Interactions of internal gravity waves with waves and mesoscale eddies

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Interaction of IGWs with an eddy

Conclusion

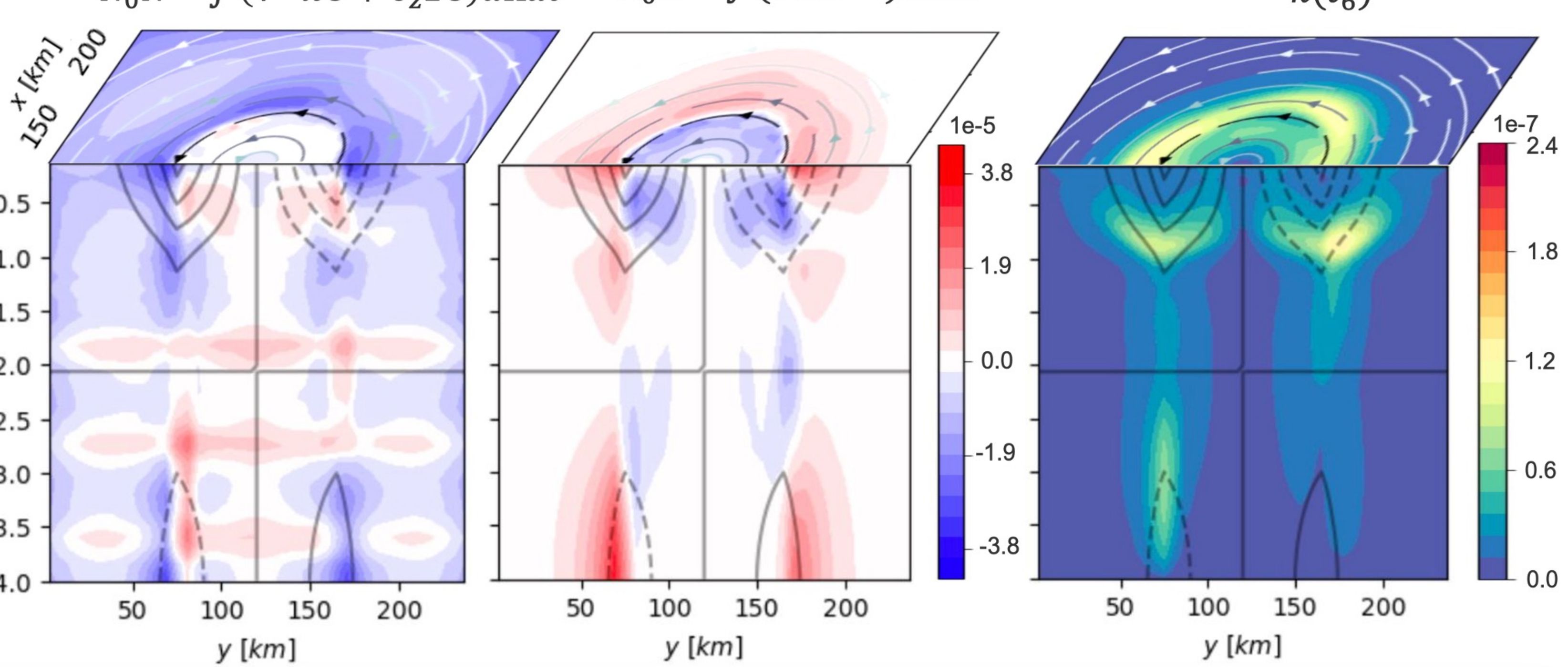
- Sebastia Saez et al. 2024 in JPO (QR-code)
- Eddy rim: vertical refraction → wave dissipation

Radiative Transfer Equation

$$\partial_t \mathcal{E} + \boldsymbol{\nabla}\cdot(\dot{\boldsymbol{x}}\mathcal{E}) + \partial_z(\dot{z}\mathcal{E}) + \boldsymbol{\nabla}_{\boldsymbol{k}_h}\cdot(\dot{\boldsymbol{k}}_h\mathcal{E}) + \partial_{k_z}(\dot{k}_z\mathcal{E}) = \omega\mathcal{S} + \frac{\mathcal{E}}{\omega}\dot{\omega}$$

