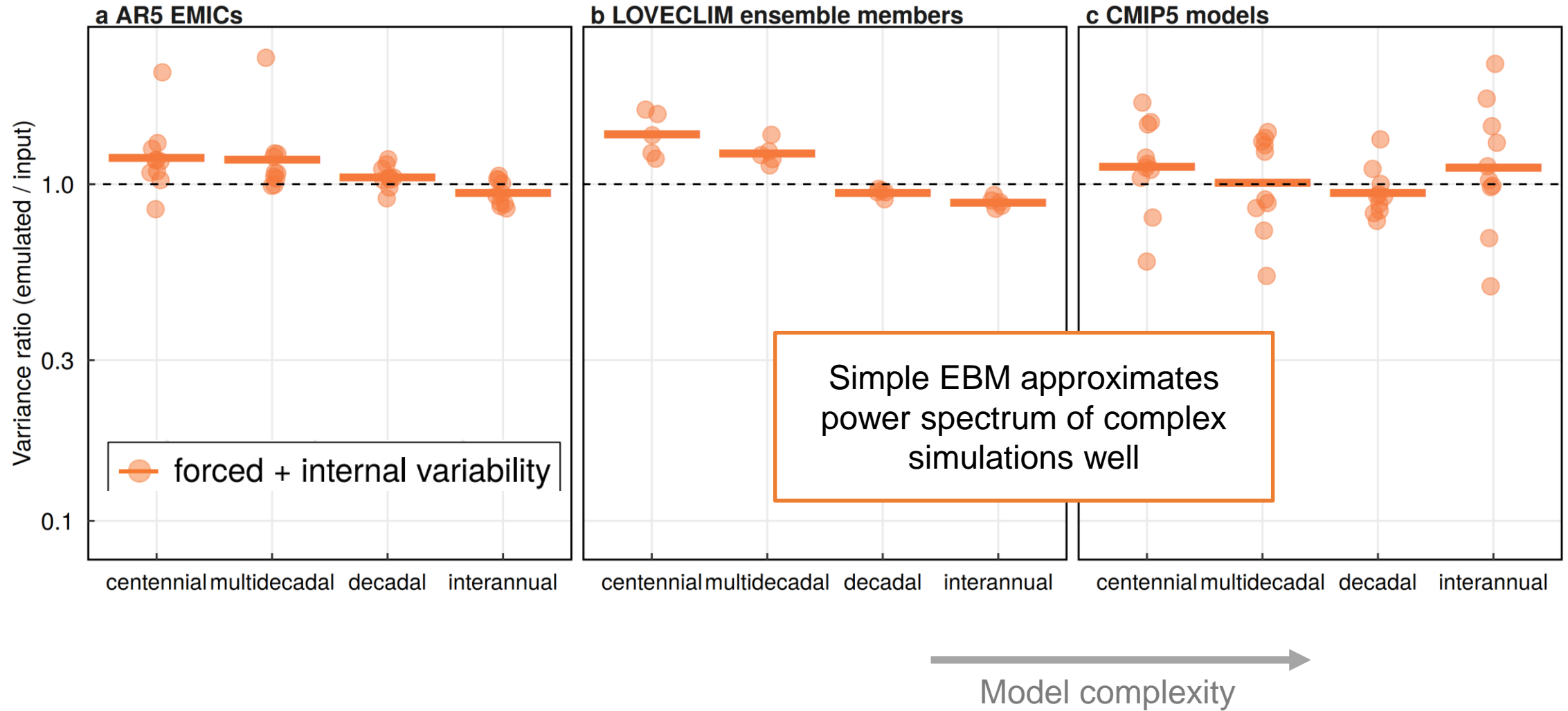


# Internal & externally-forced contribution to global temperature variability

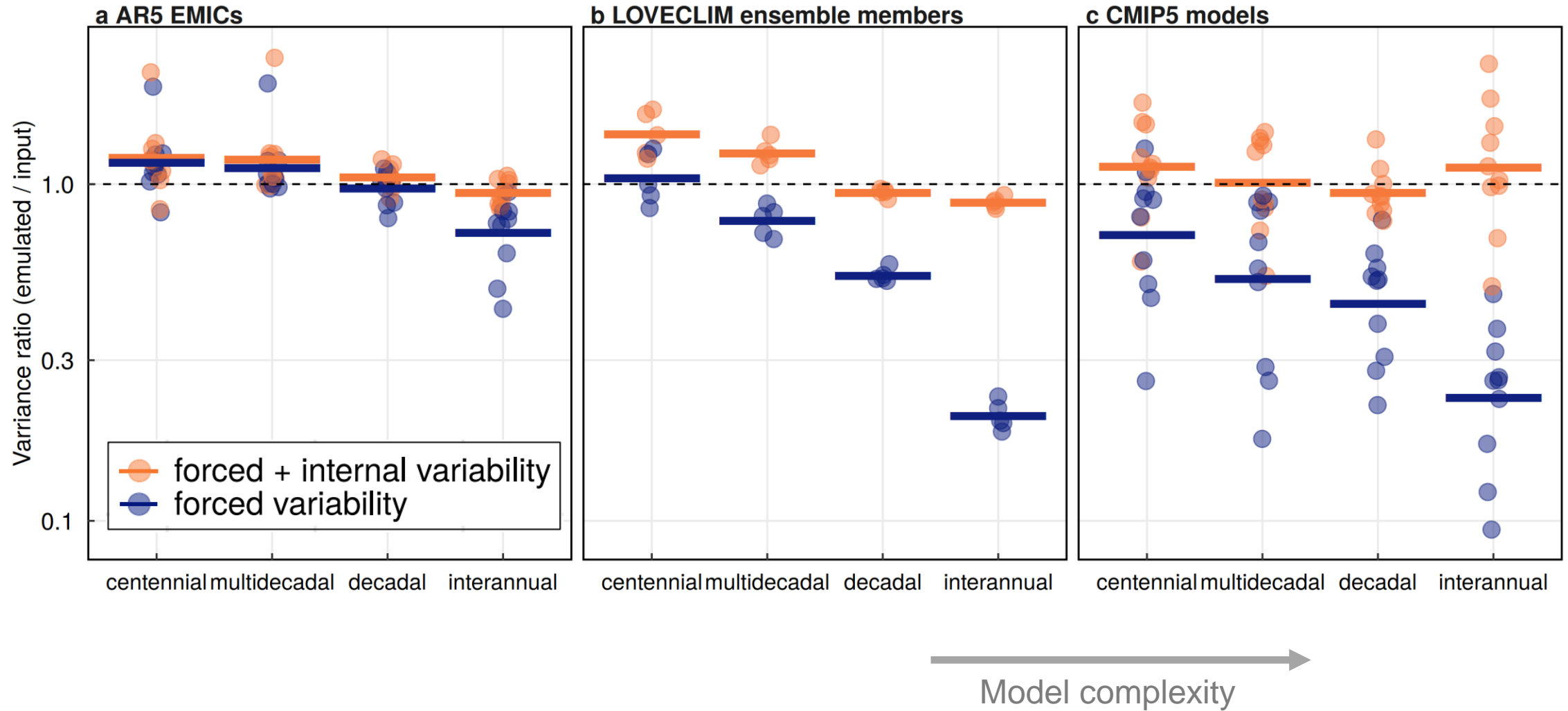




