A STATISTICALLY RIGOROUS COMPARISON OF OBSERVED AND MODELLED FLOW DIRECTIONS OF THE LAST BRITISH-IRISH ICE SHEET

Rosie E. Archer[1], Jeremy C. Ely[1], Timothy J. Heaton[2], Frances E.G. Butcher[1], Anna L.C. Hughes[3], Chris D. Clark[1]

[1]Department of Geography, University of Sheffield, [2] Department of Statistics, University of Leeds, [3] Department of Geography, University of Manchester

rarcher4@sheffield.ac.uk

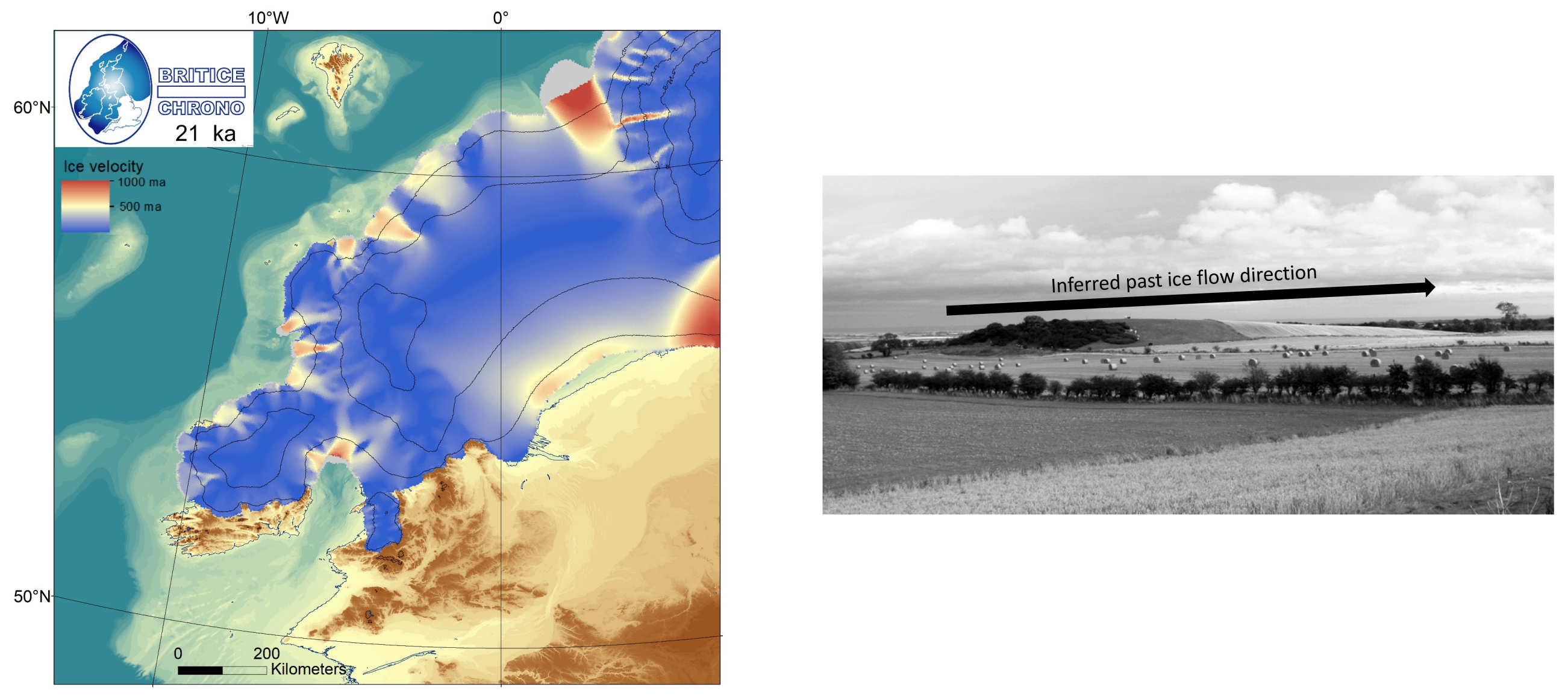

Fig. 1: *Left:* Example of numerical model output with modelled ice velocities shown with red indicating faster flow and blue showing slower flow (Clark et al. 2022). *Right:* Picture of a drumlin with the inferred past ice flow direction indicated.

