Interpolation of hydrological time series via Dynamic Mode Decomposition

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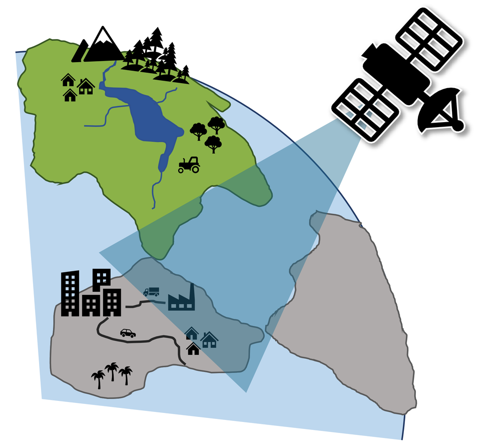
INTRODUCTION

SATELLITE DATA FOR SUSTAINABLE WATER MANAGEMENT

Satellite technologies can offer massive support to the development of adaptation and mitigation strategies to face the effects of climate change on water availability all around the world. They provide data with improved spatial coverage and time resolution, but the continuity of these datasets, and therefore their applicability, is frequently compromised by gaps in the observations series.



RECONSTRUCT AND BRIDGE GAPS IN HYDROLOGICAL TIME SERIES

Several studies have been dedicated to this issue, with an increasing interest in machine learning methods in the last decades. Recent developments in data-driven methods have opened up the possibility to reconstruct series learning a model directly from observations. The majority of the proposed methods take advantage of the correlations between the variable of interest and some other predictors, but there exist methods where even other predictors databases are not needed.

METHOD

DYNAMIC MODE DECOMPOSITION

Dynamic mode decomposition (DMD) originated in the fluid dynamics community, as a suitable technique for the discovery of high-dimensional, nonlinear dynamical systems, that exhibit rich multiscale phenomena in both space and time, directly from the data.

DMD decomposes high-dimensional datasets from complex dynamical systems into a simple representation based on spatiotemporal coherent structures.

Given an appropriate selection of observables, DMD can be viewed as a finite-dimensional linear approximation of the linear, infinite-dimensional Koopman operator, that represents the action of a nonlinear dynamical system on the Hilbert space of measurement functions of the state.

DMD is algorithmically a regression of data, collected from a dynamical system at a number of times, onto locally linear dynamics.

- equation-free
- data-driven
- simple formulation

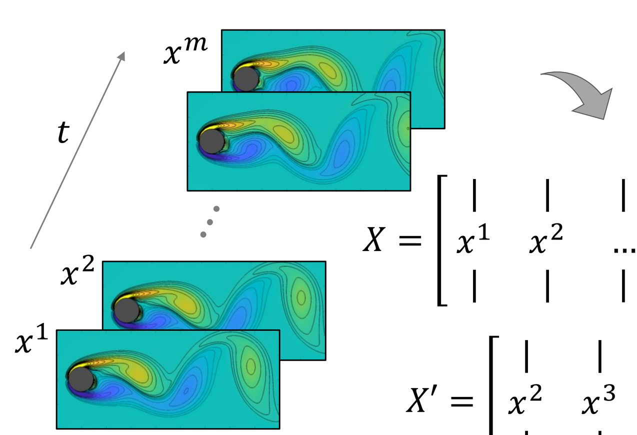
This method is closely related to ARIMA models, commonly used in time-series analysis, but has the advantage of automatically embedding seasonal variations and capturing trends in the data. Similar ideas also hold in Linear Inverse Modeling, a method developed in the climate science community.

DMD could represent a new viable approach to detect patterns, extract reduced order models and predict climate-related time series based on previous observations, especially from high-dimensional satellite datasets.

ALGORITHM

Given a dynamic system $\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t; mu)$, and denoting the discrete-time flow map by evolving for Δt as $\mathbf{x}^k = \mathbf{F}(\mathbf{x}^{k-1})$, the goal is to define a DMD reduced-order model (ROM), $\mathcal{L}$, of the dynamic system, $\mathbf{f}$, using m snapshots in time of the solution. If the true solution at time t^k induced by the flow map $\mathbf{F}$ is $\mathbf{x}^k$, the correspondent DMD approximation is $\mathbf{x}^k_{\mathcal{L}}$: $\mathbf{x}^k = \mathbf{F}(\mathbf{x}^{k-1}) \approx \mathbf{x}^k_{\mathcal{L}} = \mathcal{L}(\mathbf{x}^{k-1}_{\mathcal{L}})$.