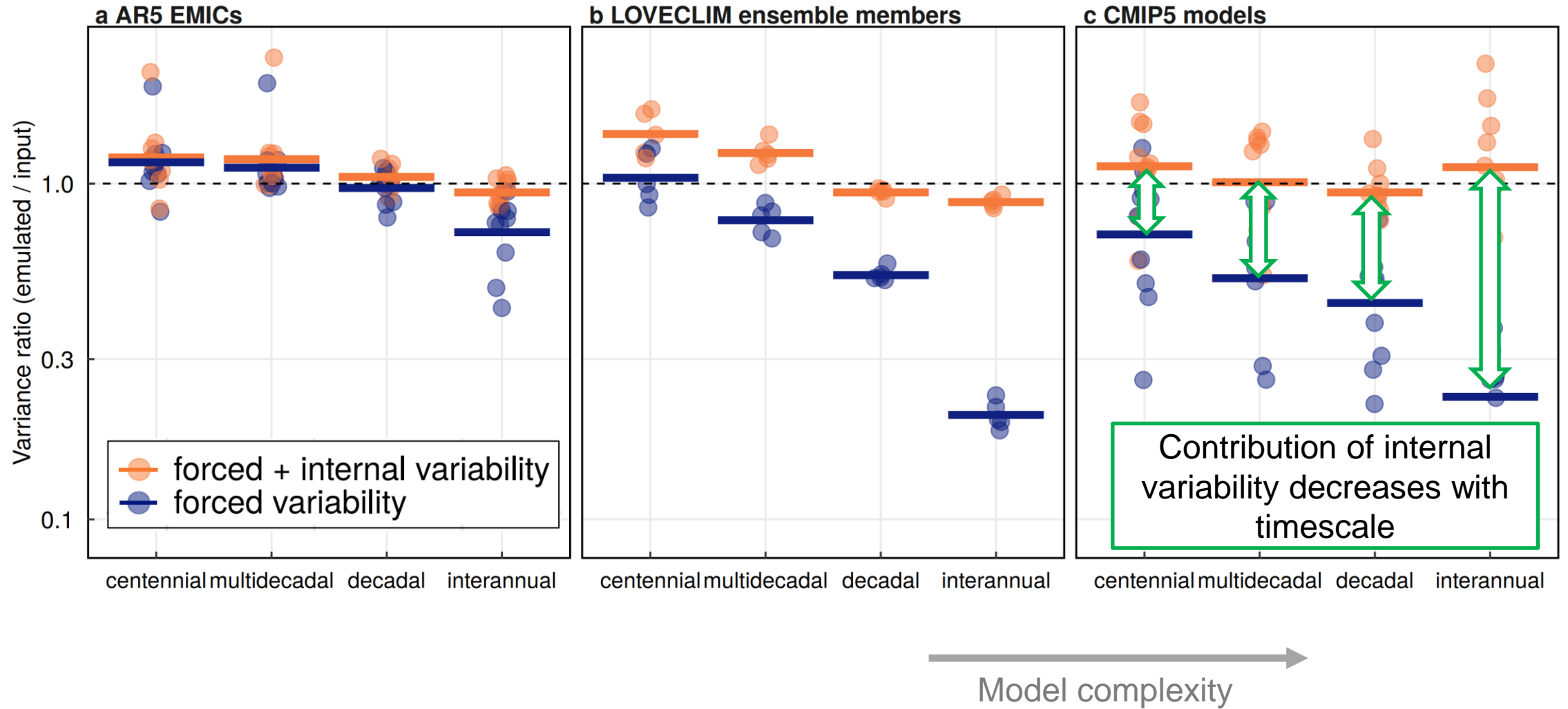
# Internal & externally-forced contribution to global temperature variability