Model-data comparison

Numerical modelling of contemporary ice sheets is important to predict their contribution to future sea level rise. Past ice sheets have left behind a record of their behaviour spanning millenia. Ice sheet models are complex, **computationally expensive**, and require multiple parameter inputs where the exact values of which are unknown. Comparing the output of model simulations to the observational record provides a powerful validation method to see which parameters can replicate the data. One abundant type of data left behind by ice sheets are lineations which indicate past flow direction. The current model-data comparison tool for matching flow direction, the Automated Flow Direction Analysis (AFDA) tool (Li et al. 2007) produces a binary fit or no-fit output, and it is difficult to compare simulations within an ensemble. This led us to create the **Likelihood of Accordant Lineations Analysis** (LALA) tool, a statistically-rigorous model-data comparison tool to compare past ice flow direction with model simulations. Here, we utilise flowsets of the last **British-Irish ice sheet** and compare them to an ensemble of numerical models from the BRITICE-CHRONO project (Clark et al. 2022) using LALA. The full details of LALA are in Archer et al. (2023).

Worked example

Here, we present a worked example to illustrate how the new tool, LALA, scores a simulation. LALA combines marked Poisson point processes which can model the occurrence of random, independent events over a defined space with an attached mark on each event. In this usage, the event is the formation of a lineation, and the mark is its orientation. A Poisson point process requires a rate of formation, which will be calculated. First, assume the simulation to be scored, M, has a 5×5 domain, $\mathcal{X}$, and we consider 3 time steps, $\mathcal{T}$, with one flowset. The flowset and modelled flow directions at each time step are shown in Figure 2. Assume that any square in the domain had the correct conditions to form lineations, and so $A_{pre}(\mathcal{X}) = 25$.

1. Calculate the integrated area over space and time for plausible lineation formation for an ideal reconstruction, $M^{\dagger}$. Say $A_{M^{\dagger}}(\mathcal{X}, \mathcal{T}) = 42$.
2. Find the rate of lineation formation across the studied time period, λ, and the pre-study rate of formation, $\lambda^{\star}$, using

$$\mathbb{E}_{M^{\dagger}}(N) = p\mathbb{E}_{M^{\dagger}}(N_{pre}) + (1-p)\mathbb{E}_{M^{\dagger}}(N_{\mathcal{T}}).$$

3. Taking $p = 0.01$ to be the probability that a flowset has formed outside of the study period, and noticing that $\mathbb{E}_{M^{\dagger}}(N) = 1 = 25\lambda^{\star} + 42\lambda$ as we are considering a single flowset, we find

$$\lambda^{\star} = 0.0004 \quad \text{and} \quad \lambda = 0.0236$$

4. Now count the area integrated over time and space where a specific simulation, M, meets the conditions to form lineations. Say $A_M(\mathcal{X}, \mathcal{T}) = 35$.
5. Calculate the log-likelihood for the number of and the location of the flowsets

$$A_M(\mathcal{X}, \mathcal{T})\lambda + A_{pre}(\mathcal{X})\lambda^{\star} = 0.8359.$$

6. Calculate the likelihood that the simulation, flowing in direction $\hat{\theta}$, could have formed the flowset with orientation θ at location x and time step, $i \in \{1, 2, 3\}$

$$f_M(\theta|\mu(x, t = i) = \frac{1}{2}(f_{vonM}(\mu = \theta - \hat{\theta}, \kappa) + f_{vonM}(\mu = (\theta + 180) - \hat{\theta}, \kappa)).$$

7. Combining the likelihoods to find the directional component and adding a term to account for unrelated flowsets gives

$$\nu(\theta|x, M) = \lambda(f_M(\theta|\mu(x, 1) + f_M(\theta|\mu(x, 2)) + f_M(\theta|\mu(x, 3))) + \frac{\lambda^{\star}}{2\pi} = 0.0104.$$