Consider a set of snapshots of the state variable as it evolves in time. Snapshots $\mathbf{x}^k$ and $\mathbf{x}^{k+1}$ are columns of $X, X' \in \mathbb{C}^{n \times m}$ respectively, with $t^{k+1} = t^k + \Delta t$ and $k = 0, \dots, m-1$.



$$X = \begin{bmatrix} | & | & & | \\ x^1 & x^2 & \dots & x^{m-1} \\ | & | & & | \end{bmatrix}$$

$$X' = \begin{bmatrix} | & | & | & | \\ x^2 & x^3 & \dots & x^m \\ | & | & | & | \end{bmatrix}$$

sDMD

The standard DMD ROM approximates the relationship between X, X′ in time with the best-fit linear operator **A** as follows:

$$\mathbf{x}^k \approx \mathbf{A}\mathbf{x}^{k-1} .$$

The operator $\mathbf{A} \in \mathbb{C}^{n \times n}$ can be computed as $\mathbf{A} = X'X^{\dagger}$, but to reduce the computational cost the truncated SVD of $X = U\Sigma V^T$ is used to obtain

$$\mathbf{A} \approx X'V\Sigma^{-1}U^T .$$

xDMD

The xDMD ROM approximates the relationship between $Y = X' - X$ and X′ in time with the best-fit linear operator as follows:

$$\mathbf{y}^k \approx \mathbf{B}\mathbf{x}^{k-1} + \mathbf{b} .$$

The best-fit linear operator can be computed as $[\mathbf{B}\ \mathbf{b}] = Y\tilde{X}^{\dagger} \in \mathbb{C}^{n \times n+1}$, but it is more computationally effective to compute

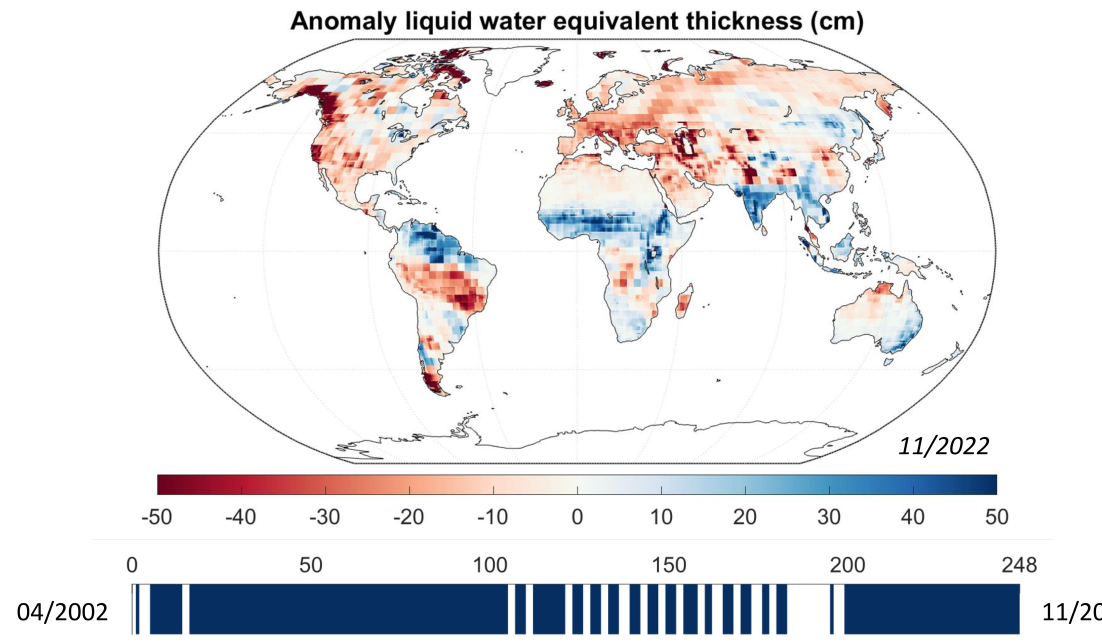
$$[\mathbf{B}\ \mathbf{b}] \approx Y\tilde{V}_r\tilde{\Sigma}_r^{-1}\tilde{U}_r^T,$$

where the truncated SVD of $\tilde{X} = [X\ 1] = \tilde{U}_r\tilde{\Sigma}_r\tilde{V}_r^T$.