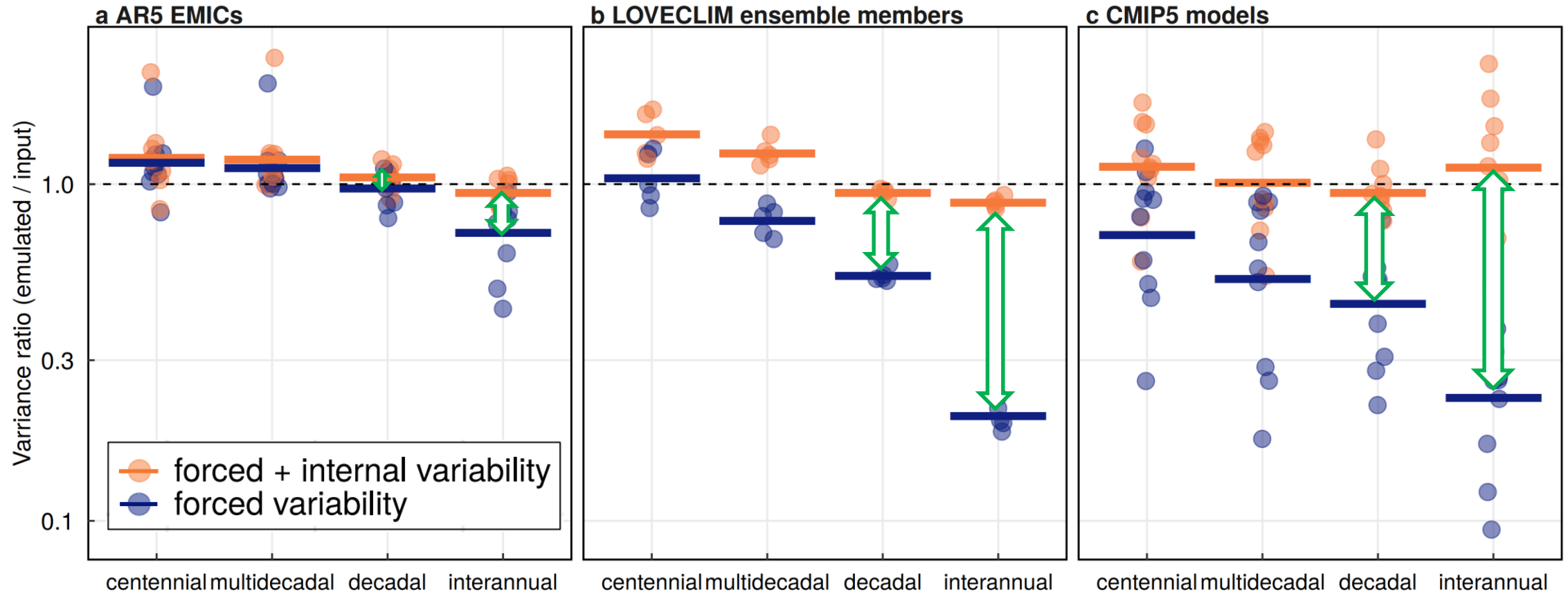
# Internal & externally-forced contribution to global temperature variability



# Internal & externally-forced contribution to global temperature variability



# Internal & externally-forced contribution to global temperature variability



Contribution of internal variability increases with model complexity

Model complexity →

# Summary

## Separating forced from internal variability



**Paper:** M. Schillinger et al.: Separating internal and externally forced contributions to global temperature variability using a Bayesian stochastic energy balance framework. *Chaos*, <https://doi.org/10.1063/5.0106123> (2022).