8. Calculate the final log-likelihood for the simulation

$$l(M|x, \theta) = log(\nu(\theta|x, M)) - 0.8359 = -4.99.$$

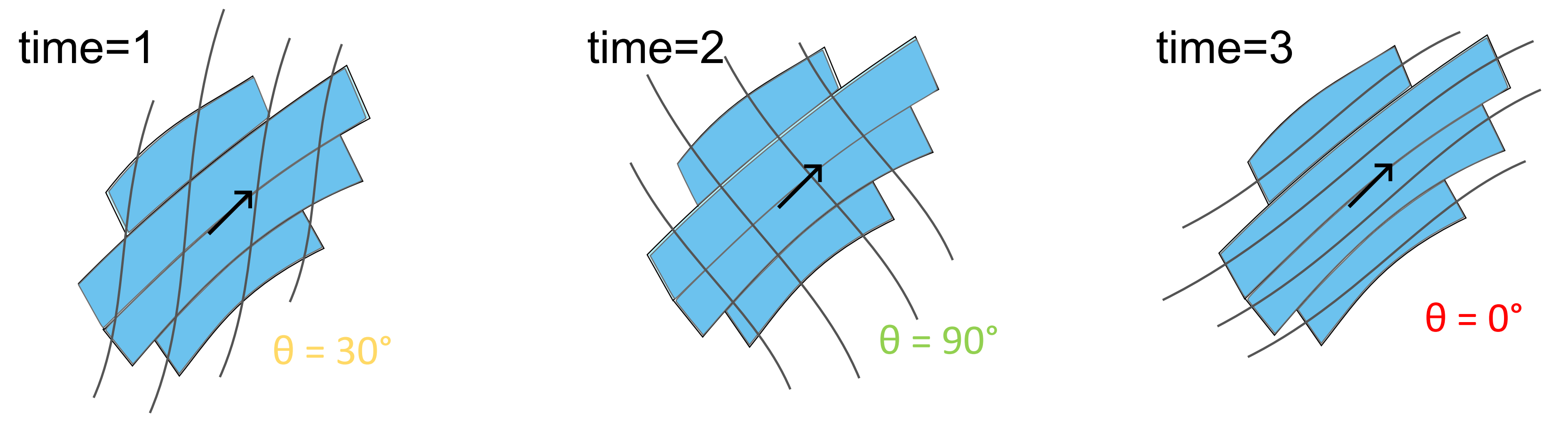

Fig. 2: A small example of a model simulation with one flowset and three time steps. The flowset is shown in blue, with a black arrow showing its orientation. The modelled flow direction are shown in grey and the difference between the observed and modelled flow directions are indicated below each time step.

Scoring Simulations of the last British-Irish Ice Sheet

The BRITICE-CHRONO project produced 200 model simulations of the last British-Irish Ice Sheet over 31 - 15 ka using the Parallel Ice Sheet Model. A huge number of model simulations would be required to explore the unknown parameter space fully. Instead, we can look to use statistical surrogates, e.g. emulators, to predict the model output based on given parameters. To use an **emulator**, a metric representing model output is required. Using output from a model-data comparison tool, the observational record can inform the best model simulations. LALA, unlike AFDA, has the capability to work within an emulator.

