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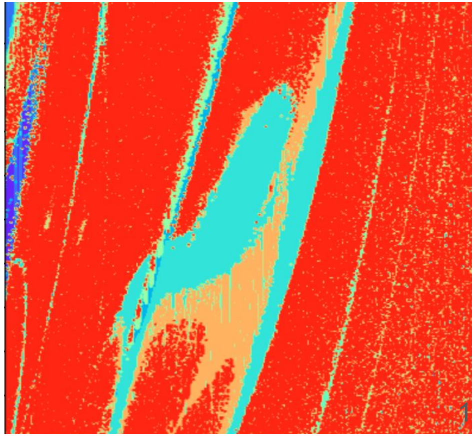
Dynamical systems view of the "El Niño Southern Oscillation" phenomenon

Julia Mindlin, Gabriel Mindlin, Pedro di Nezio



Vienna - 19.04.2024

Years of research have led to a rich model hierarchy for ENSO

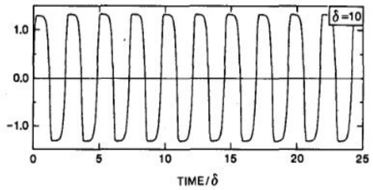
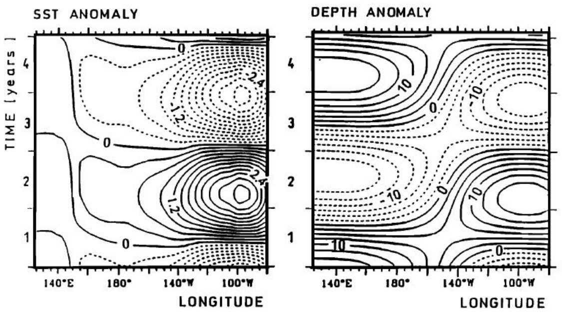


FIG. 4. Behavior of the nonlinear oscillator. (a) $\alpha = 0.75$, $\delta = 2$, (b) $\alpha = 0.75$, $\delta = 6$, and (c) $\alpha = 0.75$, $\delta = 10$. The time axis is scaled in units of the delay.

$$dT/dt = T - T^3 - \alpha T(t - \delta), \quad (2)$$

Suarez & Schopf model



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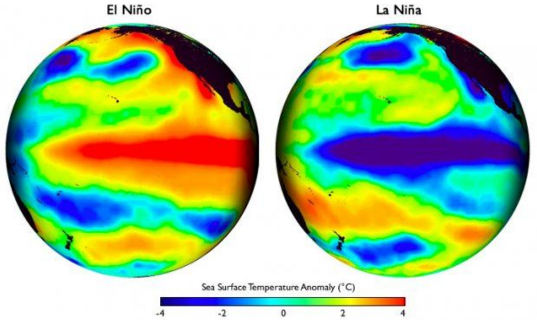

Global Climate Models

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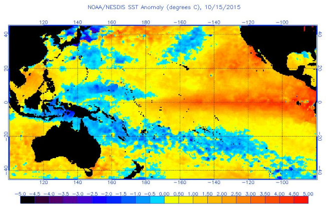
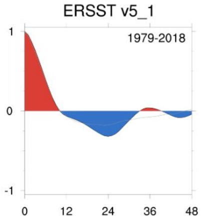
But what does "reproducing ENSO" ***actually mean*?**



Spatial patterns?



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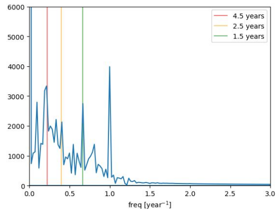
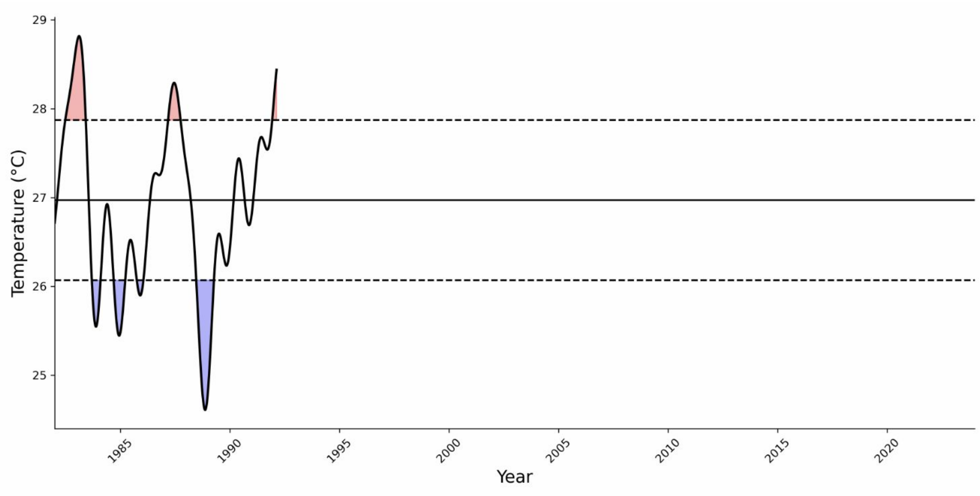
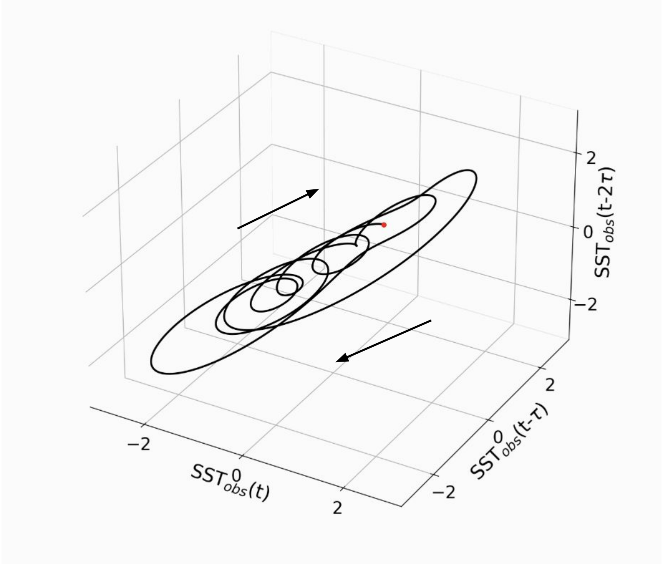


Power spectrum?