**Package ClimBayes:** <https://github.com/m-schillinger/ClimBayes>



Questions?



# References I

Roger W. Bodman, Peter J. Rayner and David J. Karoly. 'Uncertainty in temperature projections reduced using carbon cycle and climate observations'. In: *Nature Climate Change* 2013 3:8 3.8 (May 2013), pp. 725–729. ISSN: 1758-6798. DOI: 10.1038/nclimate1903

M. I. Budyko. "On the origin of glacial epochs.". In: *Meteorol. Gidrol.* (1968).

M. I. Budyko. "The effect of solar radiation variations on the climate of the Earth". In: *Tellus* 21.5 (1969), pp. 611–619. doi: 10.3402/tellusa.v21i5.10109. eprint: <https://doi.org/10.3402/tellusa.v21i5.10109>

Hege Beate Fredriksen and Martin Rypdal. 'Long-range persistence in global surface temperatures explained by linear multibox energy balance models'. In: *Journal of Climate* 30.18 (Sept. 2017), pp. 7157–7168. ISSN: 08948755. DOI: 10.1175/JCLI-D-16-0877.1

O. Geoffroy et al. "Transient Climate Response in a Two-Layer Energy-Balance Model. Part I: Analytical Solution and Parameter Calibration Using CMIP5 AOGCM Experiments". In: *Journal of Climate* 26.6 (2013), pp. 1841 –1857. doi: 10.1175/JCLI-D-12-00195.1.

W. K. Hastings. 'Monte Carlo sampling methods using Markov chains and their applications'. In: *Biometrika* 57.1 (Apr. 1970), pp. 97–109. ISSN: 0006-3444. DOI: 10.1093/BIOMET/57.1.97. URL: <https://academic.oup.com/biomet/article/57/1/97/284580>.

## References II

K. Hasselmann. “Stochastic climate models Part I. Theory”. In: *Tellus* 28.6 (1976), pp. 473–485. doi: <https://doi.org/10.1111/j.2153-3490.1976.tb00696.x> eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.2153-3490.1976.tb00696.x>.

Nicholas Metropolis and ; S Ulam. ‘The Monte Carlo Method’. In: *Journal of the American Statistical Association* 44.247 (1949), pp. 335–341

Eirik Myrvoll-Nilsen et al. ‘Statistical estimation of global surface temperature response to forcing under the assumption of temporal scaling’. In: *Earth System Dynamics* 11.2 (Apr. 2020), pp. 329–345. ISSN: 21904987. DOI: 10.5194/esd-11-329-2020.

Martin Rypdal and Kristoffer Rypdal. ‘Long-memory effects in linear response models of Earth’s temperature and implications for future global warming’. In: *Journal of Climate* 27.14 (2014), pp. 5240–5258. ISSN: 08948755. DOI: 10.1175/JCLI-D-13-00296.1. arXiv: 1305.5080

William D. Sellers. “A Global Climatic Model Based on the Energy Balance of the Earth-Atmosphere System”. In: *Journal of Applied Meteorology and Climatology* 8.3 (1969), pp. 392 –400. doi: 10.1175/1520-0450(1969)008<0392:AGCMBO>2.0.CO;2.

# Data sources I

Janica C. Bühler et al. 'Comparison of the oxygen isotope signatures in speleothem records and iHadCM3 model simulations for the last millennium'. In: *Climate of the Past* 17.3 (2021), pp. 985–1004. ISSN: 18149332. DOI: 10.5194/cp-17-985-2021.

T. J. Crowley and M. B. Unterman. 'Technical details concerning development of a 1200 yr proxy index for global volcanism'. In: *Earth System Science Data* 5.1 (2013), pp. 187–197. ISSN: 18663508. DOI: 10.5194/essd-5-187-2013.

M. A. Giorgetta et. al, "Climate and carbon cycle changes from 1850 to 2100 in MPI-ESM simulations for the Coupled Model Intercomparison Project phase 5: Climate Changes in MPI-ESM," *Journal of Advances in Modeling Earth Systems* 5, 572–597 (2013).

Keywan Riahi Peter Kolp. RCP Database. 2009. URL: <http://www.iiasa.ac.at/web-apps/tnt/RcpDb>.

IPCC. 'Summary for Policymakers (SPM)'. In: *Fifth Assessment Report of the Intergovernmental Panel on Climate Change* ()

Colin P. Morice et al. "Quantifying uncertainties in global and regional temperature change using an ensemble of observational estimates: The HadCRUT4 data set". In: *Journal of Geophysical Research: Atmospheres* 117.D8 (2012). doi: <https://doi.org/10.1029/2011JD017187>. eprint: <https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2011JD017187>.