* Between all the **optimizations** developed to adapt the method to different modeling situations, xDMD is an extension of the standard algorithm (sDMD) to face the problem of **inhomogeneity**

APPLICATION

GRACE: GRAVITY RECOVERY AND CLIMATE EXPERIMENT

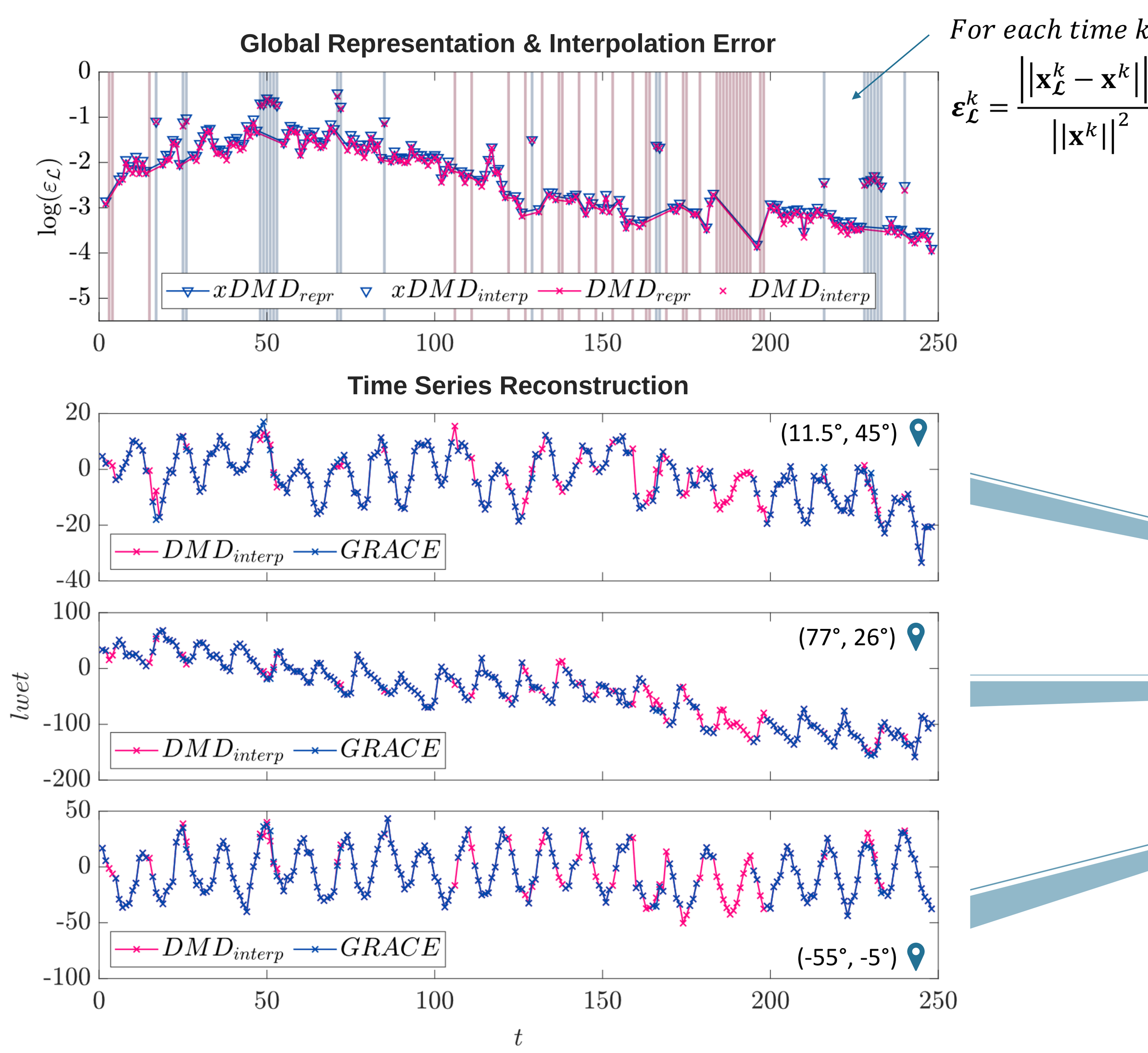


The Earth's gravity field variations detected by GRACE can be used to derive estimates of water distribution on the planet.