Dynamical systems view of ENSO

Nino 3.4 1982-2023 (Butterworth filtered)
NOAA OI SST V2 High Resolution Dataset



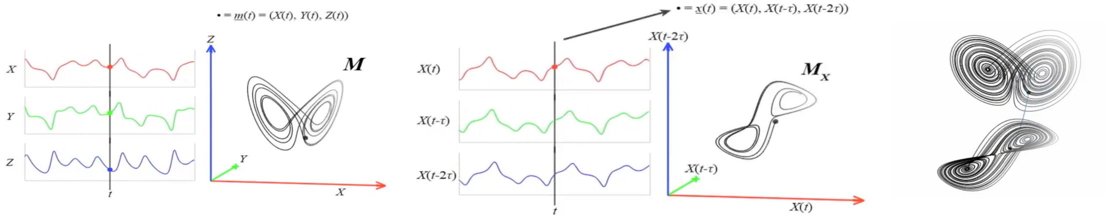


Embedded time series

Taken's theorem allows us to recover a system's phase space by embedding observable (X), taking delayed samples of one only time series. The theorem proves that this embedding will have the same topological properties as the *real* phase space.

Topological equivalence in the Lorenz attractor

SUGIHARA et al. (2012)



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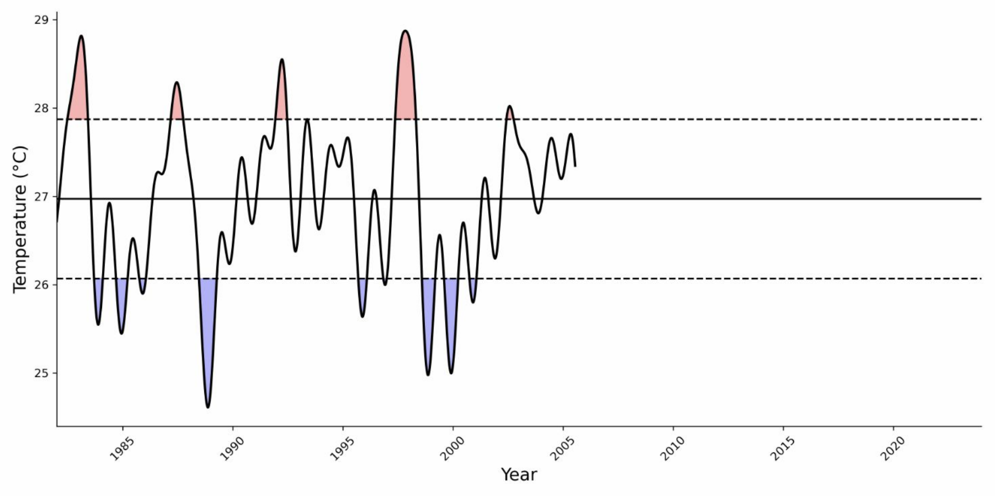

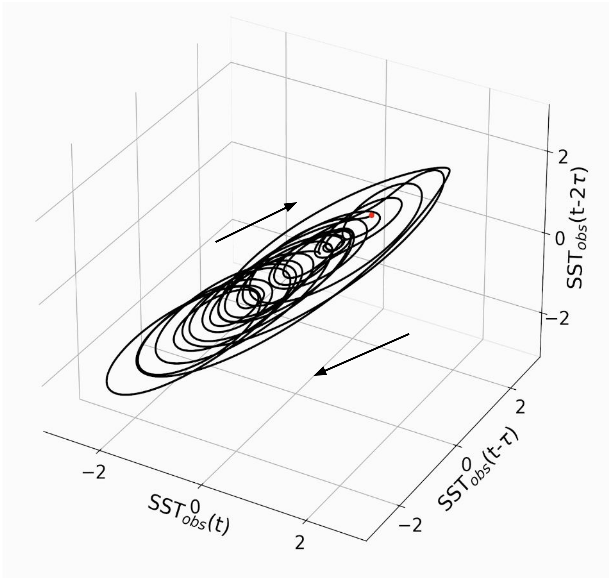

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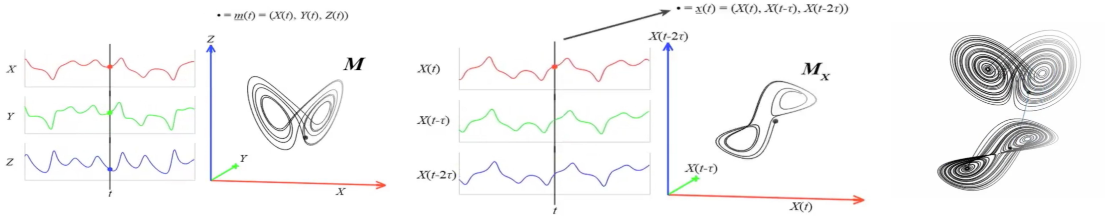

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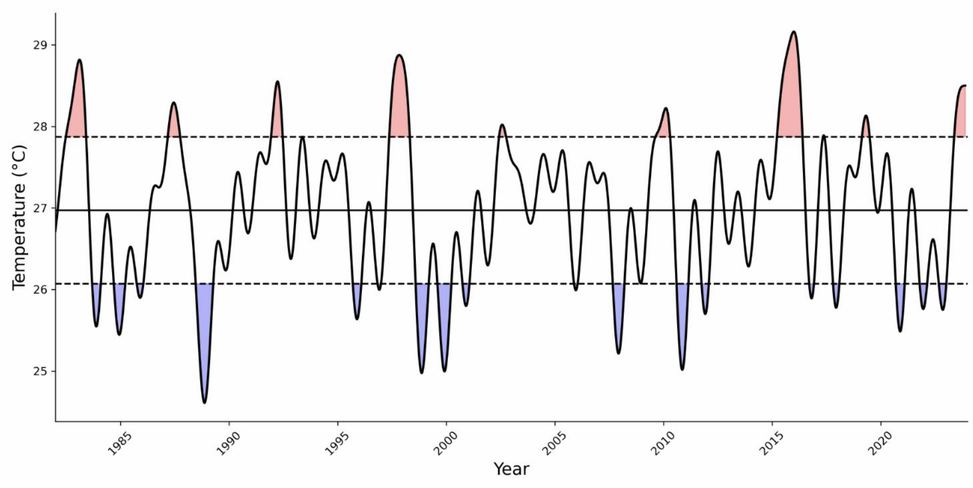