Morice, C.P., J.J. Kennedy, N.A. Rayner, J.P. Winn, E. Hogan, R.E. Killick, R.J.H. Dunn, T.J. Osborn, P.D. Jones and I.R. Simpson (in press) An updated assessment of near-surface temperature change from 1850: the HadCRUT5 dataset. *Journal of Geophysical Research (Atmospheres)* doi:10.1029/2019JD032361. eprint: <https://agupubs.onlinelibrary.wiley.com/doi/full/10.1029/2019JD032361>



## Data sources II

Raphael Neukom et al. 'Consistent multidecadal variability in global temperature reconstructions and simulations over the Common Era'. In: Nature Geoscience 12.8 (Aug. 2019), pp. 643–649. ISSN: 17520908. DOI: 10.1038/s41561-019-0400-0.

G. A. Schmidt et al. "Climate forcing reconstructions for use in PMIP simulations of the Last Millennium (v1.1)". In: Geoscientific Model Development 5.1 (2012), pp. 185–191. doi: 10.5194/gmd-5-185-2012.

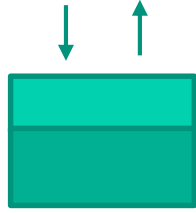
F. Steinhilber, J. Beer, and C. Fröhlich. "Total solar irradiance during the Holocene". In: Geophysical Research Letters 36.19 (2009). doi: <https://doi.org/10.1029/2009GL040142>. eprint: <https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2009GL040142>.