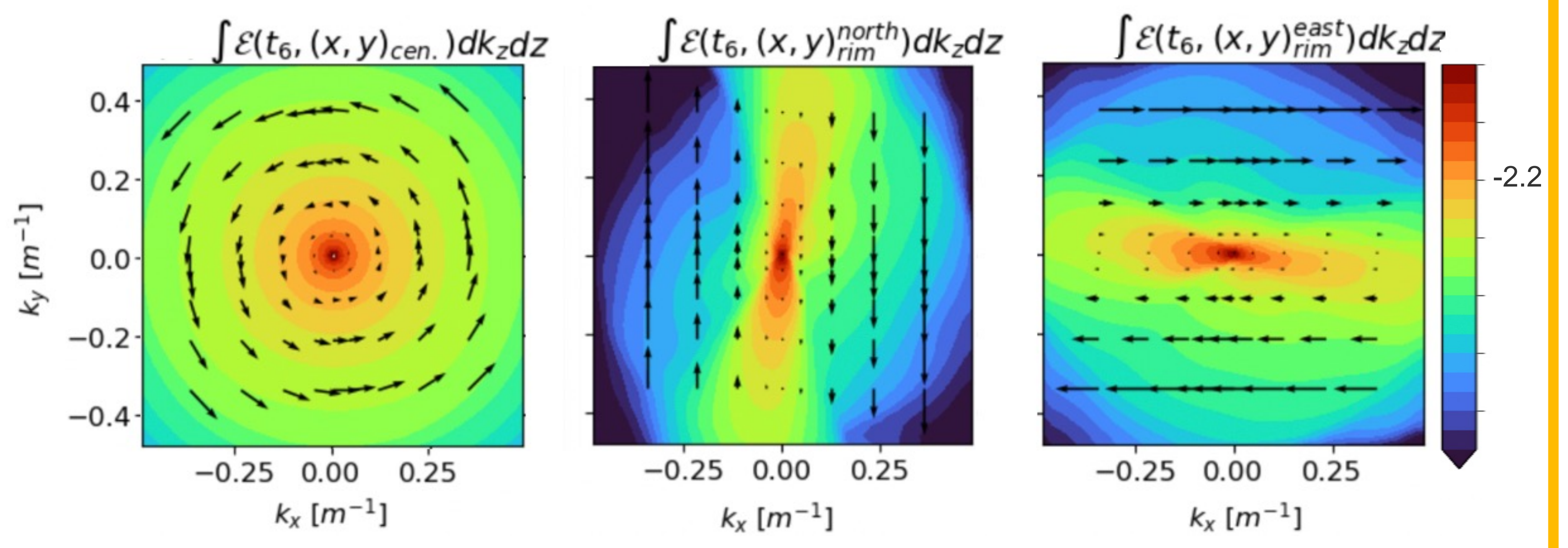
Tendency | Advection | Refraction | Wave-mean flow int. | Source/ Sink

$-N_0N^{-1}\int(\nabla\cdot\dot{x}\mathcal{E} + \partial_z\dot{z}\mathcal{E})d\mathbf{k}dt$ $\quad -N_0N^{-1}\int(\mathcal{E}\dot{\omega}\omega^{-1})d\mathbf{k}dt$ $\quad \kappa(t_6)$



- Eddy center: energy loss by wave-mean flow int.
- Eddy rim: energy gain by horizontal wave-mean flow int. after developing spectral anisotropy

Anisotropy by refraction outside the eddy: $\partial_y U > -\partial_x V > 0$. Isotropy within the eddy: $\partial_y U = -\partial_x V$. And $\partial_x U = \partial_y V = 0\text{s}^{-1}$.



Interaction of IGW with an eddy field

Conclusion

- Barotropic eddies → horizontal isotropy
- Baroclinic eddies → wave diffusion along ω-cone