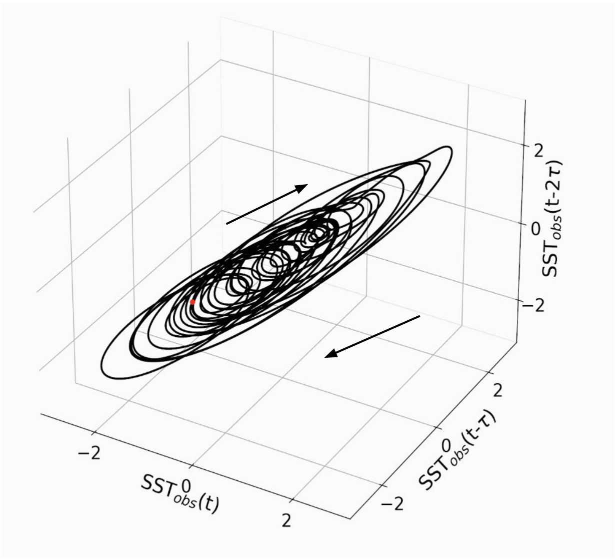

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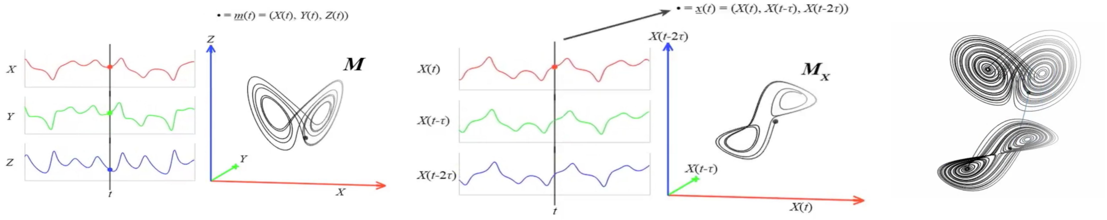
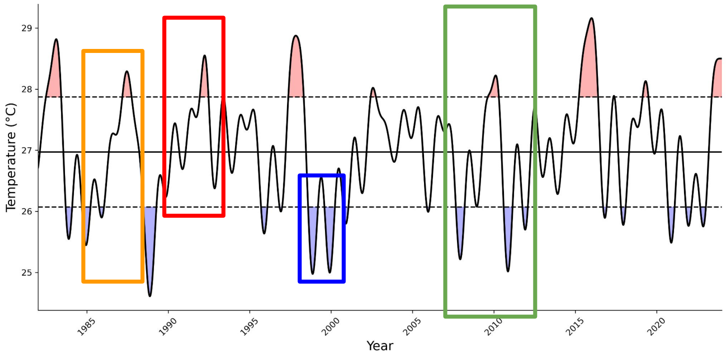
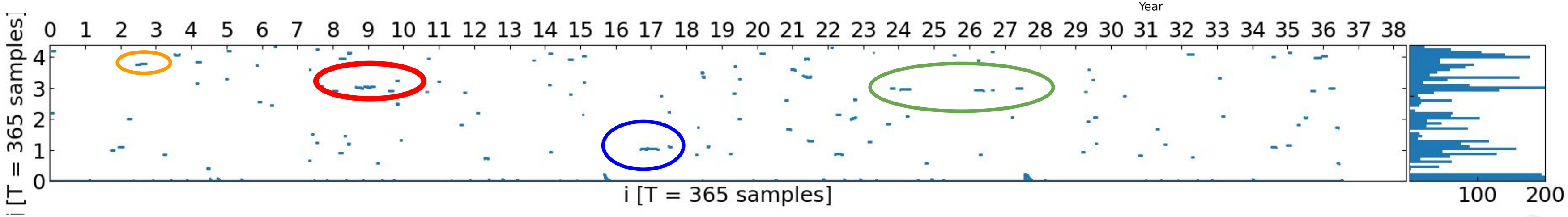
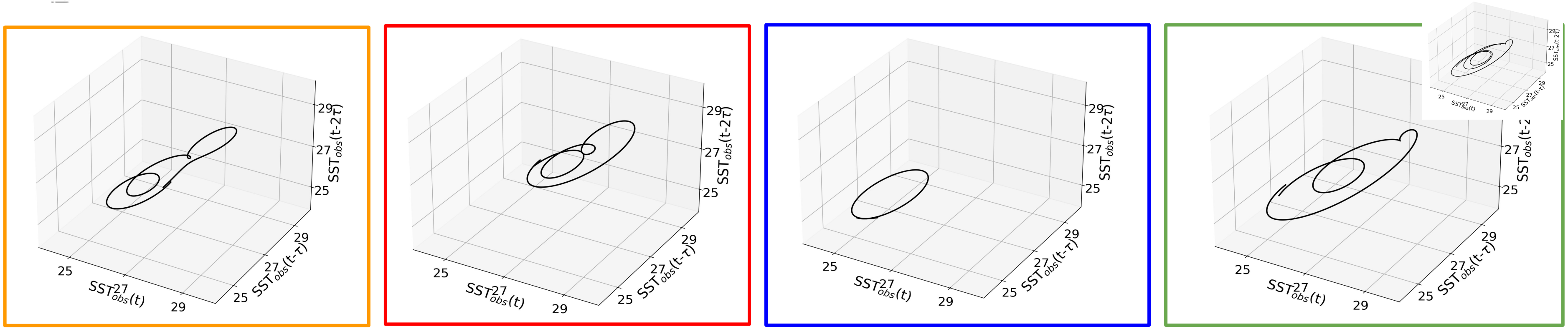