# Appendix

# Detailed Workflow of the ClimBayes package

## Linear stochastic two-box EBM

$$C \frac{dT}{dt}(t) = K T(t) + F(t) + \xi(t)$$



$$T(t) = \int R(t-s)(F(s)ds + dW(s))$$

forced      internal

$$R(t) = w_1 e^{-\lambda_1 t} + w_2 e^{-\lambda_2 t}$$



## Data

Temperature observations  $Y$

Prior information on uncertain parameters

$$X = (\lambda_1, \lambda_2, w_1, T_0, F_0)$$

Deterministic forcing  $F$

## Bayesian Inference

Approximate posterior  $p_{X|Y=y}(x)$

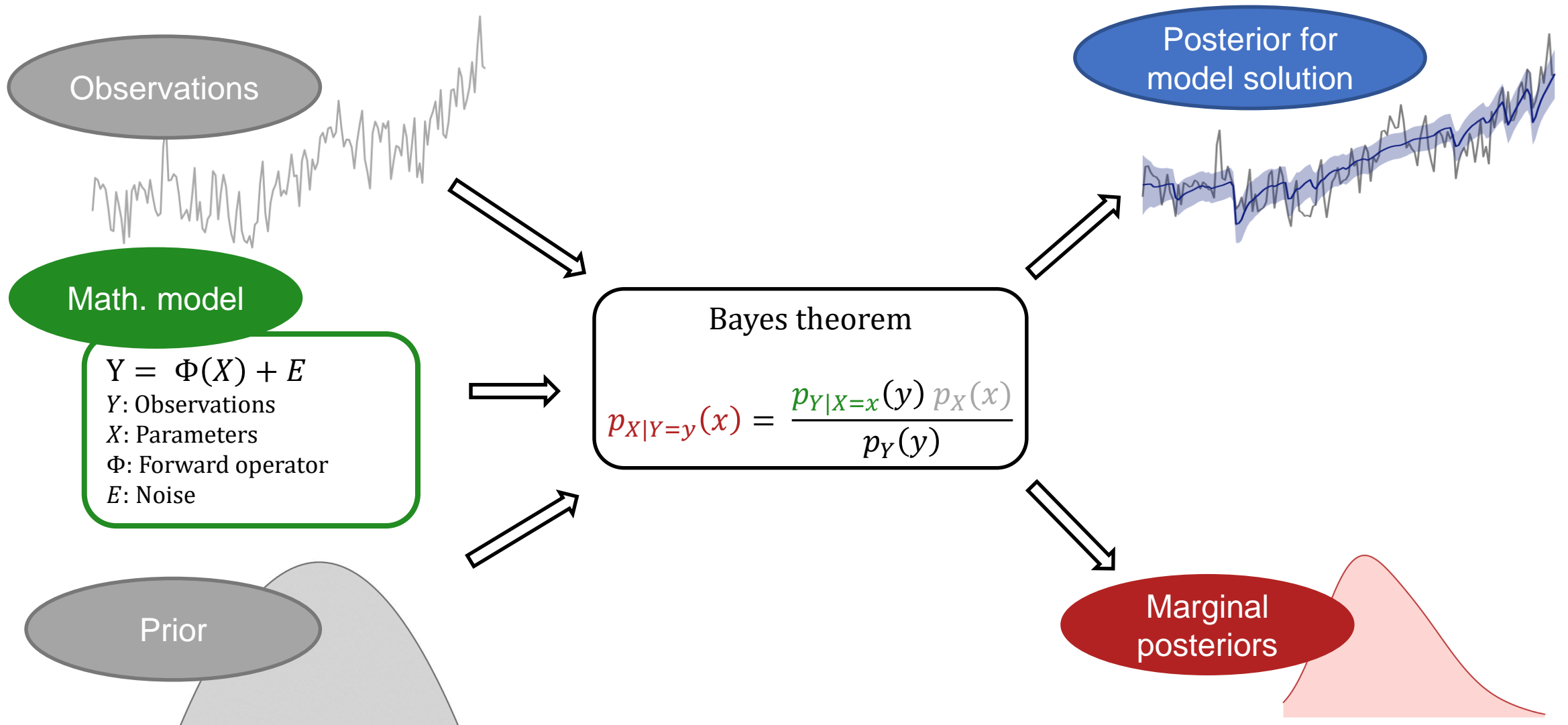


## Output

Posterior distributions for  $X$

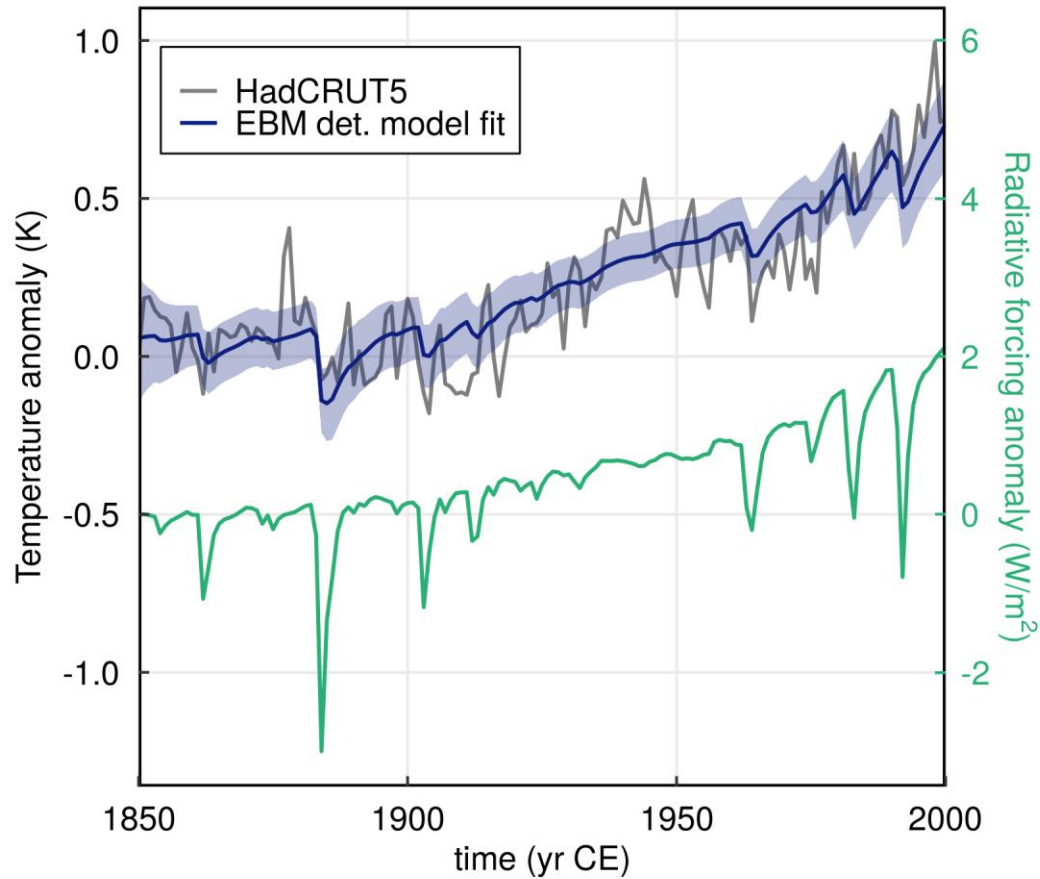
Best estimate of forced and internal temperature variations