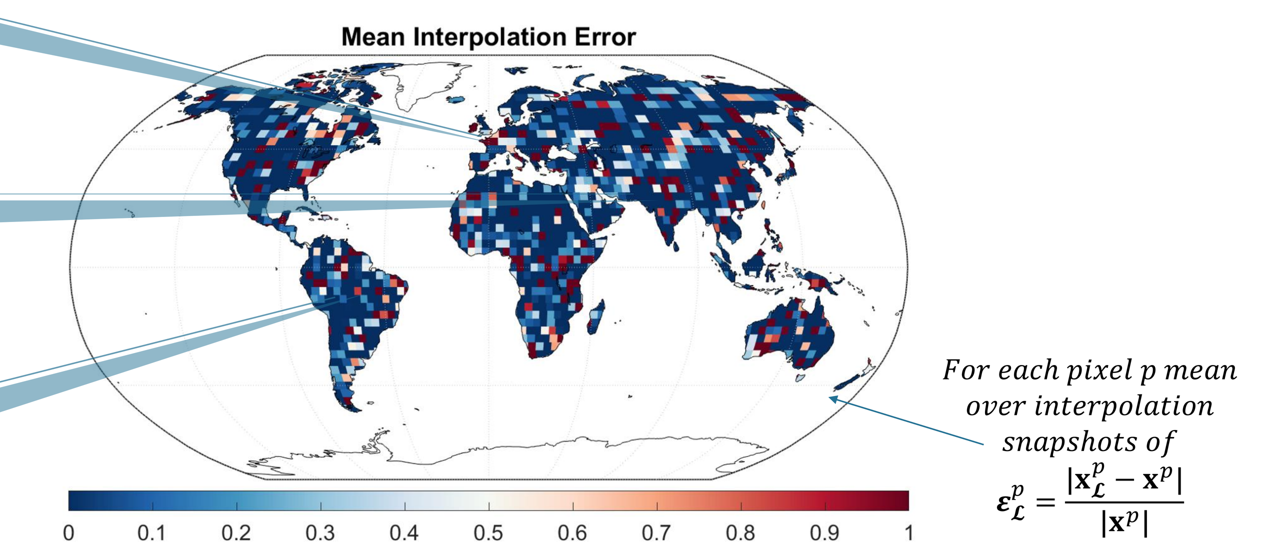
Data are provided on a monthly scale as cm of anomalies of equivalent water thickness relative to the baseline mean 1/2004-12/2009.

The series is affected by some short-term gaps and a major interruption of approximately 1 year, due to the transition between the first GRACE mission (2002-2017) and the GRACE Follow-On mission (2018-present).

sDMD and xDMD are applied to the latest GRACE mascon solution (RL06M.MSCNv03) from the Jet Propulsion Laboratory (JPL). The DMD method is used to capture the information embedded in the large amount of data collected and then use it to bridge the gaps.



- Random 23 snapshots are removed from the original GRACE dataset to simulate time series gaps.
- sDMD and xDMD are trained (truncation at 90% energy) on the remaining 192 snapshots and representation performance is tested on them.
- Interpolation performance is tested on the 23 snapshots previously removed.
- sDMD is used to interpolate existing GRACE gaps (31 snapshots).



CONCLUSIONS AND FUTURE PERSPECTIVES

This preliminary study demonstrates the viability of interpolating high-dimensional satellite datasets through the DMD method. Compared to other ML methods in the literature, this approach still has higher errors, but the model is global and built solely on GRACE available data, without correlations to other variables.

Future research will focus on improving the method performance, eventually moving from the global to the basin/regional scale. Further important advantages are also expected to come in the second part of the work, where we plan to perform the decomposition of the GRACE dataset into spatio-temporal structures to analyze and interpret patterns and trends. Evidence from this specific application of DMD could also have positive effects on a wide range of other hydrological applications.

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- The GRACE and GRACE-FO mascon solution used in this study is available at: https://grace.jpl.nasa.gov/data/get-data/jpl_global_mascons/