Periodic orbits are found in the phase space of the observed attractor. The periods are multiples of 365 days

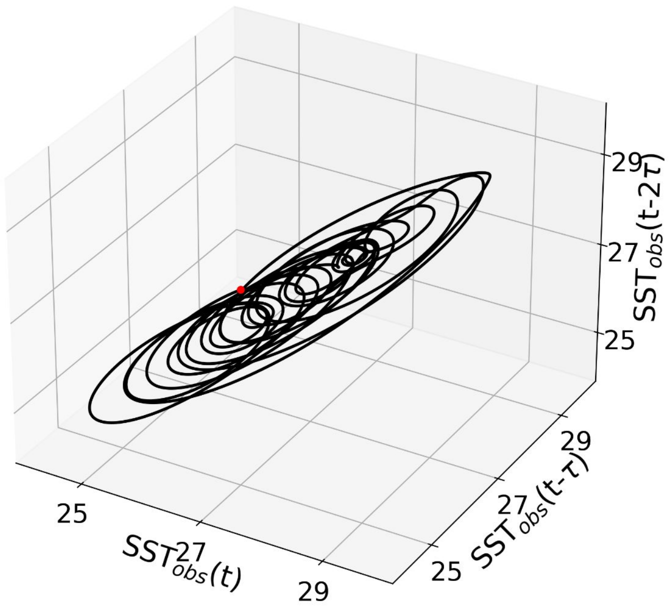
In agreement with an annual periodic forcing being an important part of the dynamics, we expect can expect subharmonics to appear.

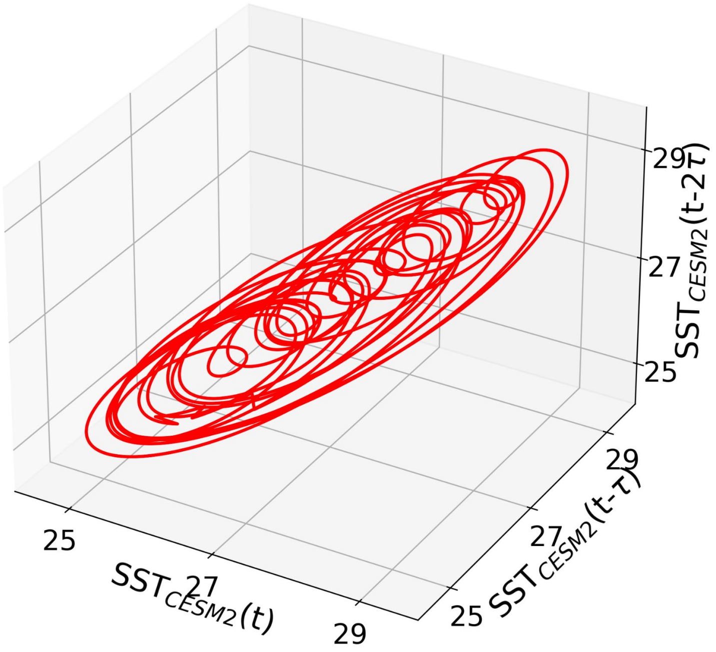






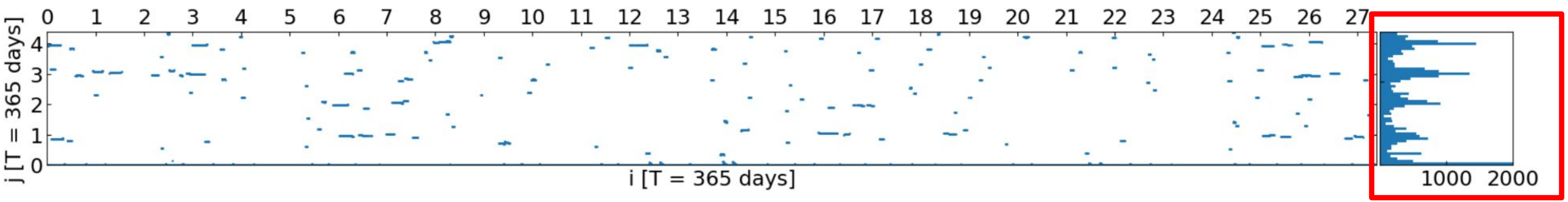
We analyze if this dynamics is reproduced by state of the art coupled models and use long and multiple simulations to have an estimate of the way these periodic orbits are visited in a fully coupled Global Climate Model.