# Bayesian parameter estimation

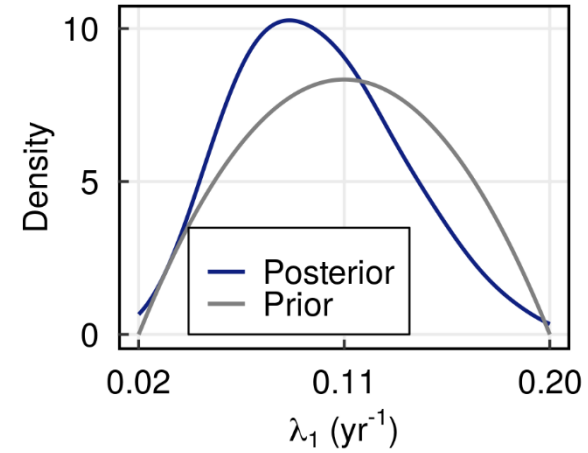


# Application to historical data

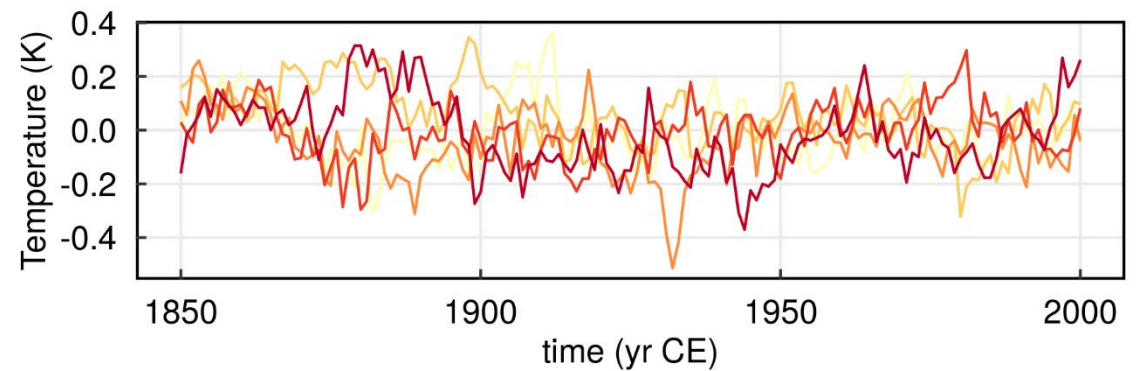
## Estimate of forced response



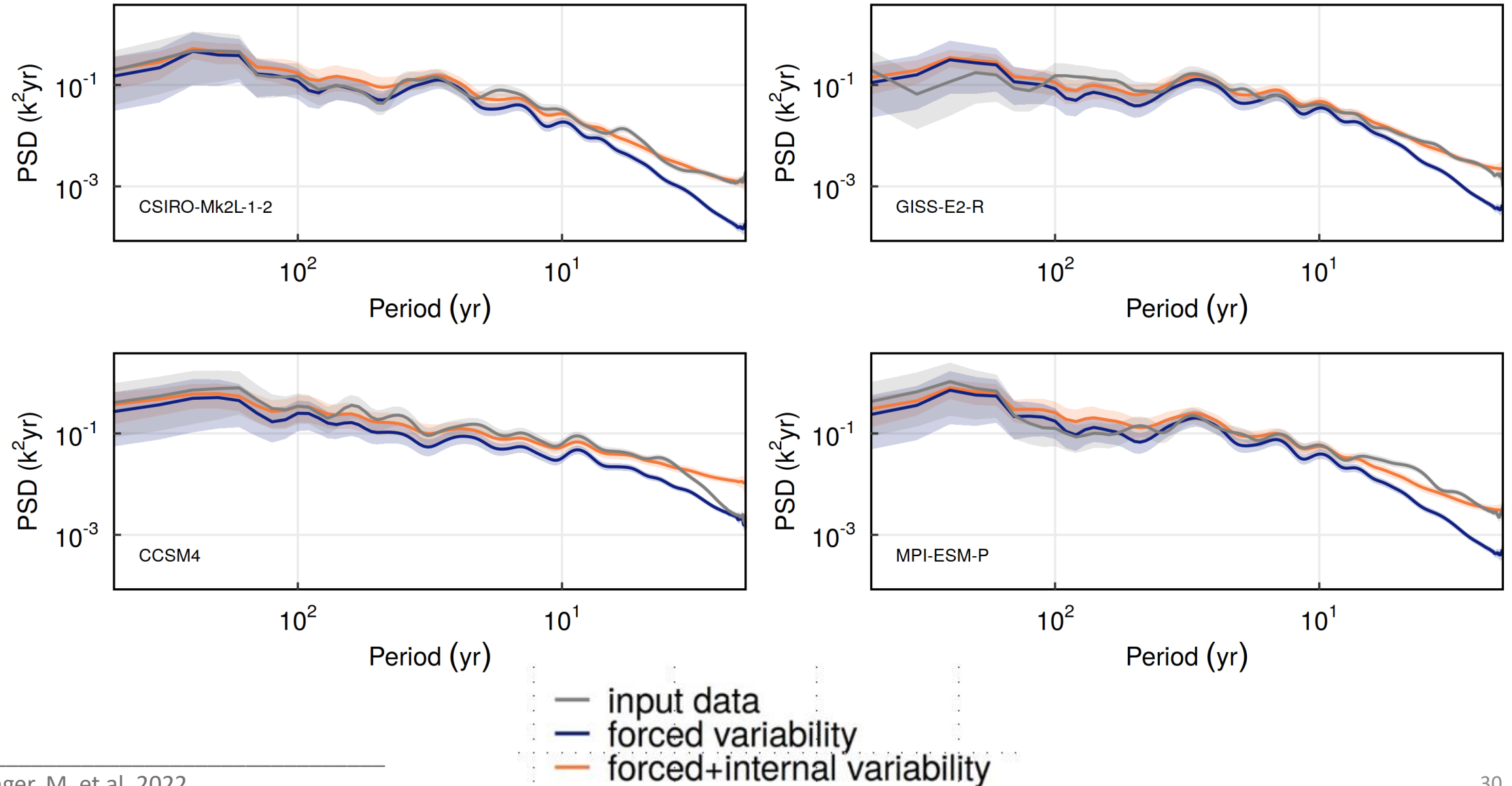
## Posteriors for $X$ , e.g.