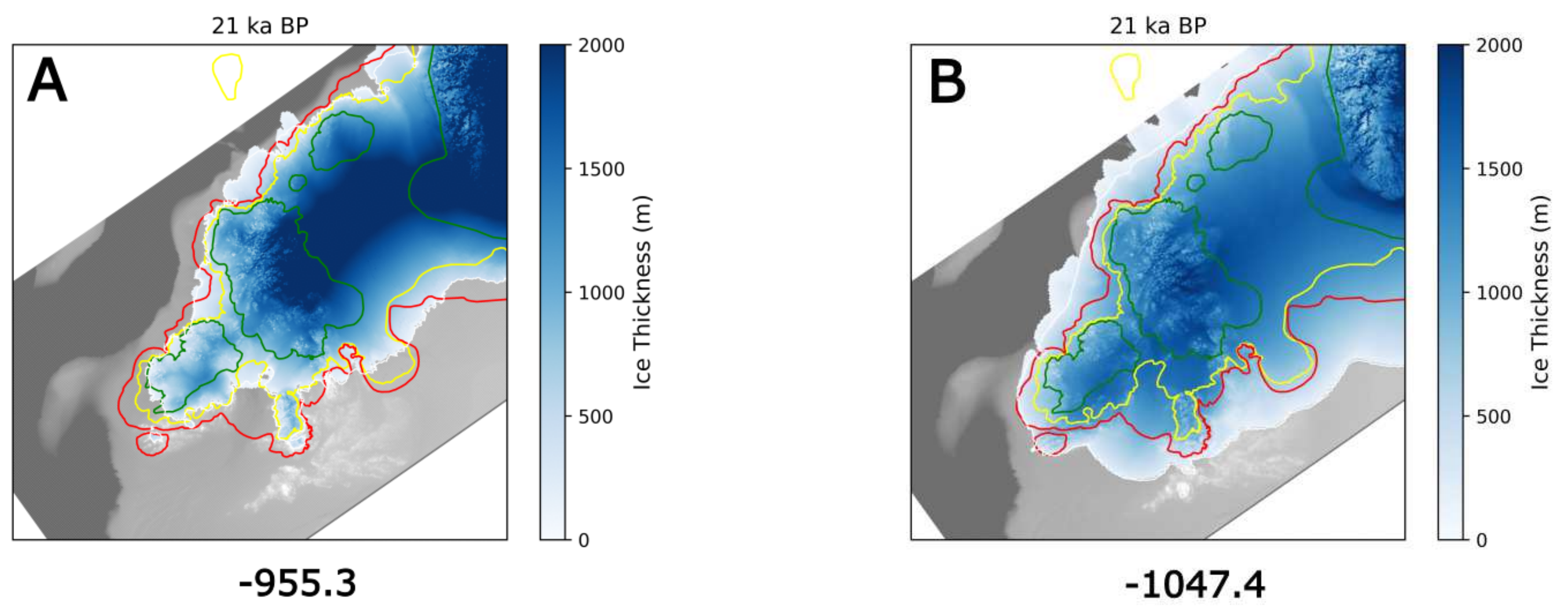

Fig. 3: The best (left) and worst (right) simulations from the BRITICE-CHRONO project when comparing to past inferred ice flow directions according to LALA. The scores for each of these simulations are given below. The red, yellow and green outlines represent the maximum, most credible and minimum extents derived by Clark et al. (2022), respectively.