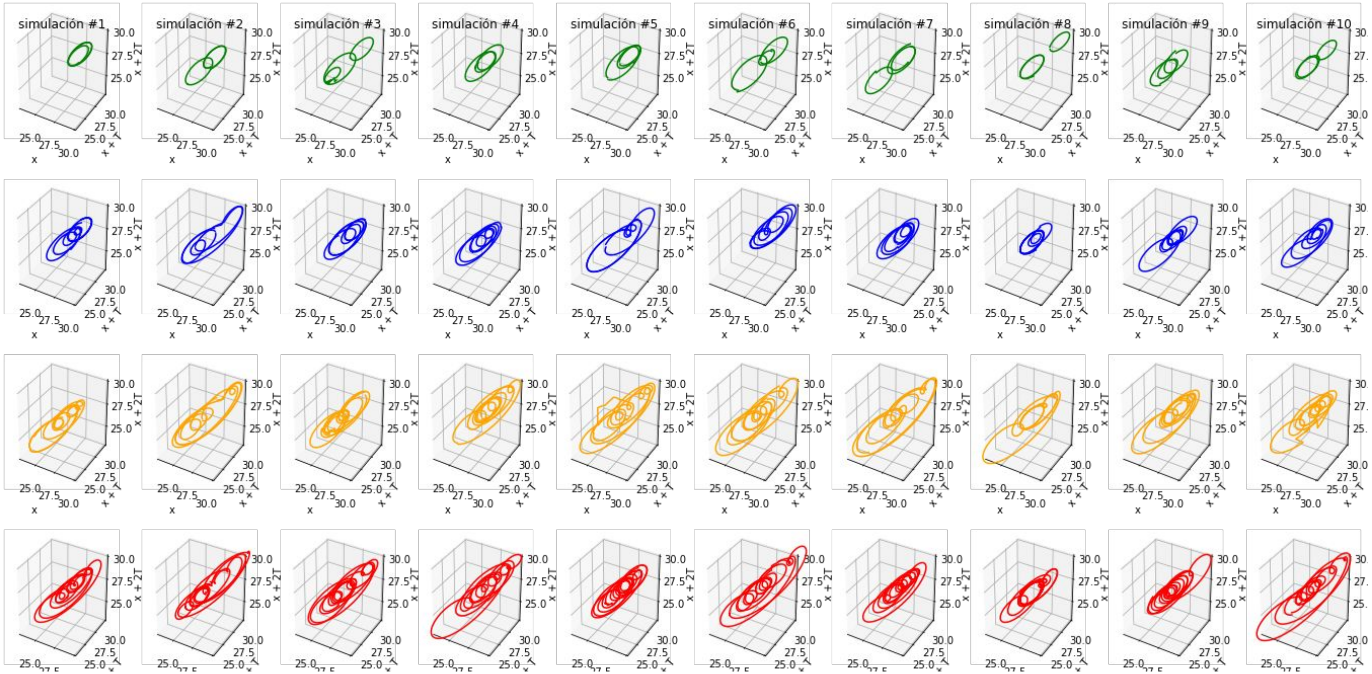




Observed system
NOAA OI SST V2 High Resolution Dataset

CESM2
historical simulations (CMIP6)





CESM2

11 simulations consistently show **periodic orbits** with periods of **nT** with **T=365** days

To sum up

From a dynamical systems perspective, a good ENSO model should have effective **dimension 3** and **unstable periodic orbits** in its attractor with periods **nT** with T = 1 year

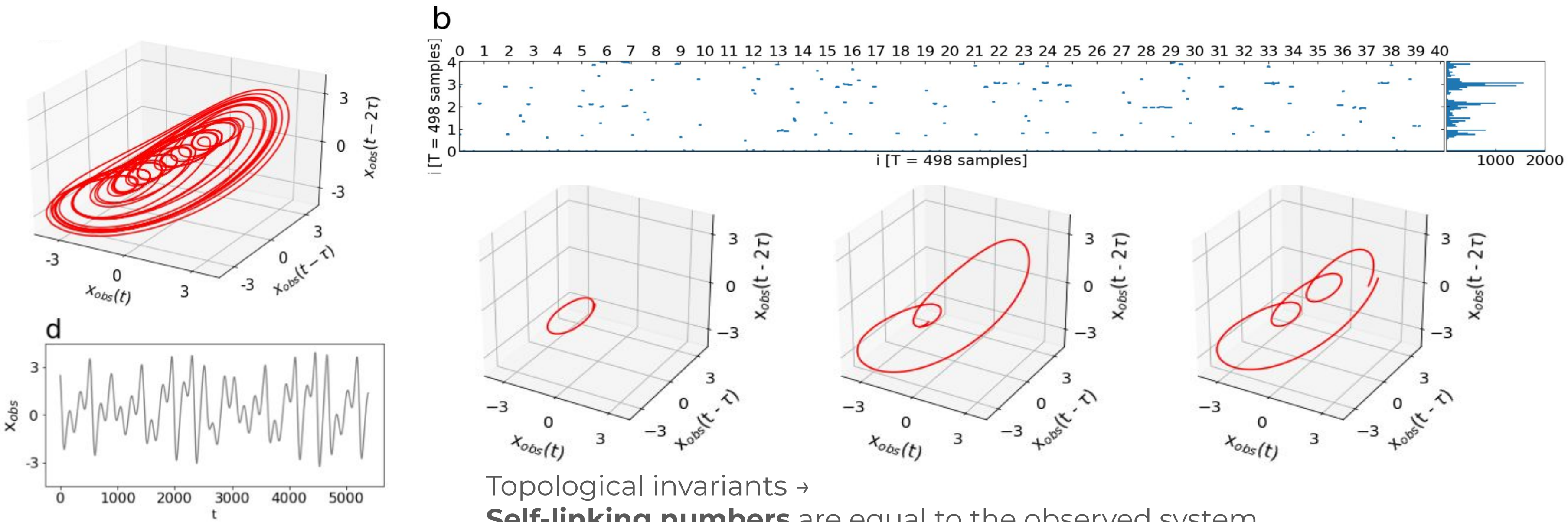
CESM2 is able to capture this dynamics

What if I told you that with this simple equation, we can recover this behaviour?

$$x' = y$$

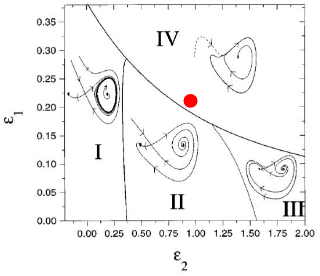
$$y' = x - y - x^3 + xy + \epsilon_1 + \epsilon_2 x^2 + Acos(\omega t)$$

Takens Bogdanov bifurcation



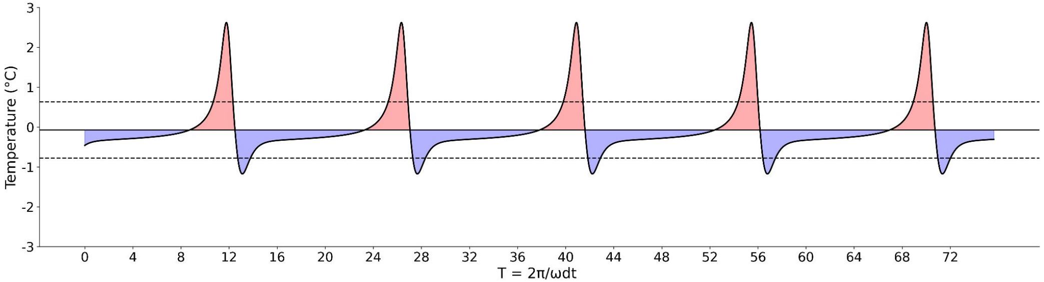
Topological invariants →
Self-linking numbers are equal to the observed system.