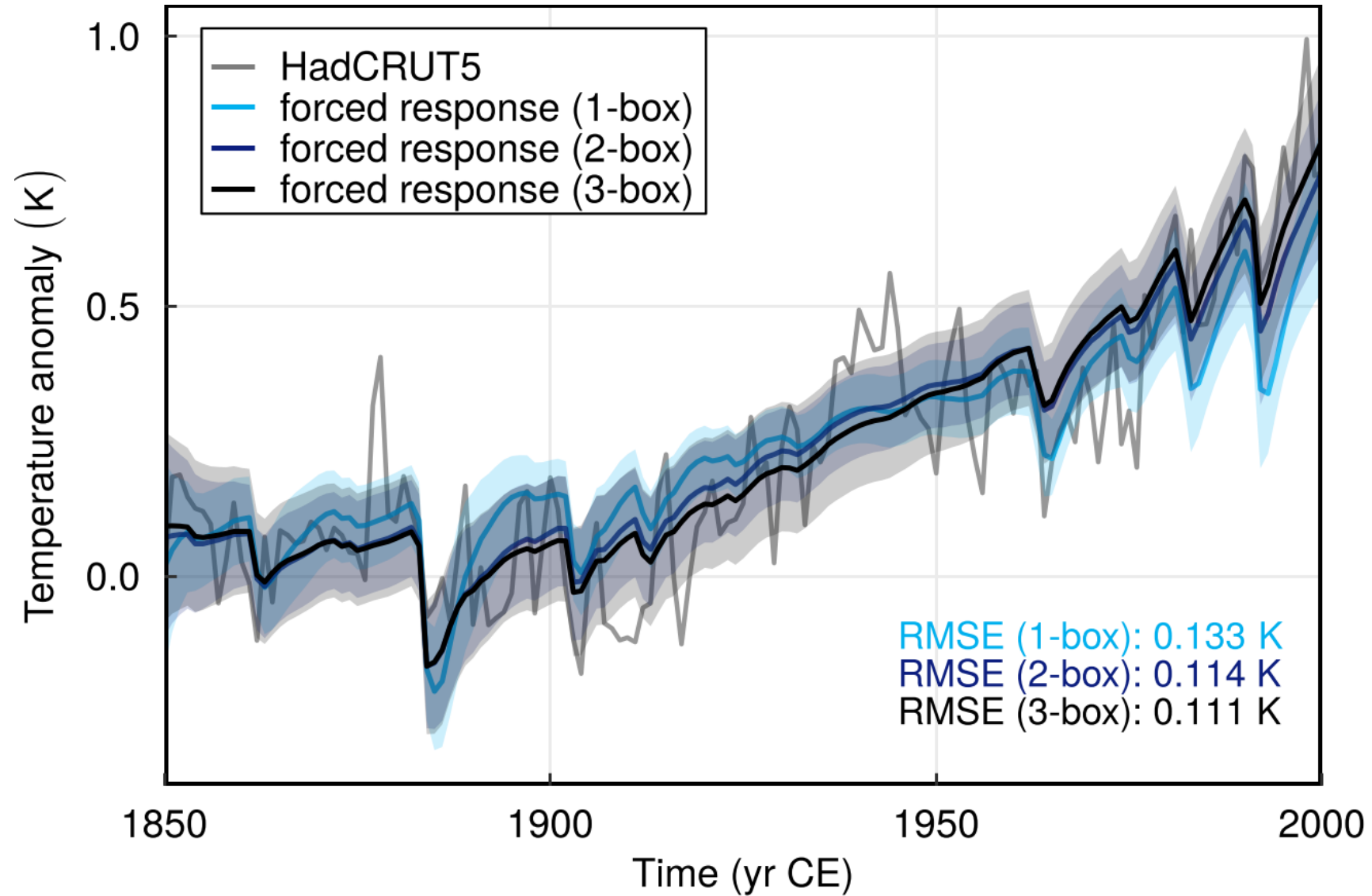
## Estimate of internal variability



# Modelling the global mean temperature spectrum



# Comparison of 1-, 2- and 3-box model



# Comparison of EBM's forced response with large ensemble