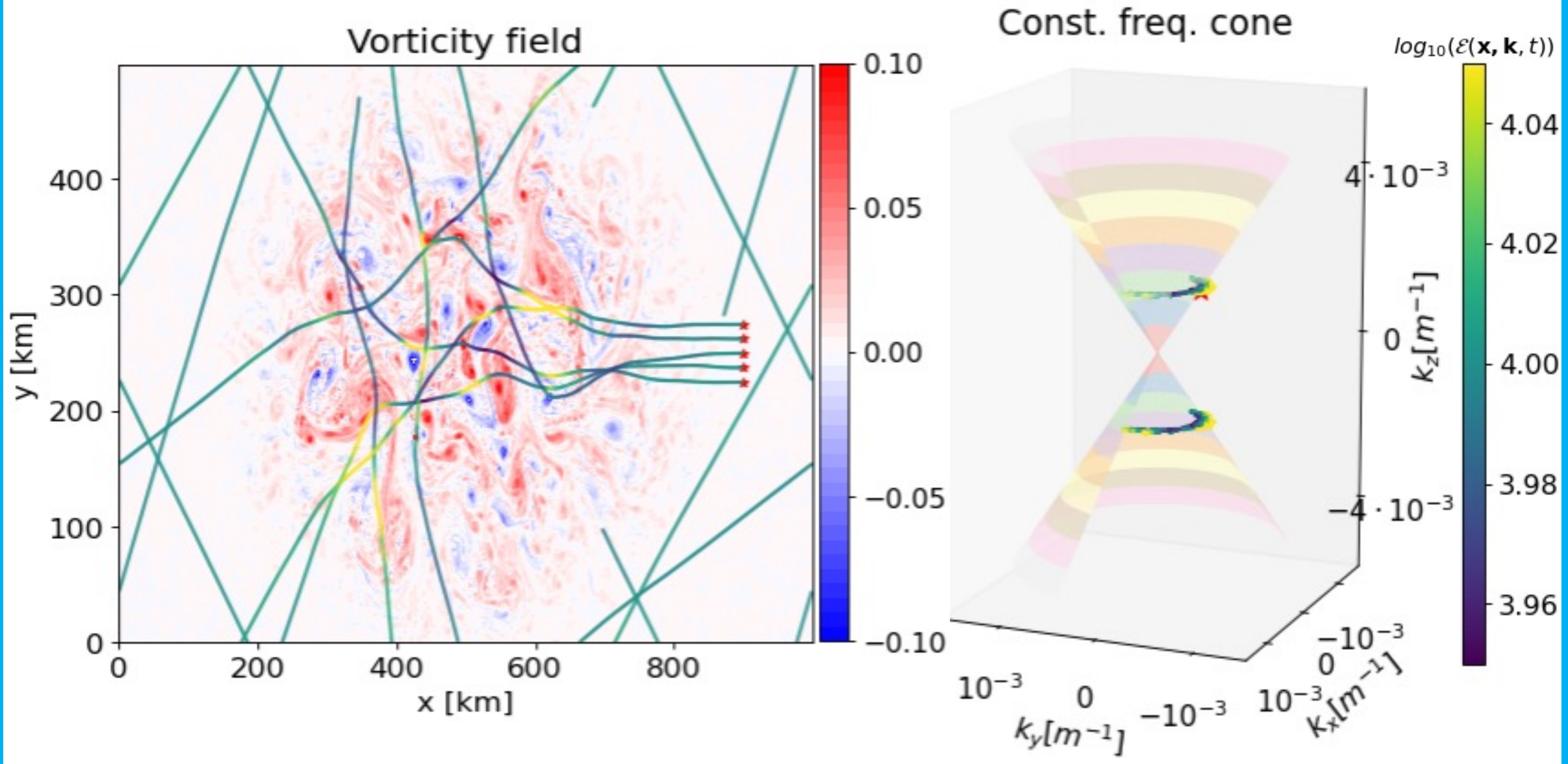
Raytracing: Wave refraction and frequency change

$$\dot{\boldsymbol{k}}_h = -\boldsymbol{k}_h\cdot\boldsymbol{\nabla}\boldsymbol{U} = -(k_x\partial_x U + k_y\partial_x V, k_x\partial_y U + k_y\partial_y V)$$