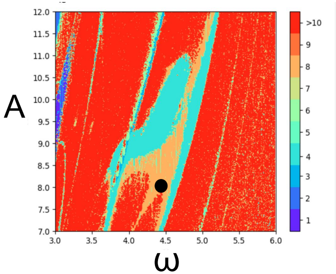
Takens Bogdanov bifurcation with periodic forcing



$$x' = y$$
$$y' = x - y - x^3 + xy + \epsilon_1 + \epsilon_2 x^2 .$$

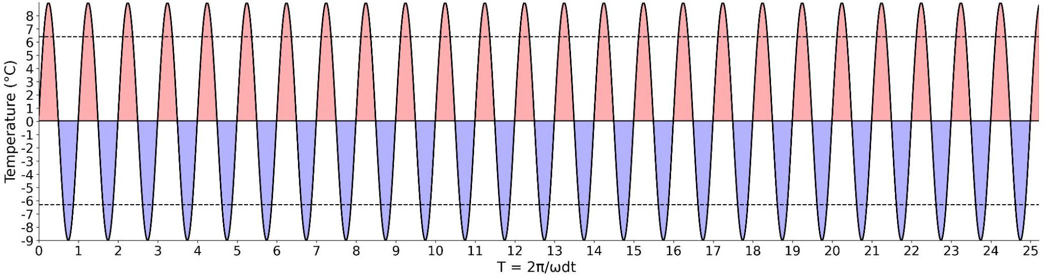
Long time in "cold phase" and rapid exploration of warm phase





$$A cos(\omega t)$$

Seasonal modulation of the Bjerknes feedback



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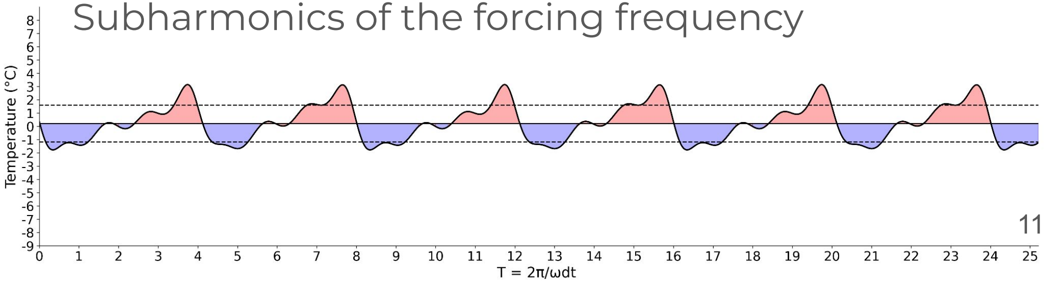
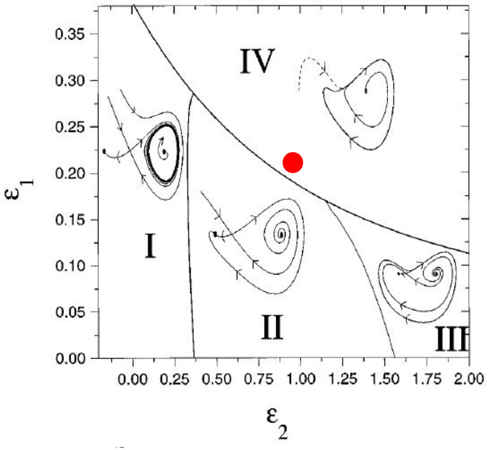
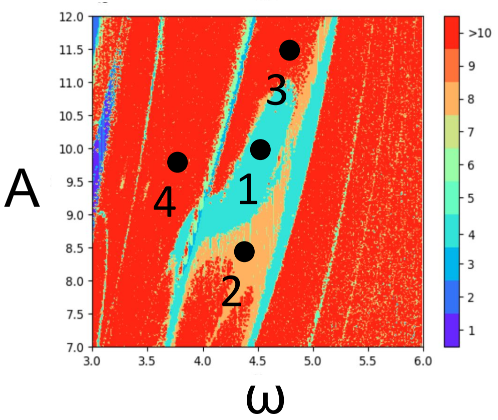
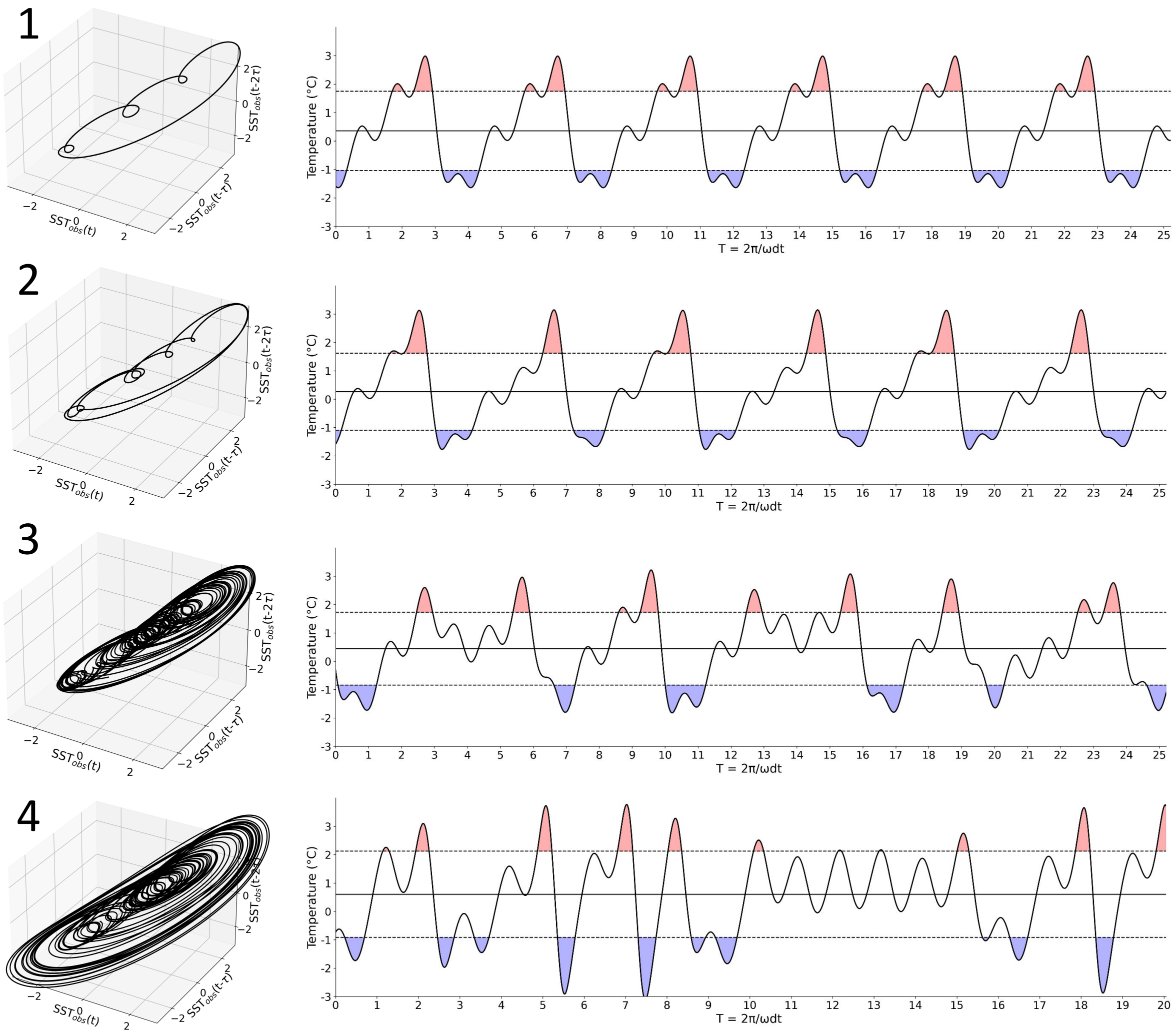
Subharmonics of the forcing frequency