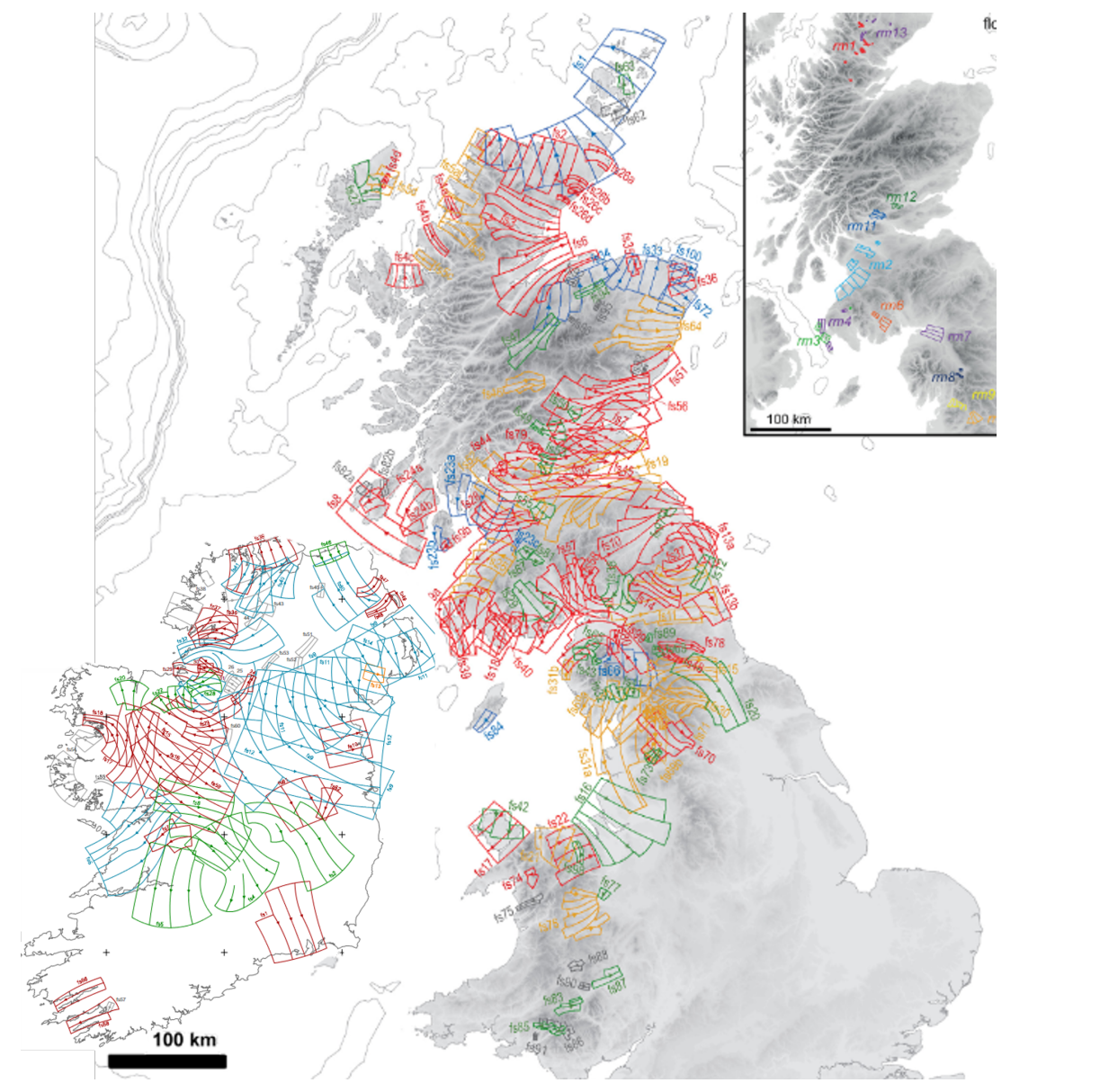

Fig. 4: Flowsets over Britain and Ireland used to compare to model output. Collated from Greenwood & Clark (2009) for Ireland and Hughes et al. (2014) for Britain.