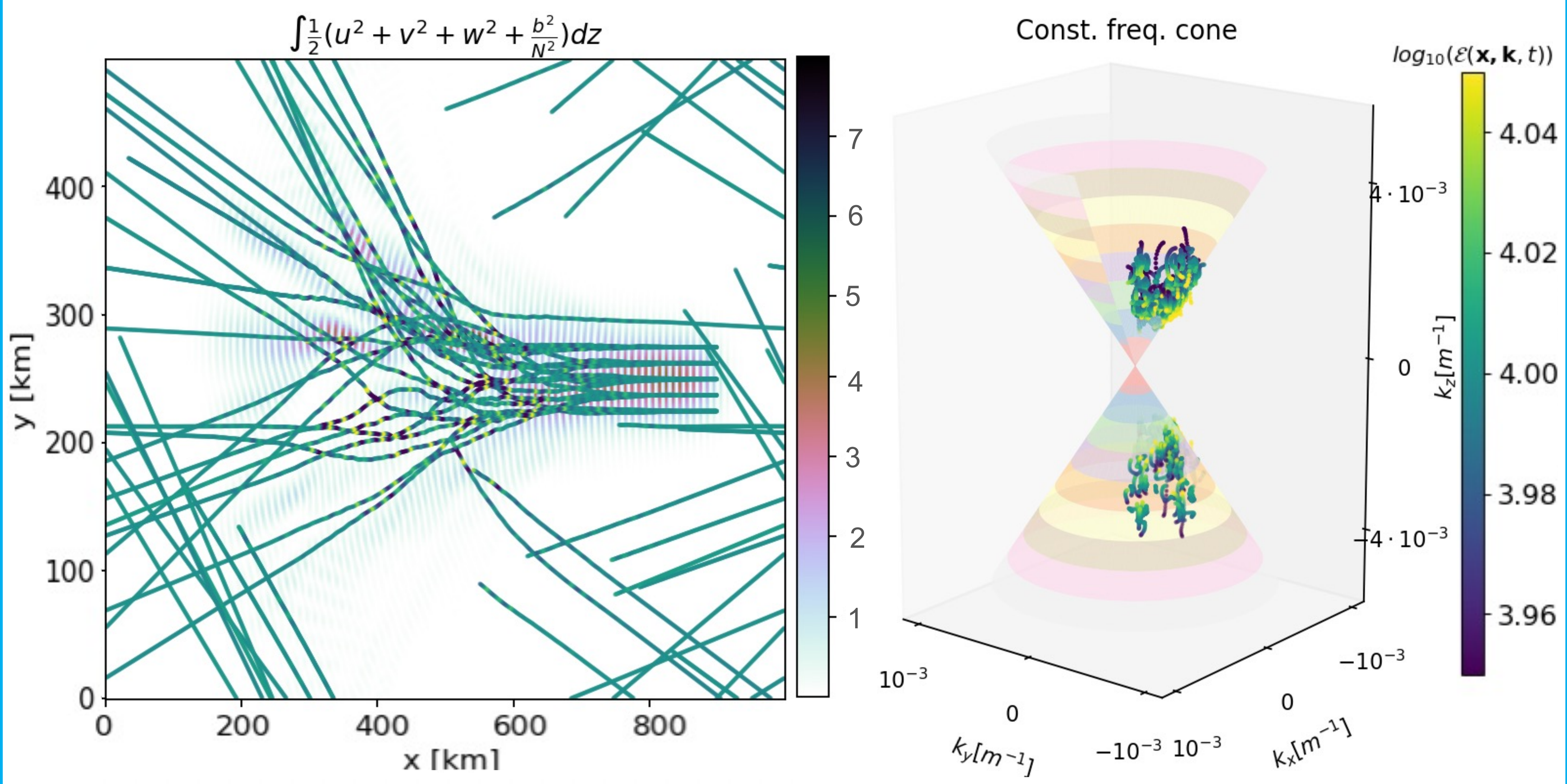
$$\dot{\omega}_h = -(\dot{\boldsymbol{x}} - \boldsymbol{U})\cdot(\boldsymbol{k}_h\cdot\boldsymbol{\nabla}\boldsymbol{U}) \sim -k_x^2\,\partial_x U - k_x k_y(\partial_y U + \partial_x V) - k_y^2\partial_y V$$

$$\dot{k}_z = -(\boldsymbol{k}_h\cdot\partial_z\boldsymbol{U}) - \partial_z\omega;\ \dot{\omega}_z = -\dot{z}(\boldsymbol{k}_h\cdot\partial_z\boldsymbol{U})$$

- Barotropic eddies refract wave rays horizontally and lead to horizontal isotropy in the spectrum:



- Baroclinic eddies refract wave rays horizontally and vertically and lead to diffusion along the constant ω-cone:



Wave-wave interactions

On-going

- Wave scattering as in Eden et al. 2019 (a,b,c)
- Include all (IG, intertial & buoyancy) wave solutions