Possible solutions







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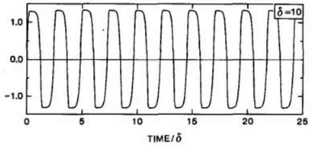

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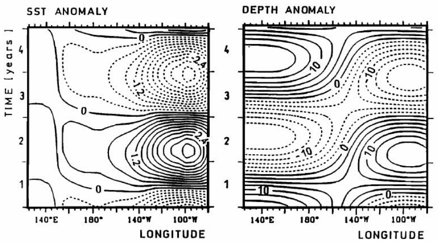

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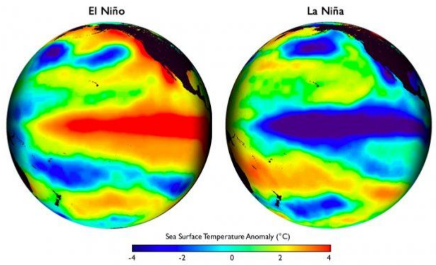
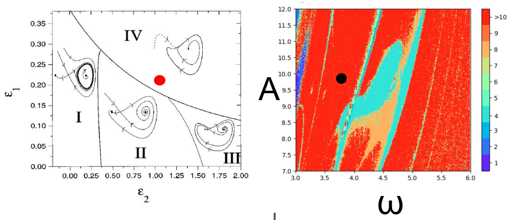


Global Climate Models



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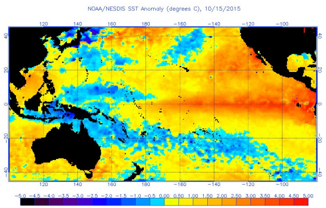
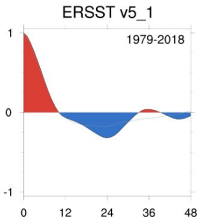


Spatial patterns?



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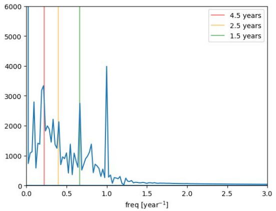
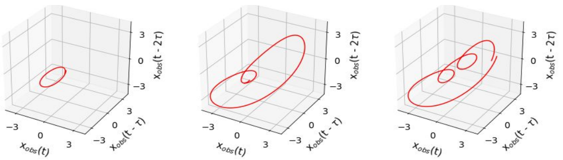


Power spectrum?



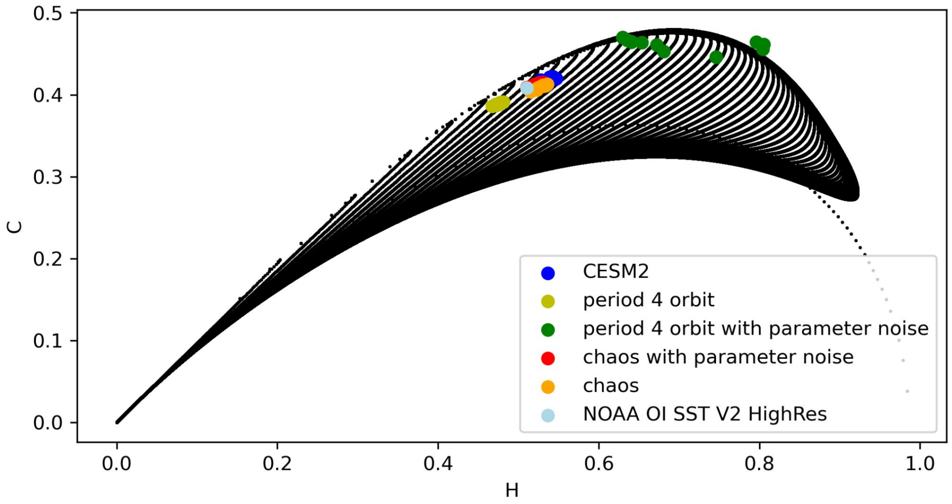
Observed topological structure

Complexity measures are a way of jointly quantifying the amount of information and the "disorder" in a complex system's behaviour

$$C = HD = -K\Sigma(p_i log(p_i))\Sigma(p_i - 1/N)^2$$

Shannon's entropy (vanishes in a crystal)

Departure from equipartition (vanishes for an ideal gas)



Is ENSO a chaotic system or a stable mode forced by noise?

We compared the complexity of the observed Nino3.4, that simulated by CEMS2 and **four integrations of our model**:

1. Chaos
2. Chaos with parameter noise
3. Stable period 4 orbit
4. Stable period 4 orbit with parameter noise

A minimalistic recipe for ENSO dynamics:

1. An oscillation with **fast and slow transitions**
   Fast: El Nino → La Nina
   Slow: La Nina → El Nino
   Consistent with La Nina events lasting multiple years

2. Periodic **annual forcing** (e.g., seasonal modulation of the Bjerknes feedback)

→ **Chaotic behaviour** arises from the interaction between the **fast and slow transitions** with the **annual forcing**, with periodic orbits of period nT (T=1year)

Implications:

- ENSO's **chaotic regime** could be sensitive to changes in climate → It it close in parameter space to a **stable periodic mode… could ENSO become more stable?**

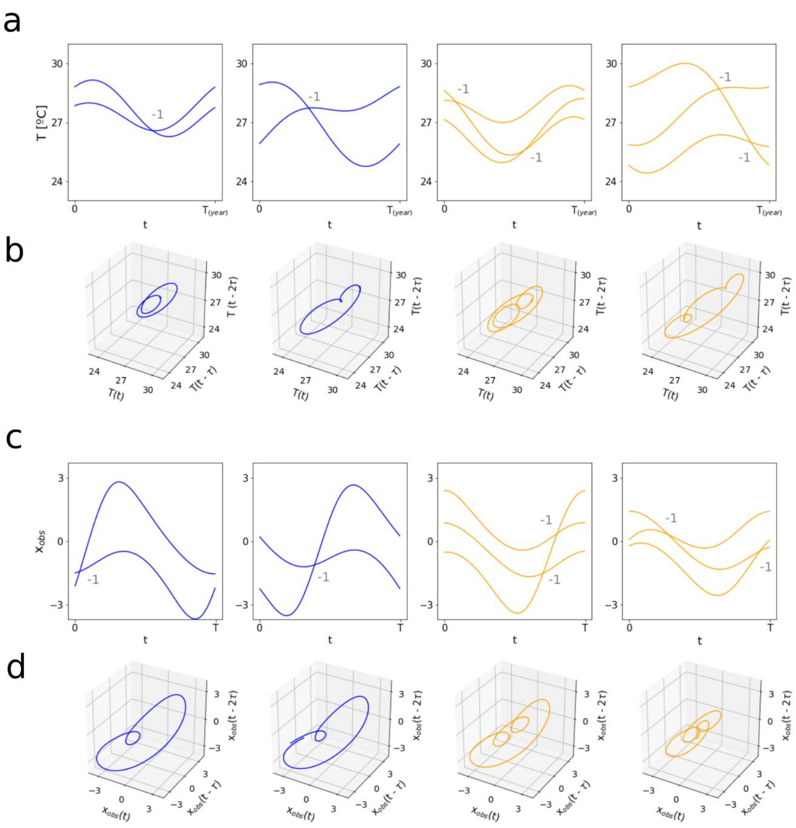
- This simple models allows us to explain different plausible ENSO behaviours (i.e., different possible natural variability states), study the predictability in different regions of the attractor

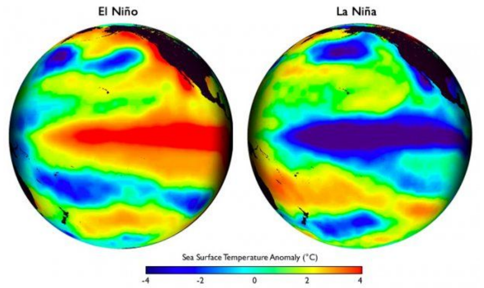
- **GCMs and machine learning models** of ENSO could be **tested** in terms of their capability to reproduce ENSO dynamics and the strange attractor structure

Thank you for your attention!

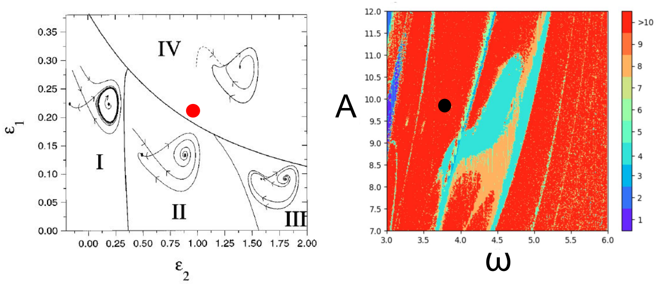
Extra slides

Self linking numbers **of periodic orbits** are topological invariants. These are quantities associated with the topological space that do not change under continuous deformations in space. Hence, any model capable of reproducing the correct dynamics should show orbits with the correct self linking number.





CESM2



Conclusions, thoughts and steps forward

A **takens-bogdanov bifurcation with a periodic forcing** can lead to **chaotic** behavior and the **complexity** of such dynamics is **comparable to that of the observed and modeled (CESM2) ENSO**.

This dynamical systems perspective can serve to evaluate interpretable dynamical models, global climate model simulations, machine learning models, etc.

Physically interpretable models in the existing literature are likely to have, in some region of their parameter space, a takens-bogdanov bifurcation. Integrating such models in a region of the parameter space where solutions are stable may lead to the conclusion that noise is needed to capture **irregular behaviour.** Some of these models are of infinite dimension (i.e., delay equation models) these results show that if realistic solutions can come from these models, they should be those where the dynamics collapses to **three dimensions**.

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Effective dimension analysis

What is the minimum dimension where this flow can live without self-crossings of the flow?