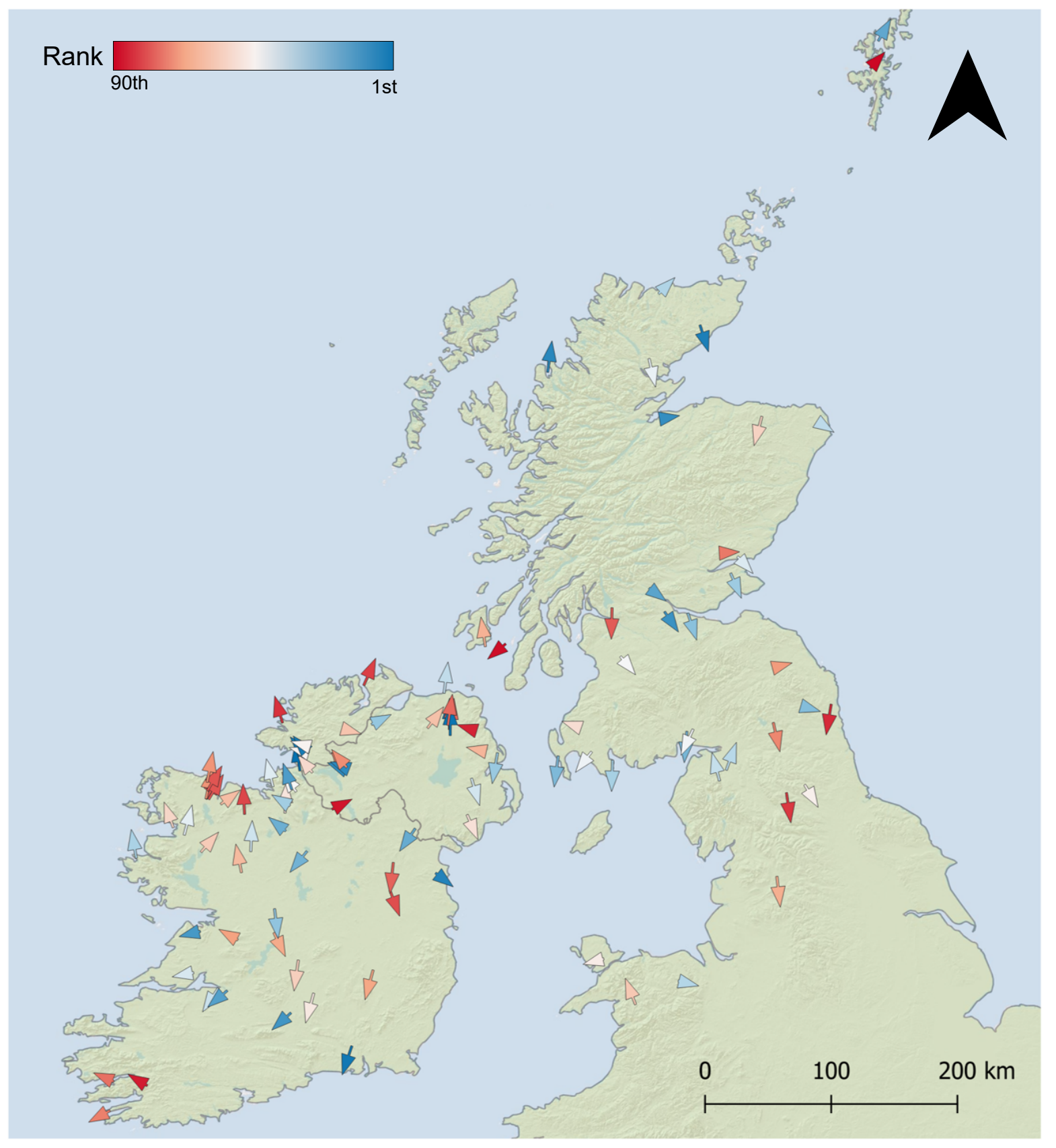

Fig. 5: Ranked fit of all flowsets over all 200 model simulations, as picked out by LALA, with the arrows indicating the direction of the past ice flow being compared to the model output, and the colour of the arrow representing the rank such that the blue matches the best and the red matches the worst.

Results

After using LALA on a 200-member ensemble, the best and worst simulations compared to the past ice flow were identified (shown in Figure 3). LALA scores each simulation over all appropriate time steps, but the ice thickness at 21 ka is shown for illustrative purposes. Visually, we can see that the best simulation largely fits within the most credible prediction derived by Hughes et al. (2016) and the worst simulation exceeds the maximum bound. LALA also has the capability to establish the rank of best flowset to model match across the whole ensemble, shown in Figure 5. The results suggest that flowsets associated with later phases of the ice sheet (those occurring inland) produce better matches than earlier phases of ice flow.

Conclusions

In this work, we have

- created a new, statistically rigorous workflow for comparing model output to observed ice flow directions;
- used the tool to rank and ascertain the best model simulations for capturing ice flow directions; and
- identified flowsets which are more easily explained by the simulations as a whole.

Future work will apply this tool to the Eurasian Ice Sheet complex and develop an emulator for finding optimal parameters for recreating ice sheet flow patterns.

References

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