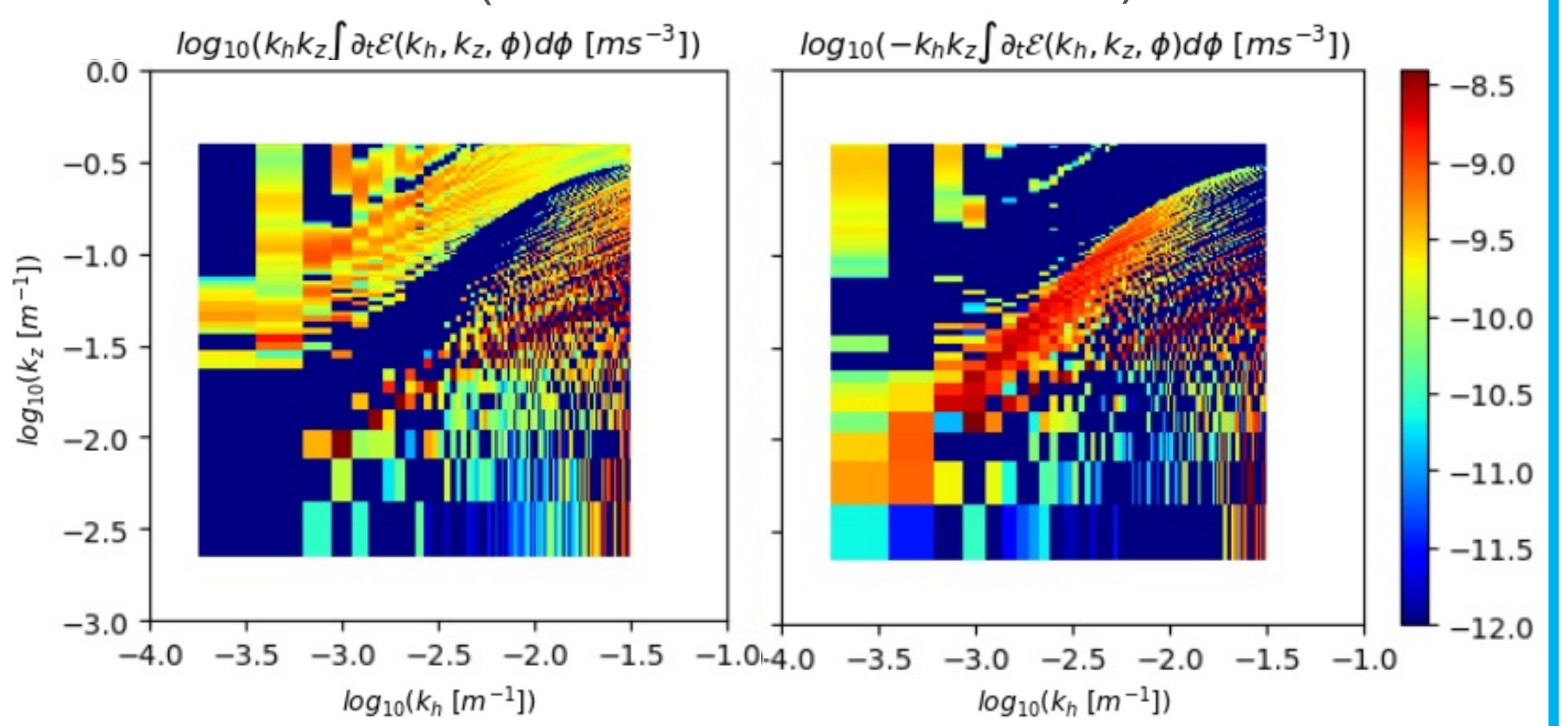
Scattering Integral

$$\partial_t E_0^{s_0} = \frac{4}{n_0^{s_0}}\int d\boldsymbol{k}_1\sum_{s_1,s_2} C^{s_0,s_1,s_2}_{\boldsymbol{k}_0,\boldsymbol{k}_1,\boldsymbol{k}_2}\left[n_1^{s_1}n_2^{s_2}E_1^{s_1}E_2^{s_2}\left(C^{s_0,s_1,s_2}_{\boldsymbol{k}_0,\boldsymbol{k}_1,\boldsymbol{k}_2}\right)^* - n_0^{s_0}n_1^{s_1}E_0^{s_0}E_1^{s_1}C^{s_2,-s_1,s_0}_{\boldsymbol{k}_2,-\boldsymbol{k}_1,\boldsymbol{k}_0} - n_0^{s_0}n_2^{s_2}E_0^{s_0}E_2^{s_2}C^{s_1,-s_2,s_0}_{\boldsymbol{k}_1,-\boldsymbol{k}_2,\boldsymbol{k}_0}\right]\times\Delta(\omega'_{0,1,2}) + c.c.$$

For finite time: $\Delta(\omega,t)|_{t=10\text{days}} = e^{i\omega t}\int_0^t dt' e^{-i\omega t'} = i\,\frac{1-e^{t\omega t}}{\omega}$

What is the role of inertial and buoyancy oscil.?

- All interactions (resonant & non-resonant):



- Near-resonant interactions with $\Delta(\omega'_{0,1,2} < 10^{-5})$: